COLLINS
QUADRANTS

183941B

THE SECTOR ON A QUADRANT,

OR

A Treatise containing the Description and Use of four several QUADRANTS; Two small ones and two great ones, each rendred many wayes, both general and particular.

Each of them Accomodated for Dyalling; for the Resolving of all Proportions Instrumentally; And for the ready finding the Hour and Azimuth Universally in the equal Limbe.

Of great use to Seamen and Practitioners in the MATHEMATICKS.

Written by JOHN COLLINS *Accountant Philomath.*

Also an Appendix touching Reflected Dyalling from a glass placed at any Reclination.

London, Printed by *J.M.* for *George Hurlock* at *Magnus* Corner, *Thomas Pierrepont*, at the Sun in *Pauls* Church-yard; *William Fisher*, at the Postern near Tower-Hill, Book-sellers; And *Henry Sutton*, Mathematical Instrument-Maker, at his House in *Thred-needle* street, behind the *Exchange*. With Paper Prints of each Quadrant, either loose or pasted upon boards; to be sold at the respective places aforesaid. 1659.

To the Reader.

Courteous Reader,

THou haſt in this Treatiſe, the Deſcription and Uſes of three ſeveral Quadrants, preſented to thy View and Acceptance; and here I am to give thee an account of their Occaſion and Original.

Being in conference with my loving friend M. *Thomas Harvie*, he told me, that he had often drawn a Quadrant upon Paper paſtboard, *&c.* derived by himſelf, and never done by any man before, as to his knowledge, from the Stereographick Projection, which for a particular Latitude, would give the Hour in the equal Limb, and would alſo perform the Azimuth very well; and but that it was ſo particular, was very deſirous to have one made in Braſs for his own uſe by an Inſtrument Maker: whereto replying, that with the acceſs of ſome other Lines to be uſed with Compaſſes, it might be rendred general for finding both the Hour and the Azimuth in the equal Limb: He thereupon intimated his deſires to M. *Sutton*, promiſing within a fortnight after their conference, to draw up full directions for the making thereof. But M. *Sutton* having very good practiſe and experience in drawing Projections, ſpeedily found out the drawing of that Projection, either in a Quadrant or a Semicircle, without the aſſiſtance of the promiſed directions, and accordingly, hath drawn the ſhape of it

for all Latitudes, and alſo found how the Horizontal Projection might be inverted and contrived into a Quadrant without any confuſion, by reaſon of a reverted tail, and let me further add, that he hath taken much pains in calculating Tables for the accurate making of theſe and other Inſtruments, in their conſtruction more difficult then any that ever were before; and the ſaid M. *Sutton* conceiving that it would be an advancement to their Trade in general, beſides ſatisfactory to the deſires of the ſtudious in the Mathematiques, to have the uſes of a good Quadrant publiſhed, prevailed with me, in regard M. *Harvey* was not at leiſure (though willing his Quadrant ſhould be made publique) to write two or three ſheets of the uſe of it, which I intended to have given M. *Sutton* (who very well underſtood the uſe as well as the making) to be publiſhed in his own name; whereto he being unwilling, and finding that therein many of the uſes of one Quadrant, much leſs of more could not be comprized, at his earneſt requeſt, I wrote what is here digeſted, *ſucciſivè & horis Antelucanis*, having little leiſure for that purpoſe, and all this performed before the Inſtruments were cut, wherefore the deſcription given of them, may not ſo nearly agree with the Inſtruments, as if they had been firſt made, nor poſſibly ſome of the examples about finding the hour of the night by the Stars; which examples were fitted from Tables of preſent right Aſcenſion, whereas the Quadrant is fitted to ſerve the better for the future, the difference notwithſtanding will be but ſmall. And thus hoping thou wilt cover my failings with the mantle of love, and kindly accept of my endeavours, tending to the publique advancement and increaſe of knowledge, I ſtill remain a Wellwiller deſirous thereof.

John Collins.

How the Projections on both the Quadrants may be Demonſtrated.

To ſatisfie the inquiſitive Reader herein, I ſhall only in this Edition quote ſuch Latine Authors and Propoſitions as will evince the truth thereof, the performance whereof in Engliſh, is hereafter intended by my loving friend M. Thomas Harvie, *in an elaborate Treatiſe, concerning all the Projections, with their Demonſtration and Application, who is accompliſhed with ſingular knowledge in that kind, as in general in the Mathematiques.*

Now the Demonſtration of theſe two Projections is as much included in the Demonſtration of the Stereographick Projection, which by Aguilonius *in his 6 Book is largely inſiſted upon, as a peculiar queſtion in Trigonometry, is included in a general Caſe, and both the Projections on theſe Quadrants being derived from the grounds of the ſaid general Projection, are neceſſarily involved in one and the ſame Demonſtration.*

Stofler *in his Aſtrolabe ſuppoſeth the eye in the South Pole Stereographically projecting upon the Plain of the Equator thoſe Circles between the North Pole Horizon and Tropick of* Capricorn, *neglecting that part under the Horizon.*

But the Projection on the Quadrant, conſidered as it may be derived from his Aſtrolabe, ſuppoſeth the eye in the ſame Poſition, and makes uſe of one half of the Projection of the other part of the Circles intercepted between the Horizon and Tropick of Capricorn, *namely, of that ſpace between the Tropick below our Horizon, only changing the names of* Cancer *for* Capricorn *in their uſe, and uſing the depreſſed Pa-*

 rallels

rallels to the Horizon, instead of the Parallels of Altitude; so that the Azimuths of the Quadrant made by this inversion, are no other then the Azimuths of Stoflers *Projection continued below the* Tropick *of* Capricorn *where he breaks them off, and the rule he prescribes to draw the parallel of* 18^d *of* Depression *for the* Twylight *serves to draw that, and all the other parallels of* Altitude *in this Quadrant.*

In like manner the Horizontal Projection supposeth the eye in the Nadir *projecting upon the Plain of the Horizon.*

That part of the Sphere intercepted between the two Tropicks, neglecting that part thereof under the Horizon.

But the Projection on the other great Quadrant, considered as derived therefrom, supposeth the eye there projecting that part of the Sphere which is there neglected with the like change of denomination; and the Parallels of Declination are no other then the continuance of the said Parallels of the Horizontal Projection round to the Midnight Meridian, and the Hour circles the continuance of the said hours, only the Index of Altitudes is fitted to the Depressed Parallels of the Horizon, in stead of the Parallels of Altitude.

Now it is evident, either from the Sphere or Analemma, that that part of either of these Projections which falls under the Horizon, will supply the use of that which hapned above, admitting only a change of denomination; for in the Horizontal Projection, that Parallel of Declination which was called the Winter Tropick, being no other then the same Circle continued about, now in its use and denomination, becomes the Summer Tropick; and the reason is, because what ever Altitude the Sun hath in any Sign upon any Hour or Azimuth reckoned from the Noon Meridian, he hath the like Depression on the like Hour and Azimuth in the opposite Signe counted from the Midnight meridian.

The

The terms of Noon and Midnight Meridian are afterwards used in relation to some general Proportions: By the Hour in general is meant the Angle between the Meridian of the Sun or Stars, and the Meridian of the place: By the Hour counted from Noon Meridian, is meant the said Angle counted from that part of the Meridian of the place which falls above the Elevated Pole, continued towards the Depressed Pole: and by the Midnight Meridian, the opposite thereto under the Elevated Pole, continued as before.

By the Azimuth counted from the Midnight Meridian, is meant an Angle at the Zenith between the Suns Vertical or Azimuthal circle, and the Meridian of the place, measured by the Horizon, counted from the Intersection of the Horizon with the Meridian under the Elevated Pole; and by the Azimuth counted from the Noon Meridian, is meant the Complement of the said Angle to a Semicircle, counted from the opposite Intersection of the Horizon with the former Meridian continued above the Elevated, and towards the Depressed Pole, according to which acceptions, the general Proportions are fitted for finding it either way in both Hemispheres, without any restriction to North or South.

A more immediate account of these Projections.

HItherto we have accommodated our Discourse, to shew how these Projections are derived from *Stoflers* Astrolabe, and from the Horizontal Projection, of which neither *Stofler* (as to my knowledge) for I have only seen his 8 Book) nor the learned M. *Oughtred*, give no peculiar Demonstration, as being particular examples of a general case, largely (as such) insisted on; and this we have done for the accommodation of Instrument makers, to whom this Derivation may seem most suitable; whereas such a deduction is not at all necessary to the Demonstration of the Projections so derived.

For in the Projection derived from the Inversion of *Stofler*, let the eye be supposed to be placed in the North Pole, projecting upon the Plain of the Equinoctial, such Circles in the Sphere, as are described in the Quadrant between the two Tropicks, a quarter of which Projection will be the same with that on the Quadrant, namely, one of those quarters between the South part of the Meridian and hour of six, which will leave out all the outward part of the Almicanters between it and the Tropick of *Cancer*, and in stead thereof, there is taken in such a like part of the depressed Parallels to the Horizon between the same Hour of six, and Tropick of *Capricorn*, which is the reverted tail; for the Parallels of Depression have the same respect to the Tropick of *Capricorn*, that the Parallels of Altitude have to the Tropick of *Cancer*, and will work the same in effect.

In like manner, the Eye in the other Projection may be

be ſuppoſed in the Zenith, Stereographically Projecting upon the Plain of the Horizon, that part only of the ſpace between the Tropicks, which falls without the Projection of the Horizontal Circle, ſave only the reverted tail, which is the Projection of ſo much of the Parallels of South Declination, as is intercepted between the prime Vertical Circle and the Horizon, and is taken in to ſerve in ſtead of that part of the Parallels of North Declination, which will fall without the Quadrant.

In any of theſe Poſitions of the Eye, all Circles paſſing through the ſame, will be projected in right lines by 91 Prop. 6 Book of *Aguilonius*, ſuch are the Azimuths on the Horizontal Quadrant, and the Hours on the other Quadrant, repreſented by the thred lying over any Ark in the Limb, ſo alſo in this latter Quadrant is the Parallel of Altitude equal to the Latitude of the place, a right line.

All Circles parallel to the Horizon and Equinoctial, will be projected in concentrick Circles by 94 Prop. 6. *Aguilonius*, ſuch are the Parallels of Altitude in the Horizontal Quadrant, and the Parallels of Declination in the other Quadrants, repreſented by the Bead, when it is rectified to the Index of Altitudes in the one, and to the Ecliptick in the other, carried in a circular trace from one ſide of the Quadrant to the other.

All other Circles in the Sphere, whatſoever and howſoever ſcituated, being projected according to the ſuppoſed poſition of the Plain and Eye, will be repreſented by Excentrick Circles by 96 Prop. 6. *Aguilonius*, and the hours in the Horizontal Projection will (if they be produced) meet with the projected Pole points, ſo alſo the Azimuths in the other Projection, which by the like parity

rity of Reaſon may be denominated *The Equinoctial Projection*, will (being produced) meet with the projected points of the Zenith and Nadir; and how in particular to project and divide any Circle however ſcituated in the Sphere, is abundantly ſhewn in the 6th Book of the aforeſaid Author, and amplified with many examples, though none of them agreeing with the particular Draughts of theſe Quadrants, yet if put in practiſe according to the propoſed Scituation of the Eye, will be found to agree with the preſcribed Directions for the making of theſe Quadrants. See alſo *Clavius* his Book of the Aſtrolabe, *Guido Ubaldus* his Theorick of the general Planiſpheres, and M. *Oughtreds* 2d Scheme B in his late Trigonometry in Engliſh.

An Appendix to the Deſcription of the Small Quadrant.

Since the Printing of the ſheet B, *we have thought fit to vary a little from the Deſcription there given of the Small Quadrant.*

The Dyalling Scale of Hours deſcribed in page 9, near the beginning, which I ſay in Page 191 may be omitted, is accordingly left out, and inſtead of it, a line of Verſed Sines of 90d put on, the uſes whereof are handled in the great Equinoctial Quadrant.

Alſo there is two Scales added to the ſmall Quadrant more then was deſcribed; namely, the Scale of Entrance, the ſame that was placed upon the Horizontal Quadrant, with a Sine of 51d 32′ put through the whole Limb ſerving to give the

Altitude

Altitude at ſix, which the thred will interſect, if it be laid over the Declination in the Limb; but enough of the uſes of theſe Scales is ſaid in the Horizontal Quadrant.

Laſtly, Thoſe that like it beſt, inſtead of having on the Small Quadrant one looſe fitted Scale for the Hour, and another for the Azimuth, may have the Hour-Scale only divided into two parts, ſerving to give the Hour and Azimuth for the Sun, and all the Stars in the Hemiſphere, the one part for South Declinations, the other for North Declinations, in imitation of the Diagonal Scale.

An Advertiſement.

ALL manner of Mathematical Inſtruments, either for Sea or Land, are exactly made in Wood or Braſs, by *Henry Sutton*, in *Thredneedle*-ſtreet, near *Chriſtophers* Church, or by *William Sutton* in Upper *Shadwel*, a little beyond the Church.

Pag.	Li.	Errors	to be thus Corrected.
8	33	apply	supply
19	25	76d 54′	79d 54′
20	7	48d	45d
23	3	50d 41′	50d 48′
23	11	58′	53′
24	25	a Letter Character.	the Character plus +
	28	is lesse	if lesse
55	14	difference of one of	differences of
55	17	of the Leggs from the Leggs	of one of the Leggs

The affections in page 57 *line* 16, 17. *wanting Braces, are expressed at large page* 140, 141. *also the last affection in that page having a mistake of* lesser *for greater in the middle brace is reprinted in page* 138

60. 14 *The last term of the* 4th *Proportion should be the Sine of the angle sought, and not the Cosecant.*

Pag.	Li.	Errors	to be thus Corrected.
92	29	a Leg and its adjacent angle	*The Hipotenusal and its adjacent angle*
	30	to find the other angle by 4 Case.	*to find the side opposite there to by* 8 *Case* See page 138.
103	34	Acquimultiplex	*Æquimultiplex*
	35	therefore by	*therefore by* 18 Prop. 7. Euclid.
121	18	Hour $3\frac{1}{4}$ Altitude 41d 31′	*Altitude for the hour* $3\frac{1}{4}$ *is* 43d 31′
158	1	Line of	*Line of Sines*
159	21	of 90d	*at* 90d
164	18	any	*some*
174	15	in the Limb	*in the lesser Sines*
181	25	between	*as also between*
	27	would find it	*would in the other Hemisphere find it*
184	21	the common	*as the common*
189	11	either	*it may be found either*
192	24	As the second including side	*As the Sine of the* 2d *including side*
207	1	great Scale	*great Quadrant*
209	7	60 parts	60 *equal parts*

The Angle C *in the Scheam* page 52 *is wrong cut, and should be* 113d 22′, See it in page 156.

Page 98 *a wrong Scheam printed, the true one is in page* 93.

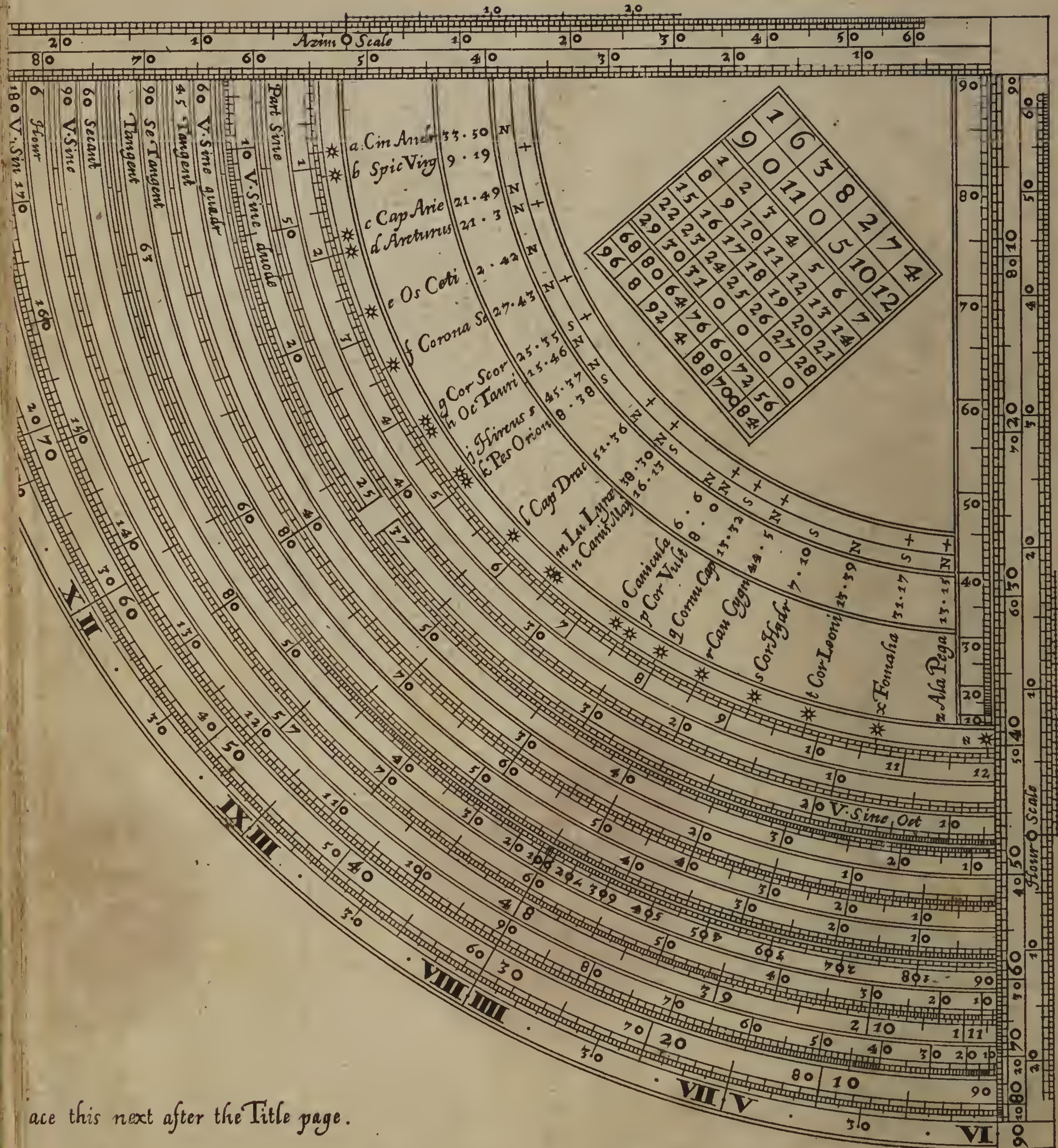
Page 102 *in the under Triangle the Angle* D *should be* 108d 37′ See page 201.

The first ten lines of calculation p. 53 are somwhat misplac'd, & should stand thus.

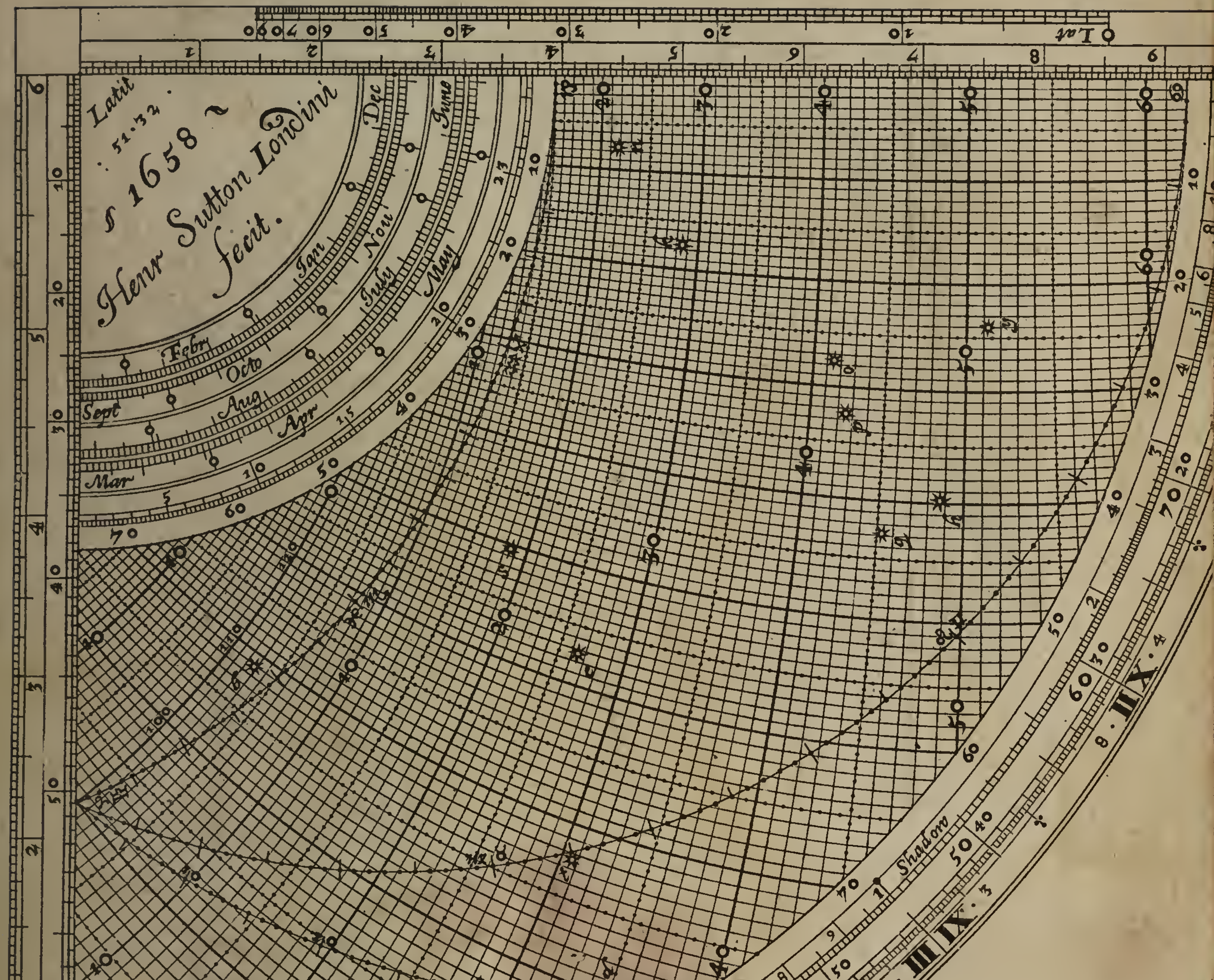
			Diffs	Logms		
B C	126	Legs	169		2,2278867	
A C	194		101		2,0043214	Sum with double Radius
A B	270	Base	25	1,3979400	24,2322081	
Sum	590					
half sum	295	— Logarith.		2 4698220	3 8677620	former Rectangle
				Residue	20,3644461	

The half is the tangent of 56d, 41′ t. ——— 10,1822230

Which Ark doubled is ——— 113,22 A C B the Angle sought.



ace this next after the Title page.



Latit 51·32
1658
Henr Sutton Londini fecit.
Dec
June
Nov
July
May
Jan
Febr
Octo
Sept
Aug
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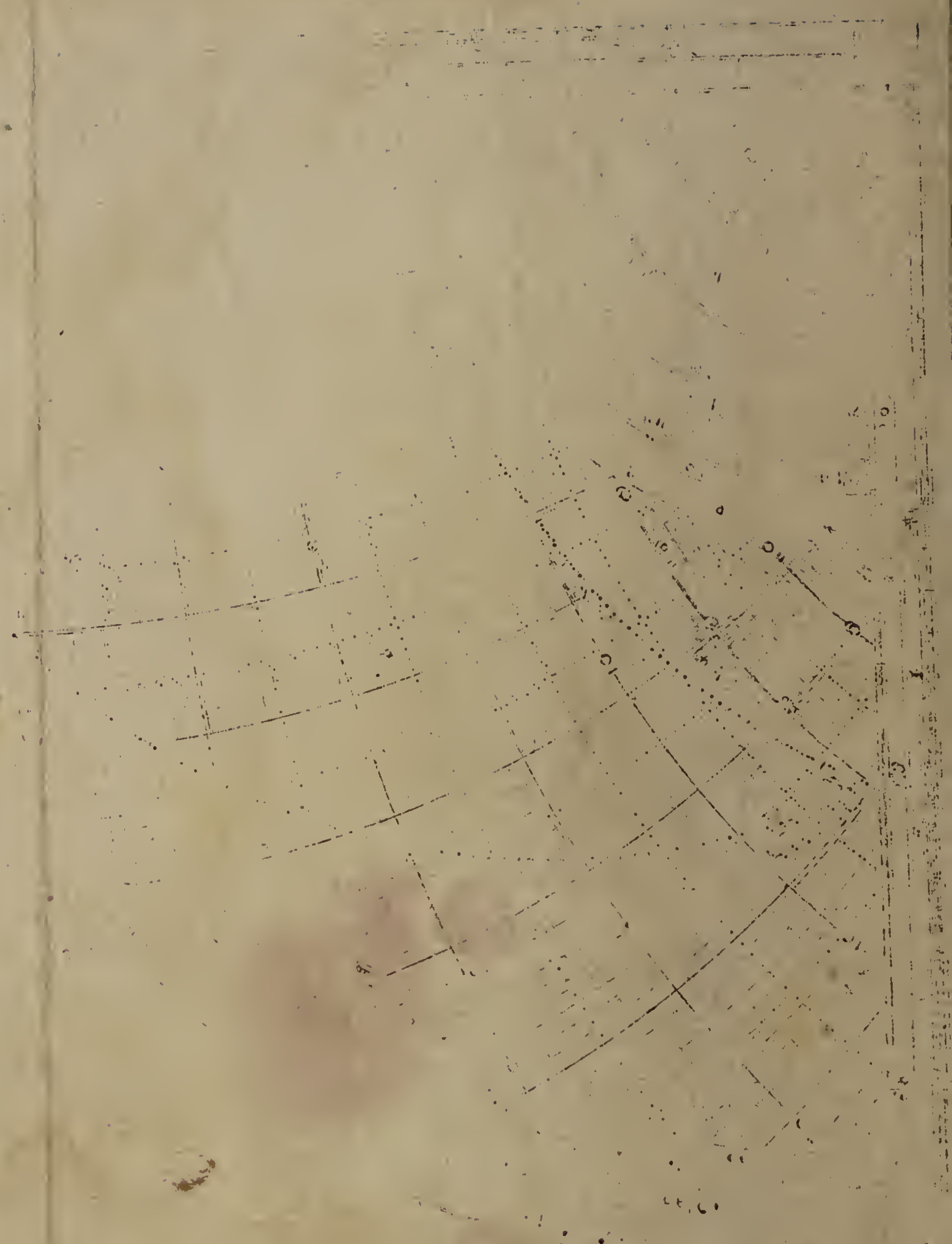
Mar
Shadow
Quadr
Hour
Lat
VI · 12
11 · V
VII · 1
10 · IIII
VIII · 2
9 · III
IX · 3
8 · II
X · 4



Of the Lines on the foreside of the

QUADRANT.

ON the right edge from the Center is placed a Line of equal parts, of 5 inches in length, divided into 100 equal parts.

On the left edge a Line of Tangents, continued to two Radii, or to 63^{d} 26^{m} the Radius whereof is $2\frac{1}{2}$ inches.

Theſe two Lines make a right Angle in the Center, and between them include the Projection, which is no other then a fourth part of *Stoflers* particular *Aſtrolabe* inverted.

Next above this Projection, towards the Center, is put on in the Quadrant of a Circle, the Suns declinations.

And above that in four other Quadrants of Circles, the days of the Moneth, reſpecting the four ſeaſons of the year.

Underneath the Projection, towards the Limbe is pnt on, in one half of a Quadrant, one of the ſides of the Geometrical Quadrat, and in the other half the Line of ſhadows.

All which is bounded in by the equal Limbe.

There ſtands moreover on the very edges of the Quadrant, two Dyalling Scales, which do not proceed from the Center; that on the

right edge is called the Line of Latitudes ; and that on the left edge the Scale of Hours (equal in length to the Sines) which is no other then a double Tangent, or two Lines of Tangents to 45^{d} each set together in the middle, and so might, if there were need, be continued, *ad infinitum.*

The Construction and making of such of these Lines as are not commonly described in other Treatises.

To inscribe the Line of Declinations, there will be given the Suns declination to find his right Ascension, which is the Ark of the Limb, that by help of a Ruler, moving on the Center of the Quadrant, and laid over the same, will in-scribe the Declination proposed.

The Canon to find the Suns right Ascension from the nearest Equinoctial point, correspondent to the Declination proposed, is

As the Radius
To the Cotangent of the Suns greatest Declination :
So the Tangent of the Declination given :
To the Sine of the Suns right Ascension.

The four Quadrants for the days of the moneth are likewise to be graduated from the Limb, by help of a Table of the Suns right Ascensions, made for each day in the year.

The Geometrical Quadrat is inscribed in half the Quadrant of a Circle, by finding in the Table of natural Tangents, what Arches answer to every equal Division of the Radius, and so to be graduated from the Limb ; so 300 sought in the Tangents gives the Ark of 16^{d} 42^{m} of the Limb against which 3 of the Quadrat is to be graduated.

The Line of shaddows is no other then the continuance of the Quadrat beyond the Radius, and so the making after the same manner ; thus having the length of the shadow assigned, annex the Ciphers of the Radius thereto, and seek in he natural Tangents, what Ark corresponds thereto ; thus the shadow being assigned thrice as long as the Gnomon, I seek 3000 in the natural Tangents, the Ark answering thereto, is 71^{d} 34′ which being counted from the left edge of the Quadrant, towards the right in the Limb, the Line of shadows may

from

from thence be graduated; the Complement of this Ark is the Suns Altitude, answering to that length of the right shadow, being 18d 26′.

The Canon to make the Line of Latitudes, will be

As the Radius
to the Chord of 90d
so the Tangents of each respective degree of the Line of Latitudes.
To the Tangents of ohter Arks:

The natural Sines of which Arks are the numbers that from a Diagonal Scale of equal parts shall graduate the Divisions of the Line of Latitudes to any Radius.

To draw the Projection.

Those Lines that cross each other, are Arches of Circles, whose Centers fall in two streight Lines.

Of the Paralels of Altitude.

All those Arks whose Aspect denotes them to be drawn from the right edge of the Quadrant towards the left, are called Paralels of Altitude, and their Centers fall in the right edge of the Quadrant, continued both beyond the Center and Limb so far as is needful.

To find the Interfections of the Paralels of Altitude, with the Meridian, that is, Points therein limiting the Semi-diameters of the Paralells.

Assume any Point in the right edge of the Quadrant, (which is called the Meridian Line) near the Limbe to be the Tropick of *Cancer*; the distance of this Point from the Center of the Quadrant, must represent the Tangent of 56d 46′ which is half the Suns greatest Declination more then the Radius; the distance of the Equator from the Center, shall be equal to the Radius of this Tangent. For the finding the Intersections of the other Paralells of Altitude, it will be best to make a Line of Semi-tangents to the same Radius, that is to number each degree of this Tangent with the double Ark, and so every half degree will become a whole one: Out of this Line of Semi-tangents

prick off from the Center of the Quadrant 66d 29′ the Complement of the Suns greateſt Declination, which will find the Interſection of the Tropick of *Capricorn* with the Meridian.

Now to fit the Projection to any particular Latitude: Out of the ſaid Line of Semi-tangents from the Center of the Quadrant, prick off the Latitude of the place, and it will find a point in the Meridian Line, where the Horizon, or Paralell of 00d of Altitude will interſect the Meridian; this Point is called the Horizontal Point, and ſerves for finding the Centers of all the Paralells.

To the Latitude of the place add each degree of Altitude ſucceſſively till you have included the greateſt Meridian Altitude; theſe compound Arks are ſuch as being prickt fromthe Center of the Quadrant out of the Line of Semi-tangents will find points in the Meridian Line, limiting the Semi-diameters of the paralells of Altitude.

Above the Horizon, and between the Circle that bounds the Projection falls a portion thereof called the Reverted Tail, which otherwiſe would if it had not been there reverted, have excurred the limits of a Quadrant.

To find the Interſections for thoſe Paralells of Altitude, ſubſtract ſucceſſively each degree out of the Latitude of theplace, and the remaining Arks prick from the Center out of the Line of Semi-tangents: The uſe of this Tail being to find the hour and Azimuth before or after 6 in the Summer time only, it need be continued no further above the Horizon then the Ark of the Suns greateſt Altitude at 6, which at *London* is 18d 12′.

To finde the Centers of the Paralells of Altitude.

Theſe are to be diſcovered by help of a Line of natural Tangents, not numbred with the double Arks, whoſe Radius muſt be equal to the diſtance of the Equator from the Center of the Quadrant, or which is all one to 90d of the Line of Semi-tangents: Out of this Line of Tangents prick off beyond the Center of the Quadrant the Complement of the Latitude, the diſtance between the Point thereby found, and the Horizontal Point is the Semi-diameter wherewith the Horizon is to be drawn.

To find the Centers of the reſt of the Paralells.

To the Complement of the Latitude add each degree of Altitude ſuc-

ſucceſſively till you have included the greateſt Meridian Altitudes; The Tangents of theſe Arks prick beyond the Center, the diſtance from the Points ſo diſcovered to the Horizontal Point,are the Semi-diameters of the Paralells of Altitude ; the extreamities of which Semi-diameters being limited in the Meridian Line ; theſe extents thence prict, finds their Centers.

Some of theſe Compound Arks will exceed 90 degrees,as generally where any Meridian Altitude is greater then the Latitude. In this caſe ſubſtract thoſe Arkes from 180^{d} and prick the Tangents of the remaining Arks from the Center of the Quadrant on the Meridian Line continued beyond the Limbe, and then as before the diſtances between thoſe Points and the Horizontal Point, are the Semi-diameters of thoſe Paralells, whoſe Extremities are limited in the Meridian Line.

To find the Centers of the Paralells of the reverted Tail.

From the Complement of the Latitude ſubſtract each degree of Altitude in order, till you have included the greateſt Altitude of 6 the Tangent of the remaining Arks prick from the Center of the Quadrant, and you will find ſuch Points the diſtances between which and the Horizontal Point are the Semi-diameters of thoſe Paralells.

To find the Centers and Semi-diameters of the Azimuths:

All thoſe Portions of Arks which iſſue from the top of the Projection towards the Limb are called Azimuths, the Centers of them all fall upon that Paralell of Altitude which is equal to the Latitude of the place whereto the Projection is fitted, which will always be a ſtreight Tangent Line.

Out of the former Line of Tangents, whoſe Radius is equal to the diſtance of the Equator from the Center of the Quadrant, prick down the Latitude of the place on the Meridian Line, and thereto perpendicularly erect the Line for finding the Centers of the Azimuths,which muſt be continued through and beyond the Projection.

Out of the ſaid Line of Tangents and beyond the Center prick down the Tangent of half the Complement of the Latitude at *London* 19^{d}. 14^{m} and it will diſcover a Point which is called the Zenith Point, becauſe in it all the Azimuths do meet ; The diſtance between this Point and the Point where the Center Line of the Azimuths interſects the

Meri-

Meridian, make the Radius of a Tangent, out of which Tangent prick down each degree successively, both within and beyond the Projection on the Line of Centers, and you have the Centers for all the Azimuths; where note, that the Centers of all Azimuths which exceed 90^d will fall within the Projection, and of all others without, the distances of these respective Points from the Zenith Point, are the Semi-diameters of the Azimuths, with which extents let them be respectively drawn.

To draw the Summer and Winter Ecliptick, and to divide them.

The Summer Ecliptick is drawn from the Point of the Equator in the left edge of the Quadrant to the Tropick of *Cancer*, and the Winter thence to the Tropick of *Capricorn* out of a Line of Tangents to the Radius equal to the distance of the Equator, from the Center prick down the Tangent of 23^d $31'$ the Suns greatest declination from the Center of the Quadrant on the Meridian Line towards the Limbe, and you shall discover the Center of the Summer Ecliptick with the same extent, being the Semi-diameter thereof, set one foot down at the Tropick of *Capricorn*, and the other will fall beyond the Center of the Quadrant on the right edge, and discovers the Center for drawing the Winter Ecliptick; to divide them use this Canon.

As the Radius
to Tangent of the Suns distance from the nearest Equinoctial Point:
So the Cosine of the Suns greatest Declination:
To the Tangent of the Suns right Ascension,

which must be counted in the Limbe, and from it the Suns true place graduated on both the Eclipticks.

To draw the two Horizons, and to divide them.

One of the Horizons is the Paralel of 00^d of Altitude, which being intersected by the Azimuth Circles, is thereby divided into the degrees of the Suns Amplitude; this is the upper Horizon, and the drawing hereof was shewed already.

The other Horizon is but this inverted, and the Divisions transferred from that, the Center of it is found by pricking the Tangent of the

the Complement of the Latitude on the Meridian Line from the Center of the Quadrant, thediſtance of the Equator being Radius.

But it may be alſo done from the Limbe by the Proportion following.

As Radius,
to Tangent of the Latitude;
So the Tangent of the Suns greateſt Declination,
to the ſine of the greateſt Aſcenſional difference

(which converted into Time, gives the time of the Suns riſing or ſetting before or after 6) by which Ark of the Limbe the Horizon is limitted; Then to divide it ſay

As the Radius,
to the Tangent of the aſſigned Amplitude:
So is the Sine of the Latitude:
To the Tangent of the Aſcenſional difference agreeing thereto,

which counted in the Limb, from it the Amplitudes may be divided on both the Horizons; and note, if theſe Amplitudes be not coincident with thoſe the Azimuths have deſigned, then are the ſaid Azimuths drawn falſe.

To inſcribe the Stars on the Projection.

Such only, and no other as fall between the two Tropicks, may be there put on.

Set one foot of the Compaſſes in the Center of the Quadrant, and extend the other to that place of either of the Eclipticks, as correſponds to the given declination of the Star, and therewith ſweep an occult Ark: I ſay then that a Thread from the Center of the Quadrant laid over the Limb to the Stars right Aſcenſion where it interſects, the former occult Ark is the place where the propoſed Star muſt be graduated.

Of the Almanack.

There is alſo graved in a Rectangular Square, or Oblong, a perpetual Almanack, which may ſtand either on the foreſide or back of the Quadrant, as room ſhall beſt permit.

On the Backside of the Quadrant there is,

1. On the right edge a Line of Signs issuing from the Center, the Radius whereof is in length 5 inches.

2. On the left edge a Line of Chords issuing from the Center.

3. On the edges of the Quadrant there are also two Scales for the more ready finding the Hour and Azimuths in one Latitude; the Hour Scale is no other then 62^d of a Line of Sines, whose Radius is made equal to half the Secant of the Latitude being fitted for *London*) to the common Radius of the Sines; the prickt Line of Declination annexed to it, and also continued beyond the other end of it, to the Suns greatest Declination is also a portion of a Line of Sines, the Radius whereof is equal to the Sine of the Latitude taken out of the other part of the Scale, or which is all one the Sine of the Suns greatest declination is made equal to the Sine of the greatest Altitude at the hour of 6 taken out of the other part of the Scale, which at *London* is 18^d 12^m

4. The Azimuth Scale is also 62^d of a Line of Sines, whose Radius is made equal to half the Tangent of the Latitude to the common Radius of the Sines, the Line of the Declination annexed to it, and continued beyond it: To the Suns greatest Declination is also a portion of a Line of Sines of such a length whereof the Sine of the Latitude is equal to the Radius of the Sines of the other part of this fitted Scale; or which is all one, the length of the Suns greatest Declination is made equal to the Suns greatest Vertical Altitude, which in this Latitude is 30^d $39'$ of the other Sine or Line of Altitudes.

The Limbe is numbred both with degrees and time, from the right edge towards the left.

Between the Limbe and the Center are put on in Circles, the Scales following.

1, A Line of Versed Sines to 180 degrees.

2. A Line of Secants to 60^d the graduations whereof begin against 30^d of the Limbe, to apply which Vacancy, and for other good uses, there is put on a Line of 90 Sines, ending where the former graduations begin; this is called the lesser Sines.

3. A Line of Tangents graduated to 63^d $26'$

4. A

4. A Line of Versed Sines to 60^d through the whole Limbe, called the Versed Sines quadrupled, because the Radius hereof is quadruple to the Radius of the former Versed Sines.

5. A Line of double Tangents, or Scale of hours, being the same Dyalling Scale as was described on the foreside.

6. A Tangent of 45^d or three hours through the whole Limbe for Dyalling, which may also be numbred by the Ark doubled to serve for a Projection Tangent, *alias* a Semi-tangent.

7. In another Quadrant of a Circle may be inscribed a portion of a Versed Sine to eight times the Radius encreased, of that of 180d called the Occupled Versed Sine, and at the end of this from the other edge, another portion of a Versed Sine to 12 times the Radius encreased may be put on.

8. Lastly, above all these is the Scale of Hours or Nocturnal with Stars names graved within it towards the Center; this is divided into 12 equal hours and their parts, and the Stars are put on from their right Ascensions, only with their declination figured against them.

All the Lines put on in Quadrants of Circles must be inscribed from the Limbe by help of Tables, carefully made for that purpose; an instance shall be given how the Line of Versed Sines to 180^d *was inscribed, and after the same manner that was put on, must all the rest:*

Imagine a Line of Versed Sines to 180^d to stand upon the left edge of a Quadrant from the Center with the whole length thereof upon the Center sweep the Arch of a Circle, and then suppose Lines drawn through each graduation or degree thereof continued parralel to the right edge till they intersect the Arch formerly swept which shall be divided in such manner as the Line of Versed Sines on this Quadrant is done.

But to do this by Calculation, A Table of natural Versed Sines must first be made, which for all Arks under 90^d are found by substracting the Sine Complement from the Radius, so the Sine of 20d is 34202 which substracted from the Radius rests 65798, which is the Versed Sine of 70^d :

And for all Arks above 90^d are got by adding the Sine of the

Arks excess above 90d unto the Radius: thus the Versed Sine of 110d is found by adding the Sine of 20d to the Radius, which will make 134202 for the Versed Sine of the Said Ark.

This Table, or the like of another kind, being thus prepared, the proportion for inscribing of it will hold.

As the length of the Line supposed to be posited on the left edge,
Is to the Radius,
So is any part of that length
To the Sine of an Arch,

which sought in the Tables, gives the Arch of the Limbe against which the degree of the Line proposed must be graduated.

But in regard the Versed Sine of 180d is equal to the double of the Radius; the Table for inscribing it will be easily made by halfing the Versed Sine proposed, and seeking that half in the Table of natural Sines, so the half of the Versed Sine of 70d is 32899 which sought in the Table of natural Sines, gives 19d 13′ *fere* of the Limb against which the Versed Sine aforesaid is to be graduated, and so the half of the Versed Sine of 110d is 67101 which answers to 42d 9′ of the sines or Limb.

So likewise the Table for puttiug on the lesser Sines was made by halfeing the natural Sines, and then seeking what Arks correspond-ed thereto in the natural Sines aforesaid; those that think these Lines to many may very well want the Versed Sines so oft repeated; And they that will admit of a Radius of 6 or 7 inches, may have the Line of Lines Superficies and Solids, put on in the Limb on the foreside, and the Segments Quadrature, Equated Bodies, Mettalls, and inscribed Bodies, or other Lines at pleasure put on upon the backside, as hath been already done upou some Quadrants.

Now to the Use.

THE

The Uses of the

PROJECTION.

Of the Almanack.

BEfore the Projection can be used, the day of the Moneth, the Suns place or Declination must be known; but these are commonly given by the knowledge thereof: Now this Almanack will as much help to the obtaining hereof, as any other common Almanack.

It consists of a Rectangular Oblong, or long Square divided into 7 Colums in the breadth, to represent 7 days of the week, accounting the Lords day first; and length ways into 9 Columns, the two uppermost represent the months of the year, accounting *March* the first, the five middlemost the respective days of each Month, and the two undermost some certain leap years, posited in such Columns, as that thereby may be known by Inspection, what day of the Week the first of *March* happened upon in the said Leap years; the contri-

vance hereof owns its original from my Worthy Friend Mr. *Michael Darie*, for the due placing of the Months over the Columns of days, take the following Rule in his own words.

First having March *assign'd to lead the round,*
The rest o'th Months are eas'ly after found;
If that you take the complement in days
To 35 *of a plac'd Month always,*
And count it from its place with due Progression
It shews you where the next Month takes possession.

Thus placing the Month of *March* first, then if I would place *April*, or the second Month, *March* having 31 days, the Complement thereof to 35 is 4 then counting four Columns from the place of *March*, it falls upon the 5 Column, where the figure 2 is placed for the 2d Moneth, then *April* being placed; if I would place *May* I take 30, the number of days in *April*, from 35 there rests 5, and counting 5 Columns from the place of *April* where it ends, which is in the 3d Column, the figure 3 is placed for the 3d Moneth or Moneth of *May*. The next thing to be known is on what day of the Week the first day of *March* falleth upon, which is continually to be remembred in using the Almanack.

This for some Leap years to come, may be known by counting in what Column the said Leap year is graved, thus in *Anno* 1660, the first of *March* falls upon a Thursday, because 60 is graved in the 5 Column, that being the fift day of the Week: But for a general Rule take it in these words.

To the number two add the year of our Lord, and a fourth part thereof, neglecting the odd remainder, when there is any; the Amount divide by 7 the remainder, when the Division is finished, shews the number of Direction, or day of the week, on which the first day of *March* falleth, accounting the Lords day the first; but if nothing remain, it falls on a Saturday.

Example

	2	
Example for the year	1657	
The even fourth thereof	414	
7)	2073	(296 quotient.
	1 remaining.	

By this Rule there will be found to remain one for the year of our Lord 1657 whence it follows that the first day of *March* fell on the first day of the Week, *alias*, the Lords day in that year; so in *Anno* 1658, there remains 2 for Munday: in 1659, rests 3 for Tuesday; in 1660 rests 5 for Thursday; so that hence it may be observed, that every 4 years the first of *March* proceeds 5 days: Upon which supposition the former Rule is built; say then

As 4 to 5, or as 1 to 1 $\frac{1}{4}$ so is the year of the Lord propounded, to the number of days, the first of *March* hath proceeded in all that Tract, caused by the odd day in each year, and the Access of the days for the Leap years; this number divided by 7, the remainder shews the fractionate part of a Week above whole ones, which the said day hath proceeded, which wil not agree with the day of the Week the first of *March* falls upon, according to common tradition, unless the number two be added thereto, which argues that the first of *March*, as we now account the days of the week fell upon Munday, or the second day of the week in the year of our Lords Nativity: This is only for Illustration of the former Rule, being to shew that the adding of the even fourth part of the year of our Lord thereto, works the proportion of 4 to 5.

The Use of this Almanack is to know for ever on what day of the Week any day of the Month falls upon.

Remembring on what day of the Week the first day of *March* fell upon in the year propounded (which doth then begin in the use of this Almanack, and not sooner or later, as upon New-years day, or Quarter day) all the figures representing the days of the Month do also represent the same day of the week in the respective Months under which they stand; and the converse, the Moneth being assigned.

signed, all the figures that stand as days under it, inform you what days of the said Month the Week day shall be the same,as it was upon the first day of *March*, and then by a due Progression it will be easie to find upon what day of the Moneth any day of the week falleth, as well as by a common Almanack, without the trouble of always one, and sometimes two Dominical Letters quite shunned in this Almanack, by beginning the year the first of *March*, and so the odd day for Leap year is introduced between the end of the old, and the beginning of the new-year.

Example.

In *Anno* 1657. looking for the figure 10 in the Column for Months, for the Month of *December*; under it I find 6, 13, 20, 27, now the first of *March* being the Lords day, I conclude also that these respective days in *December*, were likewise on the Lords day; and from hence collect, that *Christmas* day, which is always the 25 of that Month, happened on a Friday.

Uses of the Projection.

THis Projection is no other then a fourth part of *Stoflers* particular Astrolabe, fitted for the Latitude of *London* inverted, that is, the Summer Tropick and Altitudes, *&c.* turned downwards towards the Limb,whereas in his Astrolabe they were placed upwards, towards the Center; thus the Quadrant thereof made, is rendred most useful and accurate when there is most occasion for it; before the projection can be used, the Bead must be rectified, and because the Thread and Bead may stretch, there may be two Beads, the one set to some Circle concentrick to the Limb, to keep the other at a certainty in stretching, and the other to be rectified for use.

To

To rectifie the Bead.

LAy the Thread over the day of the Month in its proper Circle, and if the ſeaſon wherein the Quadrant is to be uſed, be in the Winter half year, ſet the Bead by removing it to the Winter Ecliptick; but in Summer let it be ſet to the lower or Summer Ecliptick, and then it is fitted for uſe,

One Caution in rectifying the Bead is to be given; and that is in Summer time if it be required to find the hour and Azimuth of the Sun by the Projection, before the hour of 6 in the morning, or after it in the evening, or which is all one, when the Sun hath leſs Altitude then he hath at 6 of the clock; then muſt the Bead be rectified to the Winter Ecliptick, and the Parralels above the Horizon in the Reverted Tail, are thoſe which will come in vſe.

To find what Altitude the Sun ſhall have at 6 of the clock in the Summer half year.

This will be eaſily performed by bringing the Bead that is rectified to the Summer Ecliptick to the left edge of the Quadrant, and there among the Paralels of Altitude it ſhews what Altitude the Sun ſhall have at 6 of the clock: It alſo among the Azimuths ſhews what Azimuth the Sun ſhall have at the hour of 6. Example, So when the Sun hath 17 degrees of North Declination, as about the 27 of *April*, his Altitude at the hour of 6 will be found to be 13^{d} 14^{m} and his Azimuth from the Meridian 79^{d} 14^{m} whence I may conclude if his obſerved Altitude be leſs upon the ſame day, and the Hour and Azimuth ſought, the Bead muſt be ſet to the Winter Ecliptick, and the Operation performed in the reverted Tail.

Here it may be noted alſo that the exacteſt way of rectifying the Bead, will be either from a Table of the Suns Declination, laying the Thread over the ſame in the graduated Circle, or from his true place, laying it over the ſame in the proper Ecliptick, or from his right Aſcenſion counted in the Limb.

Or

Or Laſtly from his Meridian Altitude on the right edge of the Quadrant, for theſe do mutually give each other the Bead, being rectified to the reſpective Ecliptick as before. for Example.

To find the Suns Declination.

The Thread laid over the day of the Moneth, interſects it upon that Circle whereon it is graduated, which in the Summer half year is to be accounted on this ſide the Equinoctial, *N*orth, and in the Winter-half year, South; ſo laying the Thread over the 27th. day of *April*, it interſects the Circle of Declination at 17 degrees, and ſo much was the Suns Declination.

To find the Suns true place.

The Thread lying as before, ſhews it on the reſpective Ecliptick, So the Thread lying over the 17 of *April*, will cut the Summer Ecliptick, in $17^{d}\ 7^{m}$ of *Taurus*; or in $12^{d}\ 53^{m}$ of *Leo*, which agrees to the 26 day of *July*, or thereabouts, the Thread interſecting both theſe days at once; and the oppoſite points of the Ecliptick hereto, are $17^{d}\,7^{m}$ in *Scorpio*, about the 20 of *October*; and $12^{d}\ 53^{m}$ of *Aquarius*, about the 22^{d} of *January*, all ſhewed at once by the Threads poſition.

To find the Suns right Aſcenſion.

Lay the Thread over the day of the Month as before, and it interſects it in the equal Limb; whence taking it in degrees and minutes of the Equator, whilſt the Sun is departing from the Equator towards the Tropicks, it muſt be counted as the graduations of the Limb, from the left edge towards the right; but when the Sun is returning from the right edge towards the left; the right Aſcenſion thus found, muſt be eſtimated according to the ſeaſon of the year.

	June 11 to *Sept.* 13		90	
From	*Sep.* 13 to *Dec.* 11	It muſt have	180	degrees added to it.
	Dec. 11 to *Mar.* 10		270	

But

But in finding the Hour of the night by the Quadrant, we need no more then 12 hours of Afcenfion, for either Sun or Star, and the Limb is accordingly numbred from the left edge towards the right, from 1 to 6 in a fmaller figure, and thence back again to 12, and the other figures are the Complements of thefe to 12, fo that when the Sun is departing from the Equator towards the Tropicks; his right Afcenfion is always lefs then 6 hours, and the Complement of it more; but when he is returning from the Tropicks towards the Equator, it is always more then 6 hours, and the Complement of it lefs; the odd minutes are to be taken from the Limb, where each degree being divided into 4 parts, each part fignifies a Minute of time, and to know whether the Sun doth depart from, or return towards the Equator, is very vifible, by the progrefs and regrefs of the days of the month, as they are denominated on the Quadrant.

Example.

So the Thread laid over 17^{d} of Declination, which will be

	27 *April*	The Suns	44^{d}	37^{m}		2^{h}	58'
about	26 *July*	right Af-	135	23	In	9	2
the	20 *October*	cenfion	224	37	time	2	58
	22 *January*	will be	315	23		9	2

But here the latter 12 *hours are omitted.*

Such Propofitions as require the ufe of the Bead, are,

To find the Suns Amplitude, or Coaft of rifing and fetting from the true Eaft or Weft.

Bring the Bead, being rectified to either of the Eclipticks, it matters not which, to either of the Horizons, and the Thread will interfect the Amplitude fought, upon both alike: Example; The Suns Declination being 17 North, or South, the Suns Amplitude, will be found to be 28^{d} 2^{m}.

The Amplitude before found for the Summer half year, is to be

accounted from East or West Northwards; and in the Winter half year from thence Southwards.

To find the time of the Suns rising or setting.

The Thread lying in the same Position, as in the former Proposition, intersects the Ascensional difference in the Limb, which may there be counted either in degrees or Time.

Example.

So the Bead lying upon the Horizon, being rectified to 17^{d} of Declination, the Thread intersects the Limb at 22^{d} 38^{m}, which is 1^{h} 30^{m} of time, and so it shews the time of Suns rising in Summer, or setting in Winter, to be at half an hour past 4; and his rising in Winter, and setting in Summer, to be at half an hour past 7.

To find the length of the Day or Night.

The time of the Suns rising and setting are one of them; the Complement of the other to 12 hours; so that one of them being known, the other will be found by Substraction; the time of Suns setting is equal to half the length of the day; and this doubled gives the whole length of the day; in reference to the Suns abode above the Horizon, the time of setting converted into degrees, is also called the Semi-diurnal Ark; the time of Sun rising (so converted is called the Semi-nocturnal Ark) doubled gives the whole length of the Night; so upon the 27th day of *April*, the Sun having 17^{d} of Declination, the length of the day is 15 hours, and the length of the night 9 hours.

To find the Suns Altitude on all Hours; or at any time proposed.

In Summer time, if the hour proposed be before 6 in the morning, or after it in the evening, lay the Thread to the hour in the Limb, the Bead being first rectified to the Winter Ecliptick, and amongst the Paralels of Altitude above the upper Horizon, it shews the Altitude sought.

Ex-

Example.

So the Sun having 16^{d} of declination Northwards, as about the 24th of *April*, laying the Thread over the Declination, I set the Bead to the Winter Ecliptick, and if it were required to find what Altitude the Sun shall have at 36 minutes past 6 in the afternoon, lay the Thread over the same in the Limb, and the Bead among the Parralels of Altitude will fall upon 7^{d},

At all other times the Operation is alike; the Bead being rectified to that Ecliptick that is proper to the season of the year: Lay the Thread over the proposed hour in the Limb, and the Bead amongst the Parralels of Altitude, sheweth the Altitude sought.

Example.

So if it were required the same day to find what Altitude the Sun should have at 19^{m} past 2 in the afternoon; Lay the Thread in the Limb over the time given, and the Bead among the Parralels of Altitude will fall upon 45^{d} for the Altitude sought.

To finde the Suns Altitude on all Azimuths.

IN the Summer half year, if the Azimuth propounded be more Northward then the Azimuth of the Sun shall have at the hour of 6; The Bead must be rectified to the Winter Ecliptick, and brought to the Azimuth proposed above the upper Horizon, and there among the Parralels of Altitude, it sheweth the Altitude sought.

So about the 24th of *April*, when the Suns Declination is 16^{d} his Azimuth at 6 of the clock will be found to be 76^{d} 54^{m} from the South; Then if it were required to find the Suns Altitude upon an Azimuth more remote, as upon 107^{d} from the South, laying the Thread over the Declination, I set the Bead to the Winter Ecliptick, and afterwards carrying it to the Azimuth proposed among the Parralels of Altitude above the upper Horizon, it falleth upon 7^{d} for the Suns Altitude sought.

In all other Cases bring the Bead rectified to the Ecliptick pro-

per to the ſeaſon of the year, to the Azimuth propoſed; and among the Parralels of Altitude it ſheweth the Altitude ſought; So far the ſame day, I ſet the Bead to the Summer Eclipticks, and if it were required to know what Altitude the Sun ſhall have when his Azimuth is 50 d 48′ from the Meridian carry the Bead to the ſaid Azimuth, and among the Parralels of Altitude it will fall upon 48d for the Altitude ſought.

The Hour of the night Propoſed to find the Suns Depreſsion under the Horizon.

IMagine the Sun to have as much Declination on the other ſide the Equinoctial, as he hath on the ſide propoſed; aud this Caſe will be co-incident with the former of finding the Suns Altitude for any time propoſed; the reaſon whereof is becauſe the Sun is always ſo much below the Horizon at any hour of the night, as his oppoſite Point in the Eclipticks is above the Horizon at the like hour of the Day.

Such Propoſitions as depend upon the knowledge of the Suns Altititude, are to find the Hour of the Day, and the Azimuth (or true Coaſt) of the Sun.

THe Suns Altitude is taken by holding the Quadrant ſteady, and letting the Sun Beams to paſs through both the Sights at once, and the Thread hanging at liberty ſhews it in the equal Limb, if this be thought unſteady, the Quadrant may reſt upon ſome Concave Diſh or Pot, into which the Plummet may have room to play; but for greate Quadrants there are commonly Pediſtalls made.

The

The Altitude supposed to find the Hour of the Day, and the Azimuth of the Sun in Winter.

REctifie the Bead to the Winter Ecliptick, and carry it along amongst the Parralels of Altitude till it cut or intersect that Parralel of Altitude on which the Sun was observed, and the Thread in the Limb sheweth the hour of the Day, and the Bead amongst the Azimuths sheweth the Azimuth of the Sun.

Example.

So about the 18 of *October*, when the Suns Declination is 13^d 20' South if his observed Altitude were 18^d the true time of the day would be found to be either 36 minutes after 9 or 24 minutes past 2 and his Azimuth would be 37 degrees from the South.

To finde the Hour of the Day, and the Azimuth of the Sun at any time in the Summer half year.

IT was before intimated, That if the question were put when the Sun hath less Altitude then he hath at the hour of 6 of the clock, that then the Operation must be performed amongst those Parralels above the upper Horizon, in the reverted Tail, the Bead being recti-

rectified to the Winter Ecliptick; and that it might be known what Altitude the Sun ſhall have at 6 of the clock, by bringing the Bead rectified to the Summer Ecliptick, to the left edge of the Quadrant.

So admitting the Sun to have 16^d of North Declination, which will be about the 24 *April*, I might finde his Altitude at 6 of the Clock by bringing the Bead rectified to the SummerEcliptick to the left edge of the Quadrant; to be $12^d\ 28^m$ whence I conclude, if his Altitude be leſs, the Bead muſt be rectified to the Winter Ecliptick, and be brought to thoſe Parralels above the upper Horizon; and it may be noted, that the Suns Altitude at 6 is always leſs then his declination.

Example.

Admit the 24th of *April* aforeſaid the Suns obſerved Altitude were 7^d laying the Thread over the Suns Declination, or the day of the month; I rectifie the Bead to the Winter Ecliptick, and bring it to the ſaid Parralel of Altitude above the upper Horizon; and the Thread interſects the Limb at $9^d\ 3^m$ ſhewing the hour of the day to be 24 minutes paſt 5 in the morning, or 36' paſt 6 in the evening, and the Bead amongſt the Azimuths ſhews the Azimuth or Coaſt of the Sun to be 107^d from the South.

Another Example.

But admitting the Sun to have more Altitude then he hath at the hour of 6, the Operation notwithſtanding differs not from the former, but only in rectifying the Bead, which muſt be ſet to the Summer Ecliptick, and then carried to the Parralel of the Suns obſerved Altitude, and the Thread will interſect the Limb at the true time of the day, and the Bead amongſt the Azimuths ſheweth the true Coaſt of the Sun.

So upon the 24th of *April* aforeſaid, the Suns obſerved Altitude being 45^d, I bring the Bead rectified to the Summer Ecliptick, to the ſaid Parralel of Altitude, and the Thread interſects the Limb at $55^d\ 15^m$ ſhewing the hour to be either 41^m paſt 9 in the morning, or 19^m paſt 2 in the afternoon; to be known which by the increa-

ſing

sing or decreasing of the Altitude, and the Bead amongst the Azimuths shews the Azimuth or true Coast of the Sun to be 50^d 41′ from the South :

Another Example.

Admit when the Sun hath 19^d 13^m of North Declination which will be about the 6th of *May*, his observed Altitude were 56^d the Beed being set to the Summer Ecliptick, and brought to that Parralel of Altitude amongst the Azimuths shews the Suns true Coast to be 23^d from the South, Eastwards in the forenoon, and Westwards in the afternoon, and the Thread in the Limb sheweth the true time of the day to be either 7′ past 11 in the forenoon, or 58^m past 12.

The Depression of the Sun supposed to find the true time of the night with us, or the hour of the day to our Antipodes; As also the true Coast of the Sun upon that Depression.

THis Proposition may be of use to know when the Twilight begins or ends, which is always held to be when the Sun hath 18^d of Depression under the Horizon, to perform this, Imagine as much Declination on the contrary side the Equinoctial, as the Declination given, and find the time of the Day, as if the Suns Altitude were 18^d

So when the Suns Declination is 16^d North, as about the 24th of *April*, laying the Thread over it I rectifie the Bead to the Winter Ecliptick, and bringing it to the Parralel of 18^d the Thread in the Limb shew the Twilight to begin at 54^m past 1 in the morning, and ends at 6′ past 10 at night, and the Azimuth of the Sun to be 28^d 58′ which in this Case is to be accounted from the North.

But if the Suns greatest Depression at night be less then 18^d as that it may be in any Latitude where the Meridian Altitude at any time in Winter or the opposite Signe is less then 18^d there is no dark night which in our Latitude of *London* will be from the 12th of *May* to the 11th of *July*.

Of

Of the Stars graduated on the PROJECTION.

SUch Stars as are between the two Tropicks only, are there inscribed, and such haue many things common in their Motion with the Sun when he hath the like Declination, as the same Amplitude, Semidiurnal, Arke, Meridian, Altitude, Ascentional difference, *&c.*

These Stars have Letters set to them to direct to the Circle of Ascensions on the back of the Quadrant, where the quantity of their right Ascension, is expressed from one of the Equinoctial points; those that have more Ascension then 12 hours from the point of *Aries*, are known by the Character *plus* + set to them; many more Stars might be there inserted, but if they have more then 23^{d} $31'$ of Declination, the Propositions to be wrought concerning them are to be performed with Compasses, by the general Lines on the Quadrant.

To find the true time of the Day or Night when any Star commeth to the Meridian.

In the performing of this Proposition we must make use of the Suns whole right Ascension in time, which how that might be known hath been already treated of, as also of the Stars whole right Ascension, which may be had from the Circle of Ascensions on the back of the Quadrant if 12 hours be added to the right Ascension of a Star taken thence that hath a Letter Character affixed to it.

Substract the Suns whole right Ascension from the Stars whole right Ascension, encreased by 24 hours when Substraction cannot be made without it, the remainder is less then 12 shews the time of the afternoon or night when the Star will be upon the Meridian; but if there remain more then 12, reject 12 out of it and the residue shews the time of the next morning when that Star will be upon the Meridian.

Example.

The 23[d] of *December* the Suns whole right Ascension is 18 hours 53' which substracted from 4 ho:16' the right Ascension of the *Bulls eye* encreased by 24 there remains 9^h 23' for the time of that Stars comming to the Meridian, and being substracted from 6 ho: 30' the right Ascension of the great Dogg, there rests 11 ho: 37' for the time of that Stars coming to the Meridian at night.

This Proposition is of good use to Sea-men, who have occasion to observe the Latitude by the Meridian Altitude of a Star, that they may know when will be a fit time for observation.

In finding the time of the Night by the Stars, we use but 12 ho: of right Ascension, nor no more in finding the time of their rising or setting, so that when it is found whether it be morning or evening is left to judgement, and may be known by comparing it with the former Proposition, if there be need so to do.

To find the Declination of any of these Stars.

This is engraven or annexed to the Stars names, yet it may be found on the Projection, by rectifying a Bead to the proposed Star, and bringing the Thread and Bead to that Eclíptick it wil intersect; and in the same Position the Thread will intersect the said Stars declination in the Quadrant of Declinations; if the Bead meet with the Summer Ecliptick the Declination is North, if with the Winter South.

To find the Amplitude and Ascensional difference of any of the Stars on the Projection.

BRing the Bead rectified to the Star to either of the Horizons, the Thread being kept in its due Extent, and where it intersects the same it shews that Stars Amplitude which varies not, and is Northward if the Star have North declination, otherwise Southwards; the Thread likewise intersecting the Limb, sheweth the Stars Ascensional difference.

Example.

So the Bead being rectified to the *Bulls eye,* and brought to the lower Horizon, ſhews the Amplitude of that Star to be $25^d\ 54'$ Northwards becauſe the Star hath North Declination; And the Thread lyeth over $20^d\ 49'$ of the Limb which is this Stars Aſcenſional difference, which in Time is 1 ho 23^m The Thread in the Limb lyeth over 4 ho 37^m from midnight for the Stars hour of riſing, and over 7 ho 23^m from the Meridian for the Stars hour of ſetting always in this Latitude which with the Amplitude varies not, except with a very ſmall allowance in many years.

To find a Stars Diurnal Ark, or the Time of its continuance above the Horizon.

When the Star hath	*North* / *South*	Declination	add / Subſtract.

the Aſcenſional difference of the Star before found in

Time	to / from	6 hours,	the	Sum / Reſidue	is half the time.

Of that Stars continuance above the Horizon, which doubled, ſhews the whole time, the Complement whereof to 24 ho is the time of that Stars durance under the Horizon.

Example.

So the Aſcenſional difference of the *Bulls eye* being in time 1 ho: 23^m added to 6 hours, and the Sum doubled makes 14 hours 46^m for the Stars Diurnal Ark or abode above the Horizon, the reſidue whereof from 24 is 9 ho: 14^m for the time of its durance under the Horizon.

To find the true time of the Day or night, when the Star riſeth or ſetteth.

THe Stars hour of riſing or ſetting found as before, being no other but the Aſcenſional difference of the Star added to, or ſubſtracted

ed from 6 hours; which the Thread ſheweth in the Limb the Bead being rectified to a Star, and brought to that Horizon it will interſect; is not the true time of the night; but by help thereof that may be come by; this we have denominated to be the Stars hour, and is no other but the Stars horary diſtance from the Meridian it was laſt upon;

If a Star have	*North* / *South*	Declination the Stars hour of riſing			
muſt be reckoned to be	before / after	6	and the time of its ſetting	after / before	6

Now the time of the Stars riſing or ſetting found by this and the former Propoſitions muſt be turned into common time by this Rule. To the Complement of the Suns Aſcenſion add the Stars Aſcenſion, and the Stars hour from the Meridian it was laſt upon, the Amount if leſs then 12 ſhews the the time of Stars riſing or ſetting accordingly; but if it be more then 12 reject 12 as oft as may be, and the remaind- ſheweth it.

So upon the 23d of December for the time of the Bulls eye riſing.

	h	m
The Complement of the Suns Aſcenſion found by the foreſide of the Quadrant is	5	7
And the ſaid Stars Aſcenſion on the backſide is	4	16
The Stars hour of riſing is	4	37
	14 hours.	

From which 12 rejected reſts 2 hours for the time of that Stars riſing, which I conclude to be at 2 in the afternoon, becauſe that Star was found to come to the Meridian at 23 m paſt 9 at night, the like Operation muſt be uſed to get the time of that Stars ſetting, which will be found to be at 4 ho 46 m paſt in the morning.

	h	m
Complement ☉ *Aſcenſion*	5	7
Stars Aſcenſion	4	16
Stars hour of ſetting	7	23
	16 h. 46′	

To find what Altitude and Azimuth a Star that hath North Declination shall have when it is 6 hours of Time from the Meridian.

RECtifie the Bead to the Star, and bring the Bead and Thread to the left edge of the Quadrant, and there among the Parralels of Altitude and Azimuths it sheweth what Altitude and Azimuth the Star shall have.

Example.

So the Bead being set to the *Bulls eye*, and brought to the left edge of the Quadrant it will be found to have 12 17' Altitude, and 80d 3' Azimuth from the South, when it is 6 hours of time from the Meridian, which Proposition is afterwards used to know to which Ecliptick in some Cases to rectifie the Bead as hath likewise been intimated before.

The Azimuth of a Star proposed, To find what time of the Night the Star shall be upon that Azimuth, and what Altitude it shall then have.

SUpposing the Azimuth proposed to be nearer the South Meridian then that Azimuth the Star shal have when it is 6 hours from the Meridian: Bring the Bead rectified to the Star, to the proposed Azimuth, and among the Parralels of Altitude it shews that Stars Altitude, and the Thread in the Limb shews that Stars hour to be turned into common time to attain the true time sought.

Example.

If the question were What Altitude the *Bulls eye* shall have when his Azimuth is 62d 48' from South, this being less Azimuth then he hath at 6 hours from the Meridian, the rectified Bead being brought to the Azimuth sheweth among the Parralels the Altitude to be 39d and the Stars hour shewn by the Thread in the Limb is either 8 ho: 56' or 3 ho: 4' from the Meridian; then if upon the 23 of *December*

ber you would know at what time the Star shall have this Altitude on this Azimuth, Change the Stars hour into common time by the former Rule.

Decemb. 23 Complement of ☉ Ascension	5^h	7'	5^h	7'
Stars Ascension————————	4	16	4	16
Stars hour————————	8	56	3	4
	18	19	12	27

And you will find it to be at 19' past 6 in the evening, or at 27^m past midnight.

For Stars of South Declination being they have no Altitude above the Horizon at 6 ho: distance from the Meridian, the operation will be the same, void of Caution.

But for Stars of North Declination when the proposed Azimuth is more remote from the South Meridian then the Azimuth of that Star 6 ho from the Meridian, another Bead must be rectified to the Winter Ecliptick, and carried to the Azimuth proposed above the upper Horizon, where amongst the Parralels it shews the Altitude sought; and the Thread in the *Limb* sheweth the Stars hour to be converted into common time.

Example.

The Azimuth of the *Bulls eye* being 107^d 53' from South, which is more then the Azimuth of 6 hours, the other Bead set to the Winter Ecliptick, and carried to that Azimuth in the Tail, shews the Altitude to be 6^d and the Stars hour to be 5 ho: 18' Or 6 ho: 42' which converted into common time, as upon the 23^d of *December*, will be either 41^m past 2 in the afternoon, or 5^m past 4 in the morning following.

December 23 Complement Suns Ascension	5 h	7	5 h	7
Stars Ascension————————	4	16	4	16
Stars hour————————	5	18	6	42
Rejecting 12 the Total is————————	2	41 Or	4	5

The Hour of the night proposed to find what Altitude and Azimuth any of the Stars on the Projection that are above the Horizon shall have at that time.

FIrst turn common time into the Stars hour, the Rule to do it is, To the Complement of the Stars Afcenfion add the Suns Afcenfion, and the time of the night propofed, the Aggregate if lefs then 12 is the Stars hour; if more reject 12 as oft as may be, and the remainder is the Stars hour fought. So the 23 of *December*, at 8 a Clock 59 minutes paft at night what fhall be the Horarie diftance of the great Dogg from the Meridian

	h	'
Complement of great *Doggs* Afcenfion	5	30
Suns Afcenfion	6	53
Time of the night	8	59
The Sum is, 12 rejected	9	22

Then for Stars of South Declination, rectifie the Bead to the Star propofed, and lay the Thread over the Stars hour in the Limb, and the Bead amongft the Parralels and Azimuths, fhews the Altitude and Azimuth of the Star fought. *Example.* So the Bead being rectified to the great *Dogg*, and the Thread laid over 9 ho 22' in the Limb, the Bead will fhew the Altitude of that Star at that time of the night to be 14^{d} and its Azimuth 39' from the South.

The Operation is the fame for Stars of North Declination when the Stars hour found as before is not more remote from the South Meridian then 6 hours on either fide.

But if it be more then 6 ho diftance from the Meridian as before 6 after its rifing, or after it before its fetting, then as before fuggefted, one Bead muft be rectified to the Star, and brought to the Summer Ecliptick, where the Thread being duly extended, another muft be fet to the Winter Ecliptick, and afterwards the Thread laid over the Stars hour in the Limb, this latter Bead will fhew the Stars Azimuth and Parralel of Altitude in the reverted Tail above the upper Horizon.

Example.

So upon the 23 of *December*, I would know what Azimuth and Altitude the *Bulls eye* fhall have at 4 a Clock 5 minutes paft the morning following.

Time

	h	'
Time propoſed	4h	5'
Complement of *Bulls eye* Aſcenſion	7	44
Suns Aſcenſion 23 of *December*	6	53
12 rejected reſts	6 h	42'

Proceed then and lay the Thread over 42' paſt 6 and the Bead among the other Paralels in the Tail ſheweth the Stars Altitude to be 6 d and its Azimuth from the Meridian 107d 53'

Theſe two Propoſitions have a good tendency in them to diſcover ſuch Stars as are upon the Projection if you know them not, but ſuppoſing them known the Propoſition of chiefeſt uſe is

By having the Altitude of a Star given to find out the true Time of the night, and the Azimuth of that Star.

If the Stars obſerved Altitude be leſs then its Altitude at 6 ho: diſtance from the Meridian; Bring the Bead, rectified to the Star, to the Summer Ecliptick and ſet another Bead to the Winter Ecliptick, Then carry it to the Parralell of Altitude above the upper Horizon in the Reverted taile and there it will ſhew the Azimuth of that Star; and the thread in the Limbe ſhews the houre.

Example.

So if the obſerved Altitude of the *Bulls eye* were 6d its Azimuth would be found to be 107d 53' from the South, and its hour 42' paſt 6 from the Meridian the true time would be found to be 5 minutes paſt 4 in the morning the 24 of *December.*

	ho:	'
Complement of ⊙ Aſcenſion the 23 of *December*	5	7
Stars hour	6	42
Stars Aſcenſion	4	16
	4	5

But

But for Stars that have South declination or north, when their Altitude is more then their Altitude being 6 hours from the Meridian, this trouble of rectifying two Beads is ſhunned; in this Caſe only bring the Bead that is rectified to the Star to the Parralel of Altitude, and there among the Azimuths it will ſhew the Stars Azimuth, and the Thread in the Limb interſects the Stars hour ſought.

Example.

December 11th *Bulls eye* Altitude 39^{d} Azimuth from the South 62^{d} 48

Hours from the Meridian	8^{h}	56'
Complement of ☉ Aſcenſion	6	00
Aſcenſion of *Bulls eye*	4	16
The true time of the night was 12' paſt 7 of the Clock	7	12

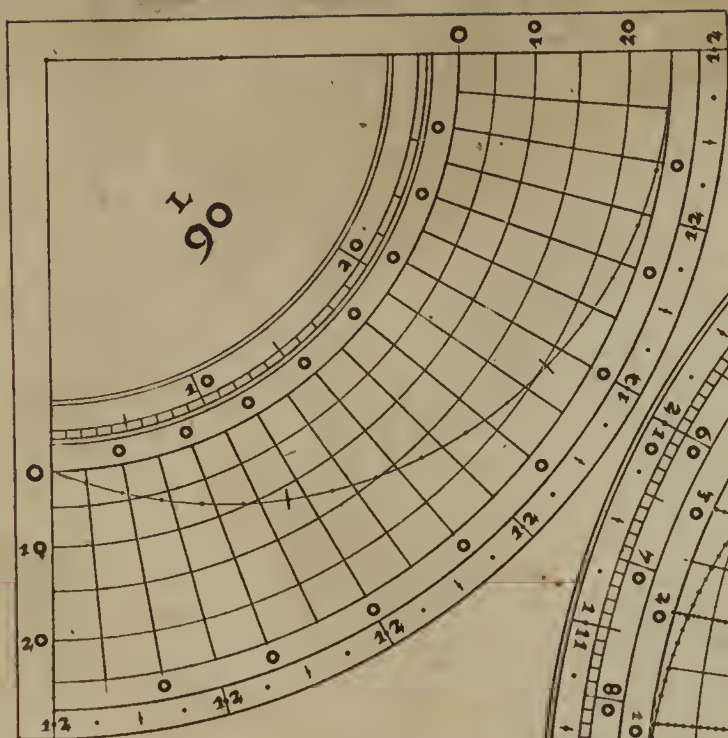
Another Example.

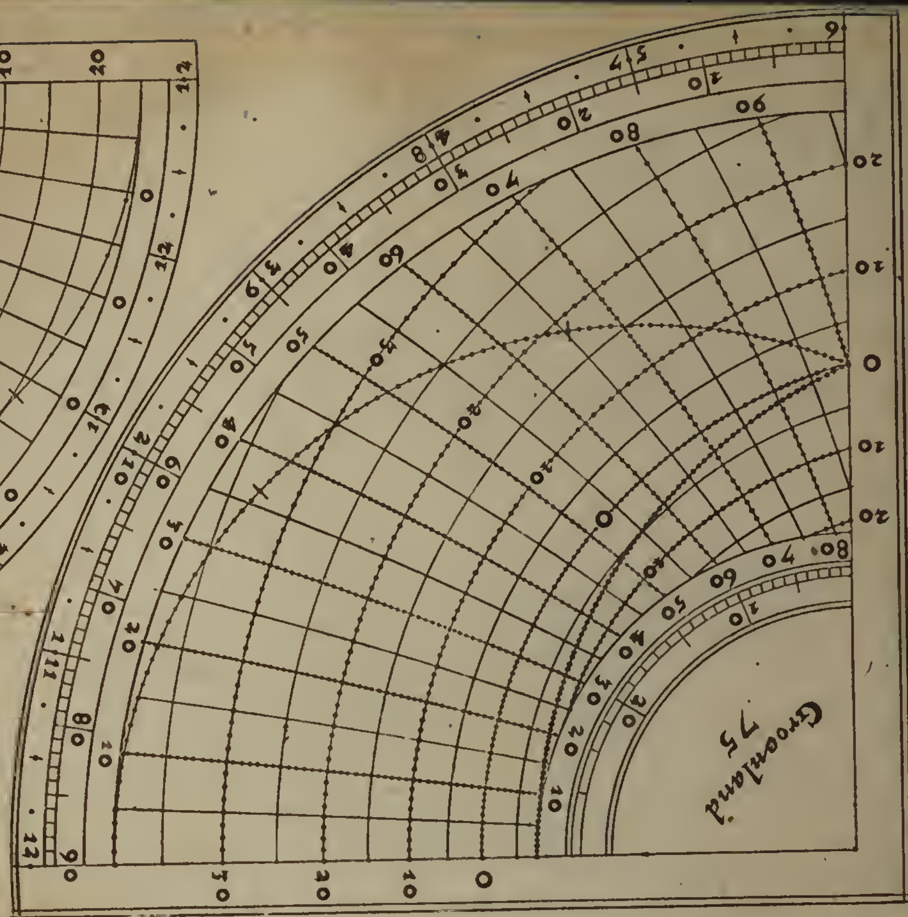
The great *Doggs* obſerved Altitude being 14^{d} his Azimuth from the South would be 39^{d}.

	h	m
And the Stars hour from the Meridian	9	22
Stars Aſcenſion	6	30
If this Obſervation were upon the 31 of *December*, the Complement of the Suns Aſcenſion would be	4	30
	8	22

And the true time of the night 22 minutes paſt eight of the Clock.

For varieties ſake there is alſo added to the Book a Draught of the Projection for the Latitude of the *Barbados*; in the uſe whereof the Reader may obſerve that every day when the Sun comes to the Meridian between the Zenith and the Elevated Pole, he will upon divers Azimuths in the forenoon (as alſo in the afternoon) have two ſeveral Altitudes, and ſo be twice before noon, and twice after




Groenland
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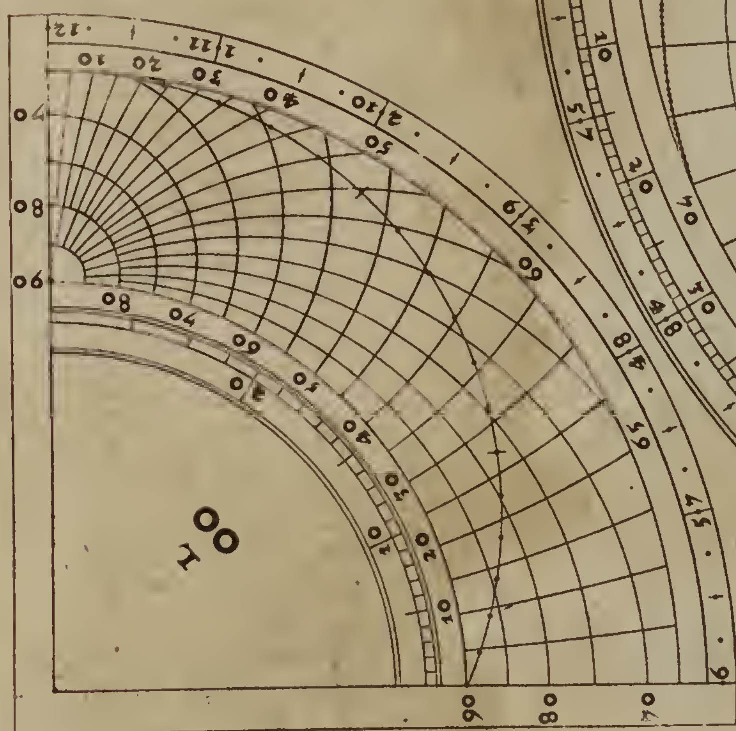
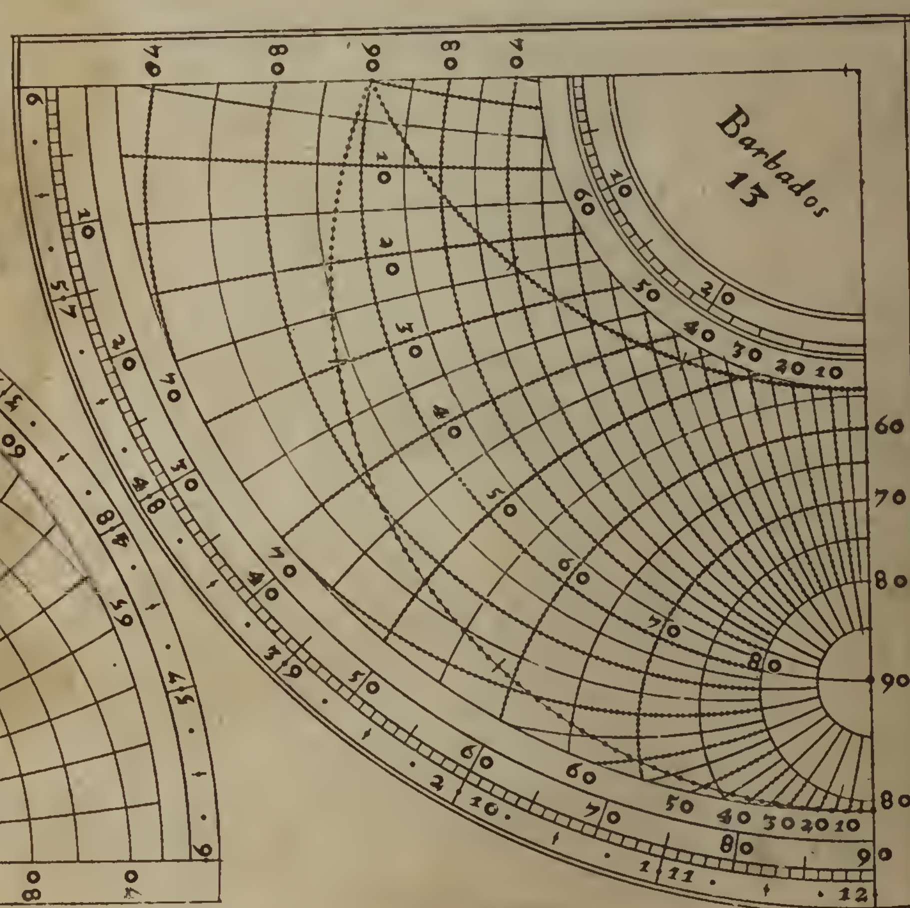
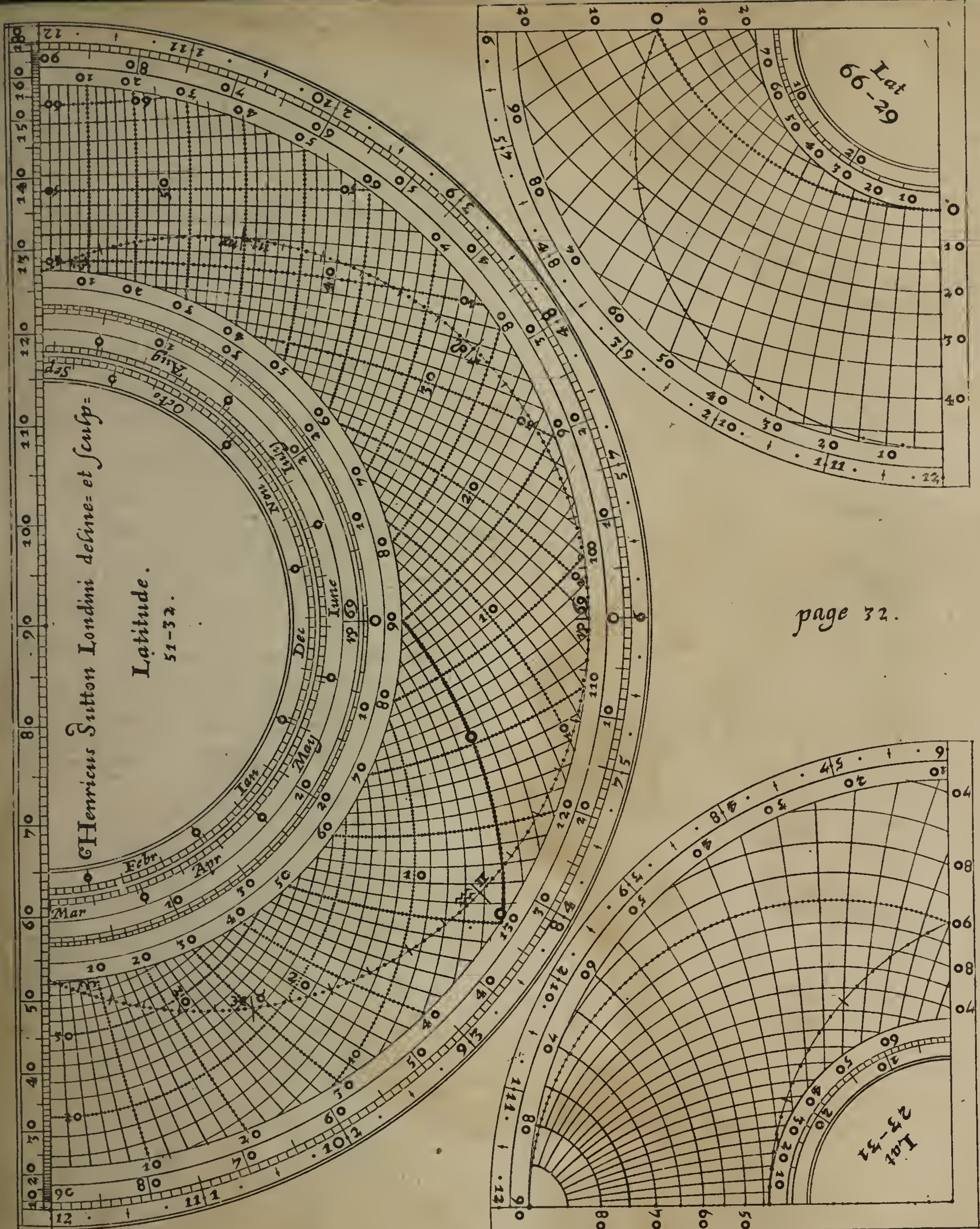



Barbados
13


Henricus Sutton Londini deline= et sculp=
Latitude.
51-32.
Lat
66-29
Lat
23-31

page 32.

afternoon, at several times of the day, upon one and the same Azimuth, *viz.* only upon such as lye between the Suns Coast of rising and setting, and his remotest Azimuth from the Meridian, which causeth the going forward and backward of the shaddow; but of this more hereafter, when I come to treat of Calculating the Suns Altitude on all Azimuths; It may also be observed that the Sun for the most part in those Latitudes hath no Vertical Altitude or Depression, and so comes not to the East or West.

Moreover there is added a Draught of this Projection for the Latitude of *Greenland*, in the use whereof it may be observed that the Sun, a good part of the Summer half year comes not to the Horizon, and so neither riseth nor sets.

And that no convenient Way that this Projection can be made should be omitted, there is also one drawn in a Semi-Circle for our own Latitude, which in the use will be more facile then a Quadrant, there being no trouble before or after six in the Summer time, with rectifying another Bead to perform the Operation in the reverted Taile, neither doth the Drawing hereof occupy near the Breadth, as in a Quadrant, and so besides the ease in the use is more exact in the performance, there being no other Rule required for rectifying the Bead, but to lay the Thread over the day of the Month, and to set the Bead to that Ecliptick the Thread intersects.

A Semi-Circle is an Instrument commonly used in Surveigh, and then it requires a large Center-hole; however this Projection may be drawn on a Semi-Circle for Surveigh, but when used at home there must a moveable round Bit of brass be contrived to stop up that great Center-hole, in which must be a small Center-hole for a Thread and Plummet to be fastned, as for a Quadrant and some have been so fitted.

The Reader will meet with variety of Lines and furniture in this Book to be put in the Limb, or on other parts of the Semicircle, as he best liketh. The Projection for the *Barbados* & *Greenland*, are drawn by the same Rules delivered in the Description of the Quadrant, and so also is the Summer part of this Semi-Circle, and the Winter part by the same Rules that were given for drawing the Reverted Taile.

Of the Quadrant of Ascensions.

The turning of the Stars hour into the Suns hour and the the converse may be also done by Compasses upon the Quadrant of Ascensions on the back side.

To turn the Stars Hour into common time, call d the Suns hour.

THe Arithmetical Rule formerly given is nothing but an abridgment of the Rule delivered by Mr. *Gunter*, and others, and the work to be done by Compasses, differeth somewhat from it, though it produce the same Conclusion which is:

To get the difference between the Ascension of the Sun and the Star by substracting the less from the greater; this remainder is to be added to the Stars hour, when the Star is before, or hath more Ascension then the Sun, but otherwise to be substracted from it; and the Sum or remainder is the true time sought.

To do this with Compasses, take the distance between the Star and the Suns Ascension, and set the Suns foot to the observed hour of the Star from the Meridian it was last upon, letting the other foot fall the same way it stood before, and it sheweth the time sought, if it doth not fall off the Quadrant.

If it doth, the work will be to finde how much it doth excur, and this may be done by bringing it to the end beyond which it falleth, letting the other foot fall inward; the distance then between the place where it now falleth, and where it stood before, which was at the Stars hour, is equal to the said excursion, which being taken, and measured on the other end of the Scale, shews the time sought.

This trouble may be prevented in all Cases, by having 12 hours more repeated after the first 12, or 6 hours more may serve turn if the whole 18 hours be also double numbred, and Stars names being set to the Additional hours, possibly the Suns Ascension and Star do not both fall in the same 12 hours, yet notwithstanding the distance is to be taken in the same 12 hours between the quantity of the Suns Ascension and the Stars, and to proceed therewith as before, and the Compasses will never excur; in the numbring of these hours, after 12 are numbred they are to begin again, and are numbred as before, and not with 13, 14, &c.

And this trouble may be shunned when there is but 12 hours by assu-

assuming any hour to be theStars hour,with such condition that the other foot may fall upon the Line;and the said assumed hour representing the Stars hour.; count from it the time duly in order, till you fall upon the other foot of the Compasses, and you will obtain the true time sought.

To turn common time, or the Suns hour into the Stars hour.

THis is the Converse of the former, take the distance between the Star and the quantity of the Suns Ascension, and set the Star foot to the Suns hour, letting the other fall the same way it stood before, and it shews the time sought.

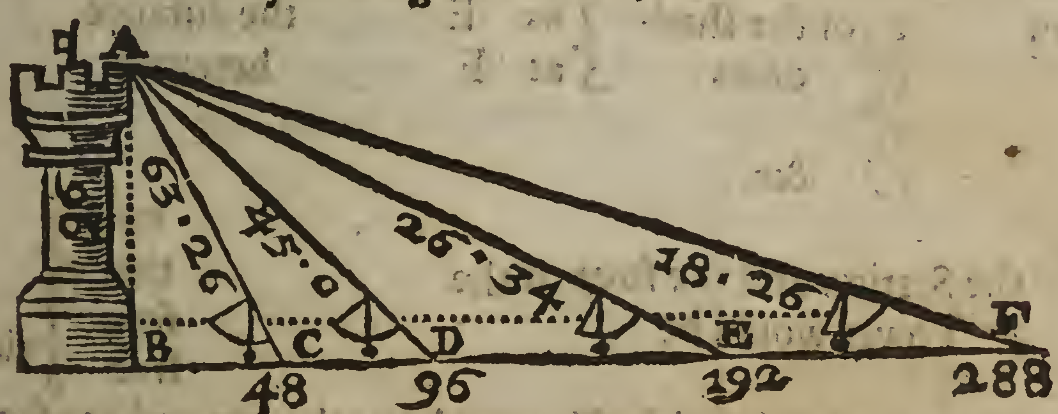
Of the Quadrat and Shaddows.

Both these as was shewed in theDescription of the Quadrant,are no other then a Table of natural Tangents to the Arks of the Limb and may supply the use of such a Cannon, though not with so much exactness, all the part of the Quadrate are to be estimated less then the Radius, till yon come against 45 ' of the Limb, where is set the figure of 1, and afterwards amongst the shadows is to be accounted more then the Radius, and so where the Tangent is in length

2	Radii as against	63^d	26^m	of the Limb.
3		71	34	
4		75	58	
5		78	42	

are set the figures 2, 3, 4, 5, and because they are of good use to be repeated on the other side of the Radius in the Quadrat, there they are not figured, but have only full points set to them, falling against the like Arks of the Limb from the right edge towards the left,as they did in the shadows from the left edge towards theright.

To find a hight at one Observation.



 Let

LEt A B reprefent a Tower, whofe Altitude you would take, go fo far back from it that looking through the fights of the Quadrant, the Thread may hang upon 45 degrees of the Limb, or upon 1, or the firft prick of the Quadrat, and the diftance from the foot of the Tower will be equal to the height of the Tower above the eye, which accordingly meafure, and thereto add the height of the eye above the ground, and you will have the Altitude of the Tower.

So if I fhould ftand at D and find the Thread to hang over 45ᵈ of the Limb, I might conclude the diftance between my Station and the Tower to be equal to the height of the Tower above my eye, and thence meafuring it find to be 96 yards, fo much would be the height of the Tower above the eye.

If I remove farther in till the Thread hang upon the

fecond / third / fourth / fifth } point of the Quadrat, then will the Altitude of the Tower above the level of the eye be

twice / thrice } as much as the diftance from the Tower is to the Station.
four / five } times

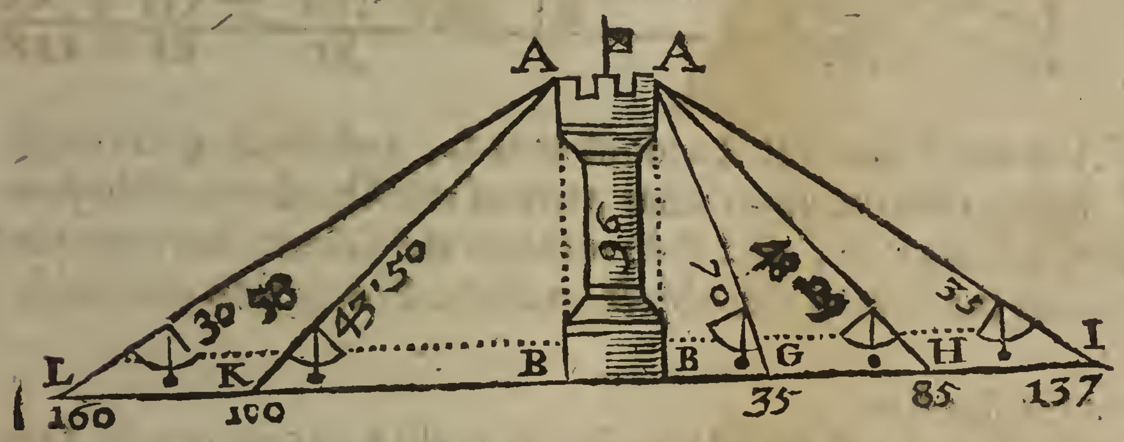
So removing to C; I find the Thread to hang upon the fecond point of the Quadrat, and meafuring the diftance of that Station from the Tower, I find it to be 48 yards, whence I may conclude the Tower is twice as high above my eye, and that would be 96 yards.

So if I fhould remove fo much back that the Thread fhould hang upon 2 / 3 / 4 / 5 } of the fhaddows &c. } as E / at F the diftance between the Station and the foot of the Tower would be twice / thrice } four / five } times as much as the height of the Tower above the eye, and confequently

if

if I should measure the distance between D and E where it hung upon 1 and 2 of the Shaddows, or between E and F, where it hung upon 2 and 3 of the shaddows, *&c* I shonld find it to be equal to the Altitude; but other ways of doing it when inaccessible will afterwards follow.

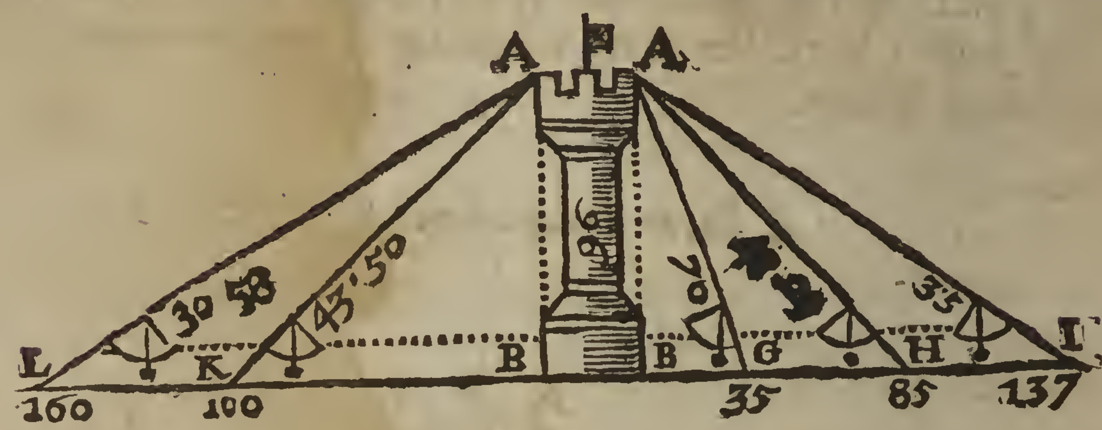
A Second way at one Station.



WIth any dimension whatsoever of a competent length, measure off from the foot of the Object, whether Tower or Tree, just 10 or 100, *&c.* of the said Dimensions, as suppose from B to K, I measure of an hundred yards; there look through the sights of the Quadrant to the Top of the Object at A, and what parts the Thread hangs upon in the Quadrat or Shadows, shews the Altitude of the Object in the said measured parts, and so at the said Station at K the Thread will hang upon 96 parts, shewing the Altitude A B to be 96 yards above the Level of the eye, and so if any other parts were measured off, they are to be multiplyed by the Tangent of the Altitude, or parts cut by the Thread, rejecting the Ciphers of the Radius, as in the next Proposition.

A

A third way by a Station at Random.



TAke any Station at Random as at L, and looking through the sights observe upon what parts of the Quadrat or Shadows the Thread falls upon, and then measure the distance between the Station and the foot of the object, and the Proportion will hold.

As the Radius
To the Tangent of the Altitude, or to the parts cut in the Quadrat or shadows,
So is the distance between the Station and the Object
To the height of the Object above the eye.

So standing at L, the Thread hung upon $30^{d} 58^{m}$ of the Limb, as also upon 600 of the Quadrat, the Tangent of the said Ark, and the measured distance L B was 160 yards, now then to work the former Proportion, multiply the distance by the parts of the Quadrat, and from the right hand of the product cut off three places, and you have the Altitude sought.

$$\begin{array}{r} 160 \\ 600 \\ \hline 96,000 \end{array}$$

In this Work the Radius or Tangent of 45^{d} is assumed to be 1000.

To measure part of an Altitude, as suppose from a window in a Tower to the top of the Tower, may be inferred from what hath been already said; first get the top of the Tower by some of the former

former ways, and then the height of the window which ſubſtract from the former Altitude and the remainder is the deſired diſtance between the window and the top of the Tower.

The former Proportion may alſo be inverted for finding of a diſtance by the height, or apprehending the Tower to lye flat on the ground, and ſo the height to be changed into a diſtance and the diſtance into a height, the ſame Rules will ſerve only the height of a Tower being meaſured, and from the top looking to the Object through the ſights of the Quadrant, what Angle the Thread hangs upon is to be accounted from the right edge of the Quadrant towards the left; but in taking out the Tangent of this Ark, after I have obſerved it, the Thread muſt be laid over the like Ark of the Quadrant from the left edge towards the right, and from the Quadrat or Shaddows the Tangent taken out by the Interſection of the Thread, and ſo to meaſure part of a diſtance muſt be done by getting the diſtances of both places firſt, and then ſubſtract the leſſer from the greater.

To find the Altitude of any Perpendicular, by the length of its ſhadow.

THis will be like the firſt Propoſition, with the Quadrant take the Altitude of the Sun, if in ſo doing the Thread hang over

the 1ſt
2d
3d
4th
5th
pricks in the Quadrat, the length of the

Shaddow is equal to
double
triple
four times
five times
the height of the Object Tree, or Perpendicular

whatever it be; But if it hang 1
2
3
4
5
in the ſhaddows

the

the highth of the object is equal to / double / triple / four, five times — the length of the

Shaddow which may happen where the Sun hath much Altitude, as in ſmall Latitudes, and ſo the length of the Shadow being forthwith meaſured, the height of the Gnomon may be eaſily attained.

If the Thread in obſerving the Altitude hang on any odd parts of the Quadrant or Shadows, the Proportion will hold as before.

As the Radius
To the length of the Shadow,
So the Tangent of the Suns Altitude, or the parts cut by the Thread
To the height of the Gnomon to be wrought as in the third Propoſition;
if the length of the Gnomon, and the length of its ſhadow were given; without a Quadrant we might obtain the Suns Altitude, for

As the length of the Gnomon
Is to the Radius
So is the Length of its Shadow
To the Tangent of the Complement of the Suns Altitude.

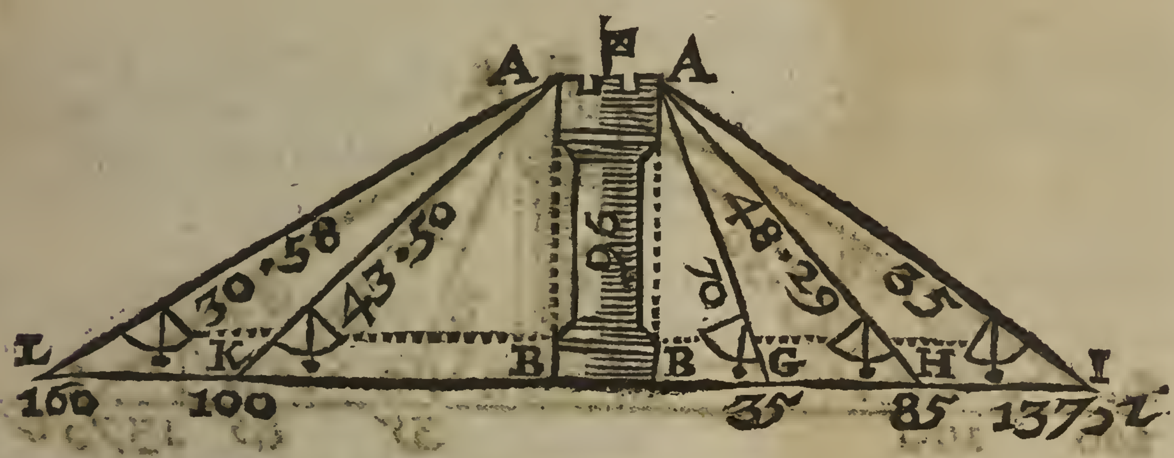
And the height of the Sun, and the length of the Gnomon aſſigned; We may find the length of the Shadow by inverting the Proportion aforeſaid.

As the Radius
To the length of the Gnomon
So the Cotangent of the Suns Altitude
To the Length of the Shadow.

To find an innacceſſible Height at two Stations.

Aſſume a Station any where, as at G, and there obſerve the Altitude of the Object A B which admit to be 70^d. Now it is well

well observed by Mr. *Phillips*, that if you remove so far back as that the Object may appear but half so high, as suppose at I it appears to have but 35^d of Altitude that then the distance between



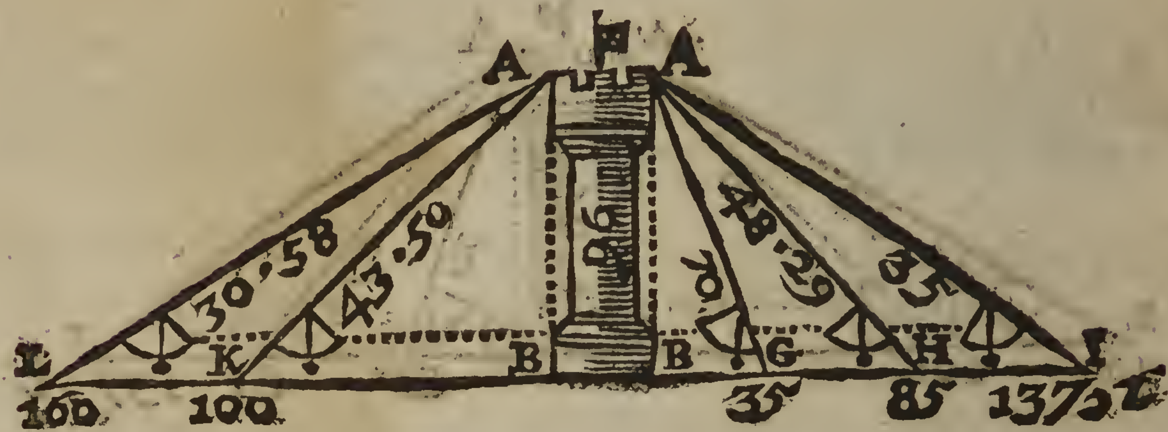
these two Stations G and I, is equal to the length of the Hipotenu Sall, or scaling Ladder A G; and this must needs be so, because the Acute Angle A G B being the Complement of the Obtuse Angle at G is equal to the Sum of the other two Acute Angles G A I and G I A, which Sum is likewise Complement of the Obtuse Angle at G to 180, but these Angles by Supposition being equal each to other will subtend equal Sides; Admit then I measure the distance G I, and find it to be 102 yards and a tenth, the Proportion to get the Altitude will hold

As the Radius
To the said measured distance
So is the Sine of the Angle at the nearest Station
To the Altitude of the Object.

To work this with the Pen, Out of the Line of Sines take the Sine of 70^d with Compasses, and measure it on the equal parts, where admit it reach to 94 parts,

Then multiply the said number by the distance 102, 1 and the Product will be 95, 974 from which cutting off three figures to the right hand the residue being 95, 974, is the Altitude sought *fere*, but should be 96 caused by omitting some fractionate parts in the distance which we would not trouble the Reader withall.

Another more general way by any two Stations taken at randome.



ADmit the first Station to be as before at G where the observed Altitude of the Object was 70^{d} and from thence at pleasure I remove to H, where observing again I find the Object to appear at $48^{d}\ 29^{m}$ of Altitude, and the measured distance between G and H to be 50 yards, a general Proportion to come by the Altitude in this case will hold.

As the difference of the Cotangents of the Arks cut at either Station.
Is to the Distance between the two Stations,
So is the Radius
To the Altitude of the Object or Tower.

To save the substracting of the two Arks from 90^{d} to get their Complements, I might have accounted them when they were observed from the right edge of the Quadrant towards the left, and have found them to have been 20^{d} and $41^{d}\ 31^{m}$; to work this Proportion lay the Thread over these two Arks in the Limb from the left edge towards the right, and take out their Tangents out of the Quadrat and Shadows, then substract the less from the greater, the remainder is the first tearm of the Proportion, being the Divisor in the Rule of three to be wrought by annexing the Ciphers of the Radius to the Distance; or as Multiplication in Decimalls; and then dividing by the first tearm the Quotient shews the Altitude sought,

41^{d}

5. *To find the Hipotenusal.*

Given both the Sides.

As one of the given Sides, To the other given Side:
So is the Radius, To the Tangent of the Angle opposite to the other side,
Then take the Complement of the Angle found for the other Angle.

Then, As the Sine of the Angle opposite to one of the given Leggs, or Sides,
Is to the given Sides:
So is the Radius, To the Hipotenusal.

6. *To find an Angle.*

Given the Hipotenusal, and one of the Sides.

As the Hipotenusal, Is to the given Side:
So is the Radius, To the Sine of the Angle opposite to the given side;
The Complement of the angle found, is the other Angle.

7. *To find an Angle.*

Given both the Sides.

As one of the given sides, To the other given Side:
So is the Radius, To the Tangent of the angle opposite to this other Side.
The Complement of the angle found is the other Angle.

Because the Sum of the Squares of the Leggs of a right angled Triangle is equal to the Square of the Hipoten[illegible] y 47 1 *Euclid.* Therefore the 2^d^ and [illegible] ormed.

To

2. Case, *To find a Side or Legg.*

Given the Hipotenusal, and other Legg.

To do it by Logarithmes, Add the Logarithm of the Sum of the Hipotenusa and Legg, to the Logarithm of their difference, the half Sum is the Logarithm of the Side fought.
Which in natural Numbers is to extract the Square root of the Product of the Sum and difference of the two Numbers given, namely, the Hipotenusal and Legg, which said root is equal to the side fought.

5. Case, *When both the Sides are given to find the Hipotenusal.*

To do it by Logarithmes, Substract the Logarithm of the lesser side from the doubled Logarithm of the greater, and to the absolute number answering to the remaining Logarithm add the lesser side the half sum of the Logarithmes of the Sum thus composed, and of the lesser side, is the Logarithm of the Hypotenusa fought,
Which work was devised, that the Proposition might be performed in Logarithmes; The same Operation by natural numbers, would be to Divide the Square of the greater of the Sides by the lesser, and to the Quotient to add the lesser Side, then multiply that Sum by the lesser Side, and extract the Square root of the Product for the Hipotenusal fought.

Cases of Oblique Plain Triangles.

1. *Data To find an Angle,*

TWo sides with an Angle opposite to one of them to find the Angle Opposed to the other side.
As one of the Sides, To the Sine of its opposite Angle :
So the other side; to the Sine of the angle opposed thereto.

If

If the Angle given be Obtuse, the Side opposite to it will be greater then either of the rest, and the other two Angles shall be Acute;

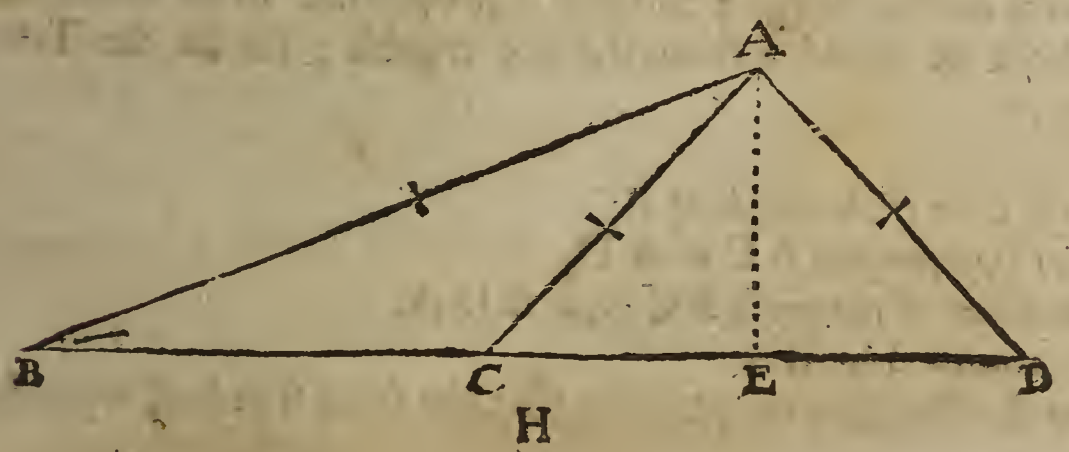
But if the given Angle be Acute, it will be doubtful whether the angle opposed to the greater side be Acute or Obtuse, yet a true Sine of the 4th Proportional.

The Sine found will give the angle opposite to the other given side, if it be Acute, and it will always be Acute when the given angle is Obtuse; But if it be fore-known to be Obtuse, the Arch of the Sine found substract from a Semicircle, and there will remain the Angle sought.

In this Case the quality or affection of the angle sought must be given, and fore-known, for otherwise it is impossible to give any other then a double answer, the Acute angle found, or its Complement to 180d, yet some in this Case have prescribed Rules to know it, supposing the third Side given which is not, if it were then in any Plain Triangle it would hold, That if the Square of any Side be equal to the Sum of the Squares of the other two Sides, the angle it subtends is a right angle, if less Acute, if greater Obtuse; but if three Sides are given we may with as little trouble by following Proportions come by the quantity of any angle, as by this Rule to know the affection of it.

Two Sides with an Angle opposite to one of them, to find the third Side.

In this Case as in the former, the affection of the angle opposite to the other given side, must be fore-known, or else the answer may be double or doubtful.



In

In the Triangle annexed there is given the ſides A B and A C with the angle A B C to find the angle oppoſite to the other given ſide, which may be either the angle at C or D, and the third ſide, which may be either B C, or B D; the reaſon hereof is becauſe two of the given tearms the Side A B, and the angle at B remain the ſame in both Triangles, and the other given ſide may be either A C or A D equal to it, the one falling as much without as the other doth within the Perpendicular A E.

The quality of the angle oppoſite to the other given ſide being known; by the former Caſe get the quantity, and then having two angles the Complement of their Sum to 180^{d} is equal to the third angle, the Caſe will be to find the ſaid ſide, by having choice of the other ſides, and their oppoſite angles as follows.

2. Two Angles with a Side oppoſite to one of them given to find the Side oppoſite to the other,

As the Sine of the angle oppoſed to the given Side,
Is to the given Side;
So is the Sine of the angle oppoſite to the ſide ſought,
To the ſide ſought.

Ricciolus in the *Trigonometrical* part of his late *Almageſtum Novum*, ſuggeſts in the reſolution of this Caſe, that if the ſide oppoſite to the Obtuſe angle be ſought, it cannot be found under three Operations; as firſt to get the quantity of the Perpendicular falling from the Obtuſe angle on its Oppoſite ſide, and then the quantity of the two Segments of the ſide, on which it falleth, and ſo add them together to obtain the ſide ſought; But this is a miſtake, and if it were true by the like reaſon a ſide oppoſite to an Acute angle, could not be found without the like trouble; for in the Triangle above,

As the Sine of the angle at B
Is to its Oppoſite ſide A C *or* A D,
So is the Sine of the angle B C A, *or* B D A;
To its oppoſite Side B A;

where the Reader may perceive that the ſame ſide hath oppoſite to it.

it both an Acute and an Obtuse angle, the one the Complement of the other to 180^d, the same Sine being common to both, for the Acute angle A C E is the Complement of the Obtuse angle B C A; but the angle at C is equal to the angle at D being subtended by equal Sides, and the Proportion of the Sines of Angles to their opposite Sides is already demonstrated in every Book of *Trigonometry*.

3. *Two Sides with the angle comprehended to find either of the other Angles.*

Substract the angle given from 180^l, and there remains the Sum of the two other Angles, then

As the Sum of the Sides given,
To tangent of the half sum of the unknown Angles:
So is the difference of the said Sides,
To Tangent of half the difference of the unknown Angles.

If this half difference be added to half sum of the angles, it makes the greater, if substracted from it, it leaves the lesser Angle

4. *Two Sides with the Angle comprehended, to find the third side.*

By the former Proposition one of the Angles must be found, and then
As the Sine of the angle found, is to its Opposite side:
So is the Sine of the angle given,
To the Side opposed thereto,

If two angles with a Side opposite to one of them be given, to find the side opposite to the third Angle is no different Case from the former, because the third angle is by consequence given, being the Complement of the two given angles to 180^l, and the like if two angles with the side between them were given.

5. *Three Sides to find an Angle.*

The Side subtending the Angle sought is called the Base.

As the Rectangle or Product of the half Sum of the three sides, and of the difference of the Base therefrom,
Is to the Square of the Radius:
So is the Rectangle of the difference of the Leggs, or conteining Sides therefrom, to the Square of the Tangent, to half the Angle sought.

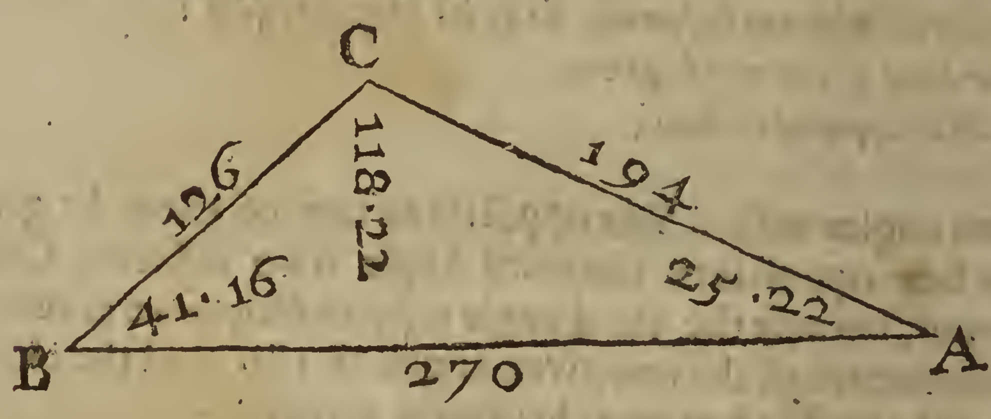
And by changing the third tearm into the place of the first.

As the Rectangle of the Differences of the Leggs from the half Sum of the 3 sides,
Is to the Square of the Radius:
So is the Rectangle of the said half sum, and of the difference of the Base there from, To the Square of the Tangent of an angle, which doubled is the Complement of the angle sought to 180^{d}, Or the Complement of this angle doubled is the angle sought.

To Operate the former Proportion by the Tables.

From the half Sum of the three Sides substract each Side severally, then from the Sum of the Logarithmes of the Square of the Radius (which in Logarithms is the Radius doubled) and of the differences of the Sides containing the angle sought:

Substract the Snm of the Logarithms of the half sum of the three sides, and of the difference of the Base therefrom the half of the remainder is the Logarithm of the Tangent of half the angle sought.



Example.

In the Triangle A B C let the three sides be given to find the Obtuse angle at C.

Diffe-

Differences from the half sum.

B C	126	Leggs	101		2, 0043214
A C	194		166		2, 2278867
A B	270	Base	25	Logarithms	
				1, 3979400 Sum with double Radius	24,2322081
Sum	590				
half sum	295			2, 4698220	3,8677620
				3, 8677620	20,3644461
					10,1822230 half

Tangent of 56^d $41'$ which doubled is 113^d $22'$ the angle sought.

For the 2^d Proportion for finding an Angle the Operation varies; to the former sum add the double Radius, and from the Aggregate substract the latter sum, the half of the Relique is the Logarithm of the Cotangent of half the Angle sought.

23, 8677620
4, 2322081
―――――
19, 6355539
9 8177769 Tangent of 23^d $19'$ the Complement whereof is 56^d $41'$ which doubled makes 113^d $22'$ as before

The Tables here made use of are Mr. Gellibrands.

It may be also found in Sines.

As the Rectangle of the Containing Sides or Leggs,
Is to the Square of the Radius;
So the Rectangle of the differences of the said Leggs from half Sum of the three Sides,
To the Square of the Sine of half the Angle sought.

Or, We may find the Square of the Cosine of half the Angle sought by this Proportion.

As

As the Rectangle of the containing Sides,
Is to the Square of the Radius:
So the Rectangle of the half sum of the three Sides, and of the difference of the Base there from,
To the Square of the Cosine of half the Angle sought, viz. *To the Square of the Sine of such an Ark, as being doubled is the Complement of the Angle sought to* 180^d, *or the Complement of the found Ark doubled is the angle sought.*

All these Proportions hold also in Spherical Triangles if instead of the bare names of Sides you say the Sines of those Sides, and this Case may also be resolved by a Perpendicular let fall; but the Reader need not trouble himself with name nor thing in no Case of Plain or Sphœrical Triangles, but when two of the given Sides or Angles are equal.

If the Square of the Radius and the Square of the Sine of an Ark be both divided by Radius the Quotients will be the Radius, and the half Versed Sine of twice that Arch, and in the same Proportion that the two tearms propounded, as will afterwards be shewed, upon this Consideration may any of these Compound Proportions be reduced to single Tearms for Instrumental Operations, by placing the two Tearms of the first Rectangle, as two Divisors in two single Rules of three, and the tearms of the other Rectangle as middle tearms.

An Example in that Proportion for finding an Angle in the Sines:

As one of the Leggs or including Sides,
Is to the difference thereof from the half sum of the three Sides:
So is the Difference of the other Legg therefrom,
To a fourth Number.

Again.

As the other Legg, To that fourth Number:
So is the Radius, To the half Versed Sine of the Angle sought;
And so is the Diameter or Versed Sine of 180^d *Or Secant of* 60^d *To the Versed Sine of the Angle sought.*

And

And upon this Consideration, that the fourth Tearm in every direct Proportion bears such Proportion to the first Tearm, as the Rectangle or Product of the two middle Tearms doth to the Square of the first Tearm, as may be demonstrated from 1 *Prop.* 6. *Euclid.* We may place the Radius or Diameter in the first place of the first Proportion, and still have the half or the whole Versed Sine in the last place as before, therefore

As the Radius; To one of the given Leggs:
So the other given Legg, To a fourth Number, which will bear such Proportion to the Radius, as the Rectangle of the two given Leggs doth to the Square of the Radius, then it holds,

As that fourth number, To the Radius:
So the Rectangle of the difference of one of the Leggs from the half sum of the three sides; To the Square of the Sine of half the Angle sought, and omitting the Raidius, it will hold;

As that fourth Number, To the differences of the Leggs from the Leggs from the half sum of the three Sides: So is the difference of the other Legg therefrom, To the half Versed Sine of the Angle sought.

And if the Diameter had been in the first place, then would the whole Versed Sine have been in the last.

This Case when 3 Sides are given to find an Angle is commonly resolved by a Perpendicular let-fall, it shall be only supposed and the Cannon will hold,

As the Base or greater Side,
To the sum of the other Sides:
So is the difference of the other Sides,
To a fourth Number, which taken out of the Base, half the remainder is the lesser Segment, and the said half added to this fourth Number is the greater Segment.

Again

Again.

As the greater of the other Sides, To Radius:
So is the greater Segment,
To the Coſine of the Angle adjacent thereto,

Or,

As the leſſer of the other Sides, To Radius:
So is the leſſer Segment,
To the Coſine of its adjacent Angle:

This for either of the Angles adjacent to the Baſe or greateſt ſide but for the angle oppoſite to the greateſt ſide, which may be ſometimes Obtuſe, ſometimes Acute; not to multiply Directions, the Reader is remitted to the former Cannons, or to find both the Angles at the Baſe firſt, and by conſequence the third angle is alſo given, being the reſidue of the ſum of the former from a Semicircle.

In right angled Spherical Triangles.

By the affection of the Angles to know the Affection of the Hipotenuſal, and the Converſe.

If one of the angles at the Hipotenuſal be a right angle	The Hipotenuſal will be	A Quadrant.	
If both be of the ſame kind,		leſſer	then a Quadr.
If of a different kind,		greater	

By the Hipotenuſal to find the affection of the Leggs, and the Converſe.

If the Hipotenuſal be	A Quadrant	one of the Legs wil be a Quadrant:
Leſs then		The Leggs, and their Oppoſite Angles wil be leſs then Quadrant:
Greater,		One Legg will be greater, & the other leſs then the Quadrant:

The

The Leggs of a right angled Sphærical Triangle are of the same Affection as their Opposite Angles and the Converse:

<table>
<tr><td rowspan="3">If a Legg be</td><td>a Quadrant</td><td rowspan="3"></td><td rowspan="3">the Angle</td><td rowspan="3">opposite thereto will be</td><td>a Quadr.</td></tr>
<tr><td>less then a Quadrant</td><td>Acute</td></tr>
<tr><td>greater then a Quadrant</td><td>Obtuse.</td></tr>
</table>

All these Affections are Demonstrated in *Snellius*; I shall add some more from *Clavius de Astrolabio*, and the Lord *Nepair*.

The three Sides of every Sphærical Triangle are less then a Whole Circle.

In an Oblique angular Triangle, If two Acute / Obtuse Angles be equal the Sides opposite to them shall be lesser / greater then Quadrants. *Reg.* 10..11,4

If an Oblique angular Triangle, if two Acute / Obtuse Angles be unequal the side opposite to the lesser / greater shall be lesser / greater then a Quadrant *Reg.* 12,13.4.

An Acute angular Triangle hath all its Angles Acute, and each side less then a Quadrant.

Two sides of any Spherical Triangle are greater then the third.

If a Sphærical Triangle be both right angled and Quadrantal, the sides thereof are equal to their Opposite Angles.

If it hath three right Angles, the three sides of it are Quadrants.

If it have two right Angles, the two sides subtending them are Quadrants and the contrary, and if it have one right angle, and one side a Quadrant, it hath two right angles, and two Quadrental sides.

Any side of a Sphærical Triangle being continued, if the other sides together

<table>
<tr><td>are equal to</td><td rowspan="3">then</td><td>A Semicircle</td><td>equal to</td><td rowspan="3">then</td><td rowspan="3">the inward opposit angle on the</td></tr>
<tr><td>lesser</td><td rowspan="2">the outward angle wil be</td><td>lesser</td></tr>
<tr><td>greater</td><td>greater</td></tr>
</table>

side continued.

If any Sphœrical Triangle have two Sides equal to
leſſer } then } a Se-
greater
micircle, the two angles at the Baſe or third Side will be
equal to
leſſer } then } two right Angles.
greater

In every right angled Spherical Triangle having no Quadrantal Side, the angle Oppoſite to that Side that is leſs then a Quadrant is Acute, and greater then the ſaid Side;

But that angle which is Oppoſite to the Side, that is greater then a Quadrant, is Obtuſe; and leſs then the ſaid Side.

In every right angled Spherical Triangle all the three angles are leſs then 4 right angles, that is the two Oblique angles are leſs then 3 right angles, or 270^{d}.

In a right angled equicrural Triangle, if the two equal angles be Acute, either of them will be greater then 45^{d}, but if Obtuſe leſs then 135^{d}.

In every right angled Sphœrical Triangle either of the Oblique angles is greater then the Complement of the other, but leſs then the difference of the ſame Complement from a Semicircle.

Two angles of any Sphœrical Triangle are greater then the difference between the third angle and a Semicircle, and therefore any ſide being continued, the outward angle is leſs then the two inward oppoſite angles.

The ſum of the three Angles of a Sphœrical Triangle is greater then two right angles, but leſs then 6.

In Spherical Triangles, that angle which of all the reſt is neareſt in quantity to a Quadrant, and the ſide ſubtending it are doubtful, Whether they be of the ſame, or of a different affection, unleſs foreknown, or found by Calculation;

But the other two more Oblique angles are each of them of the ſame kind as their Oppoſite Sides, which Mr *Norwood* thus propounds, *Two Angles of a Spherical Triangle, ſhall be of the ſame affection as their Oppoſite Sides*, and to this purpoſe,

If any Side of a Triangle be nearer to a Quadrant then its oppoſite Angle, two Angles of that Triangle (not univerſally any two) ſhall

ſhall be of the ſame kind, and the third greater then a Quadrant.

But if any Angle of a Triangle be nearer to a Quadrant then its oppoſite ſide, two Sides of that Triangle (not univerſally any two) ſhall be of the ſame kind, and the third leſs then a Quadrant.

In any Sphœrical Triangle, if one of the angles be ſubſtracted from a Semicircle, and the reſidue ſo found ſubſtracted from a Whole Circle, the Ark found by this latter Subſtraction, will be greater then the Sum of the other two Angles.

In every Sphœrical Triangle the difference between the ſum of two angles howſoever taken, and a whole Circle or 4 right angles is greater then the difference between the other Angle and a Semicircle; The demonſtration of moſt of theſe Affections are in *Clavius* his Comment on *Theodoſius*, or in his Book *de Aſtrolabio*, where ſhewing how to project in Plano all the Caſes of Sphœrical Triangles, and ſo to meaſure the ſides and Angles, he delivers theſe Theorems to prevent ſuch Fictitious Triangles as cannot exiſt in the Sphere.

The 16 Cases of right Angled Sphœrical Triangles, Translated from *Clavius de Astrolabio.*

1. *To find an Angle.*

Given the Hipotenusal and side opposite to the Angle sought.

As the Sine of the Hipotenusal, To Radius: So the Sine of the given side To the Sine of the Angle sought.

Or, *As Radius, To sine of the Hipotenusal: So the Cosecant of the side, To the Cosecant of the Angle.*

As Radius, To sine of the side: So Cosecant of the Hipotenusal, To sine of the angle.

As Cosecant of the side, To Radius: So Cosecant of the Hipotenusal, To Cosecant of the angle.

As Cosecant Hipotenusal, To Radius: So Cosecant of the side, To Cosecant of the angle.

As the Sine of the side to Radius: So sine of the Hipotenusal, To Cosecant of the Angle.

The Angle found will be Acute if the Side given be less then a Quadrant, Obtuse if greater.

2. *To find an Angle.*

Given the Hipotenusal and side adjacent to the Angle.

As Radius, To Cotangent Hipotenusal: So tangent of the side, To Cosine of the angle.

As tangent Hipotenusal. To Radius, So tangent of the Side. To Cosine of the angle.

As the Cotangent of the side, To Radius: So Cotangent Hipotenuse, To Cosine of the angle.

As Radius, To Cotangent ſide : So tangent Hipotenuſa, To Secant of the angle.

As Cotangent Hipotenuſal : To Radius : So Cotangent ſide, To Secant of the angle.

As the tangent of the ſide, To Radius :: So tangent Hipot: To Secant of the angle.

The Angle found will be Acute, if both the Hipotenuſal and the given ſide be greater, or leſs then a Quadrant, but Obtuſe if one of them be greater, and the other leſs.

3. *To find an Angle.*

Given the Hipotenuſal, and either of the Oblique Angles

As the Radius, To Coſine Hip: :: So Tangent of the given angle : To the Cotang of ſought angle

As the Cotangent of the given angle : To Radius :: So Coſine of the Hipot: To the Cotang of the ſought angle.

As the Coſine of the Hipot: To Radius :: So Cotang: of the given angle : To the tang. of the ang: ſought

As the Radius, To Secant Hip: :: So Cotangent of the given angle : To the tang of the angle ſought

As Secant of the Hipotenuſal : To Radius :: So the tangent of the angle given : To the Cotan: of the angle ſought

As the tangent of the given angle : To Radius :: So Secant of the Hipotenuſal : To tangent of the angle ſought.

The angle found will be Acute, if the Hipotenuſal be leſs then a Quadrant, and the given angle Acute; or if the Hipotenuſal be greater then a Quadrant, and the given angle Obtuſe; And the ſaid angle will be Obtuſe, if the Hipotenuſal be leſs then a Quadrant, and the given angle Obtuſe; Or if the Hipotenuſal be greater then a Quadrant, and the given angle Acute.

4. *To find an Angle.*

Given the Side oppoſite to the Angle ſought, and the other Oblique Angle.

As

As the Radius	*: To ſine of the angle given*	*:: So is the Coſine of the given ſide*	*: To the Coſine of the angle ſought*
As the Radius	*: To Coſecant of the given angle*	*:: So Secant of the ſide given*	*: To Secant of the angle ſought*
As the ſine of the given angle	*: To Radius*	*:: So the Secant of the given ſide*	*: To the Secant of the angle*
As the Coſine of the given ſide	*: To Radius*	*:: So the Coſecant of the given angle*	*: To the Secant of the angle ſought*
As the Coſecant of the given angle	*: To Radius :*	*So Coſine of the given ſide*	*: To the Coſine of the angle ſought*
As the Secant of the given ſide	*: To Radius ::*	*So ſine of the given angle*	*: To the Coſine of the angle ſought*

The angle found will be Acute, if the ſide given be leſs then a Quadrant, Obtuſe if greater.

5. *To find an Angle.*

Given a ſide adjacent to the angle ſought, and the other Oblique angle, if it be foreknown whether the angle ſought be Acute or Obtuſe, or whether the Baſe or other ſide not given be greater, or leſſer then a Quadrant.

As the Coſine of the given ſide	*: To Radius ::*	*So Coſine of the given angle*	*: To the ſine of the angle ſought*
As the Radius	*: To Secant of the given ſide*	*:: So Coſine of the given angle*	*: To Sine of the angle ſought*
As the Radius	*: To Secant of the given angle*	*:: So Coſine of the given ſide*	*: To Coſecant of ſought angle*
As the Coſine of the given angle	*: To Radius ::*	*So Coſine of the given ſide*	*: To the Coſecant of the angle ſought*
As the Secant of the given ſide	*: To Radius ::*	*So is the Secant of the given angle*	*: To the Coſecant of the angle ſought*
As the Secant of the given angle	*: To Radius ::*	*So is the Secant of the given ſide*	*: To the Sine of the angle ſought*

The angle found will be Acute, if the ſide not given be leſs then a Quadrant, Obtuſe if greater.

In

In like manner if the Hipotenuſal be leſs then a Quadrant, and the given angle Acute; Or if the Hipotenuſal be greater then a Quadrant, and the given angle Obtuſe, the angle found will be Acute.

But if the Hipotenuſal be leſs then a Quadrant, and the given angle Obtuſe, or if the Hipotenuſal be greater then a Quadrant, and the given angle Acute, the angle found will be Obtuſe,

6. *To find an Angle.*

Given both the Leggs.

As Radius :	*To ſine of the ſide adjacent to ſought angle ::*	*So Cotang of the ſide oppoſite to the angle ſought :*	*To Cotang of the angle ſought*
As the ſine of the ſide adjacent to ſought angle :	*To Radius ::*	*So the tangent of the ſide oppoſite to the ſought angle :*	*To the tang: of the angle ſought*
As the Tangent of the ſide oppoſite to ſought angle :	*To Radius ::*	*So the ſine of the ſide adjacent to the ſought angle :*	*To Cotang: of the angle ſought*
As the Radius :	*To Coſecant of the ſide adjacent to ſought angle ::*	*So the tangent of the ſide oppoſite thereto :*	*To the tang: of the angle ſought*
As the Coſecant of the ſide adjacent to the angle ſought :	*To Radius ::*	*So is the Cotang of the ſide oppoſite to the angle ſought :*	*To Cotang: of the ſought angle*
As the Cotangent ſide oppoſite to the ſought angle :	*To Radius ::*	*So Coſecant of the ſide adjacent to the angle ſought :*	*To the tang: of the angle ſought*

The Angle found will be Acute, if the ſide oppoſite to the angle ſought be leſs then a Quadrant, but Obtuſe if greater.

7. *To find a Side or Legg.*

Given the Hipotenuſal and the other Legg.

As the Coſine of the given ſide :	*To Radius ::*	*So Coſine of the Hipotenuſal :*	*To Coſine of the ſide ſought*
As Radius :	*To Secant of the ſide ::*	*So Coſine of the Hipotenuſal :*	*To Coſine of the ſide ſought*

As

As Coſine Hipotenuſal:	*To Radius ::*	*So Coſine of the : given ſide*	*To Secant of the ſide ſought.*
As Radius :	*To Secant of the :: Hipotenuſal*	*So Coſine of the : given ſide*	*To Secant of the ſide ſought*
As the Secant of the : Hipotenuſal	*To Radius ::*	*So Seca't of the : given ſide*	*To Coſine of the ſide ſought*
As the Secant of : the given ſide	*To Radius : :*	*So Secant of the : Hipotenuſal*	*To the Secant of the ſide ſought*

The ſide ſought will be leſs then a Quadrant, if both the Hipotenuſal and given ſides be leſs then Quadrants, but greater then a Quadrant if either the Hipotenuſal be greater, and the given ſide leſs, Or if the Hipotenuſal be leſs, and the given ſide greater.

8. *To find a Side.*

Given the Hipotenuſal, and an Angle oppoſite to the ſide ſought.

As the Radius :	*To ſine of the : : Hipotenuſal*	*So ſine of the : given angle*	*To the ſine of the ſide ſought.*
As Radius :	*To Coſecant of the :: Hipotenuſal*	*So the Coſecant of : the given angle*	*To Coſecant of the ſi'e ſought*
As the ſine of the : Hipotenuſal	*To Radius ::*	*So Coſecant of the : given angle*	*To Coſecant of the ſide ſought*
As the Coſecant of. the given angle	*To Radius ::*	*So the ſine of the : Hipotenuſal*	*To the ſine of the ſide ſought*
As the ſine of the : given angle	*To Radius ::*	*So the Coſecant of : the Hipotenuſal*	*To the Coſecant of the ſi le ſought*

The ſide found will be leſs then a Quadrant, if the Angle oppoſite thereto be Acute, but greater if Obtuſe.

9, *To find a Side.*

Given the Hipotenuſal and Angle adjacent to the Side ſought

As the Rad us :	*To Coſine of the :: given angle*	*So tangent of the : Hipotenuſal*	*To the tangent of ſide ſought.*

As

As the Coſine of the : *given angle*	*To Radius* ::	*So Cotang nt of* : *the Hipotenuſal*	*To Cotang of the ſide ſought*
As the Cotangent of : *Hipotenuſal*	*To Radius* ::	*So the Coſine of* : *the given angle*	*To tangent of the ſide ſought*
As the Radius :	*To Secant of* : *the angle*	; *So Cotangent of* : *the Hipotenuſal*	*To Cotang of the ſide ſought*
As the Secant of the : *given angle*	*To Radius* ::	*So tangent of the* : *Hipotenuſal*	*To the tangent of the ſide ſought*
As the tangent of : *the Hipotenuſal*	*To Radius* ::	*So Secant of the* : *given angle*	*To Cotang* : *of the ſide ſought*

The *Side* ſought will be leſs then a Quadrant, if the Hipotenuſal be leſs then a Quadrant, and the given angle Acute: Or if the Hipotenuſal be greater the*n* a Quadrant, and the given angle Obtuſe; But it will be greater then a Quadrant, if the Hipotenuſal be leſs then a Quadrant, and the given angle Obtuſe: Or if the Hipotenuſal be greater then a Quadrant, and the given angle Acute.

10. *To find a Side.*

Given a Side, and an Angle adjacent to the ſought ſide.

Provided it be foreknown whether the ſide ſought be greater or leſs then a Quadrant, or whether the other angle not given be Acute or Obtuſe; or finally whether the Hipotenuſal be greater or leſs then a Quadrant.

As the Radius :	*To the Cotangent of* : *the given angle*	: *So tangent of the* : *given ſide*	*To the Sine of the ſide ſought*
As the tangent of the : *given angle*	*To Radius* ::	*So tangent of the* : *given ſide*	*To the ſine of the ſide ſought*
As the Cotang of the : *given ſide*	*To Radius* ::	*So the Cotang of* : *the given angle*	*To the ſine of the ſide ſought*
As Radius :	*To Cotangent of the* ; *given ſide*	*So tangent of the* : *given angle*	*To Coſecant of the ſide ſought*
As the tangent of : *the given ſide*	*To Radius* ::	*So tangent of the* : *given angle*	*To the Coſecant of the ſide ſought*

As

As the Cotangent of the given angle	*: To Radius ::*	*So Cotang of the given ſide*	*: To the Coſecant of the ſide ſought*

The Side found will be leſs then a Quadrant, if the angle oppoſite thereto and not given be Acute, but greater if it be Obtuſe; In like manner it will be leſs, if the Hipotenuſal be leſs then a Quadrant, and the ſide given alſo leſs then a Quadrant:

Or if the Hipotenuſal be leſs then a Quadrant, and the given ſide greater, the ſide found will be greater then a Quadrant; Laſtly, if both the Hipotenuſal, and the ſide given be greater then a Quadrant, the ſide found will be leſs then a Quadrant; but greater if the Hipotenuſal be greater then a Quadrant, and the given ſide leſs.

11. *To find a Side.*

Given a Side, and an Angle oppoſite to the Side ſought

As Radius	*: To ſine given ſide ::*	*So tangent given angle*	*: To tangent of the ſid ſought*
As the ſine of the given ſide	*: To Radius ::*	*So Cotangent of the given angle*	*: To Cotangent of the ſide ſought*
As the Cotangent of the given angle	*: To Radius ::*	*So ſine of the given ſide*	*: To the tangent of the ſide ſought*
As the tangent of the given angle	*: To Radius ::*	*So Coſecant of the given ſide*	*: To Cotangent of the ſide ſought*
As the Radius	*: To Coſecant of the given ſide ::*	*So Cotang of the given angle*	*: To Cotangent of the ſide ſought*
As the Coſecant of the given ſide	*: To Radius ::*	*So tangent of the given angle*	*: To tangent of the ſide ſought*

The ſide found will be leſs then a Quadrant, if the given Angle oppoſite thereto be Acute, but greater if Obtuſe.

12. *To find a Side.*

Given both the Oblique Angles.

As the ſine of the angle adjacent to ſide ſought :	*Is to Radius ::*	*So Coſine of the angle oppoſite to ſide ſought :*	*To Coſine of the ſide ſought*
As the Radius :	*To Secant of the angle oppoſite to ſide ſought ::*	*So ſine of the angle adjacent to the ſide ſought :*	*To Secant of the ſide ſought*
As Radius :	*To Coſecant of the angle adjacent to ſide ſought ::*	*So Coſine of the angle oppoſite to ſide ſought :*	*To Coſine of the ſide ſought*
As the Coſine of the angle oppoſite to ſide ſought :	*To Radius ::*	*So ſine of the angle adjacent to the ſide ſought :*	*To Secant of the ſide ſought*
As the Secant of the angle oppoſite to ſide ſought :	*To Radius ::*	*So Coſecant of the angle adjacent to the ſide ſought :*	*To Coſine of the ſide ſought*
As Coſecant of the angle adjacent to ſide ſought :	*To Radius ::*	*So Secant of the angle oppoſite to ſide ſought :*	*To Secant of the ſide ſought*

The ſide found will be leſs then a Quadrant, if the given angle Oppoſite thereto be Acute, but greater if Obtuſe.

13. *To find the Hipotenuſal.*

Given a ſide and au Angle adjacent thereto.

As the Radius :	*To Coſine of the given angle ::*	*So Cotangent of the given ſide :*	*To Cotang of the Hipotenuſal*
As the Coſine of the given angle :	*To Radius ::*	*So tangent of the given ſide :*	*To tangent of the Hipotenuſal*
As the tangent of the given ſide :	*To Radius ::*	*So Coſine given angle :*	*To Cotangent of the Hipotenuſal*
As Radius :	*To Secant of the given angle ::*	*So tangent of the given ſide ;*	*To the tangent of the Hipotenuſal*
As the Secant of the given angle :	*To Radius ::*	*So Cotang of given ſide.*	*To the Cotang of the Hipotenuſal*
As the Cotangent of given ſide :	*To Radius ::*	*So Secant of the given angle :*	*To tangent of the Hipotenuſal*

The Hipotenuſal found will be leſs then a Quadrant, if the given ſide be leſs then a Quadrant, and the angle given adjacent thereto Acute; As alſo if the given ſide be greater then a Quadrant, and the given angle adjacent thereto be Obtuſe.

But it will be greater then a Quadrant, if the given ſide be greater then a Quadrant, and the given Angle adjacent thereto Acute; As alſo when the given ſide is leſs then a Quadrant; and the given Angle Obtuſe.

14. *To find the Hipotenuſal*

Given a Side, and an angle Oppoſite thereto.

If it be fore-known whether the Hipotenuſal be greater or leſs then a Quadrant, or whether the other angle not given be Acute or Obtuſe; Or laſtly, whether the other ſide not given, be greater or leſs then a Quadrant.

As the ſine of the given angle :	*To Radius* ::	*So ſine of the given ſide* :	*To the ſine of the Hipotenuſal*
As the Radius :	*To Coſecant of angle given* ::	*So ſine of the given ſide* :	*To the ſine of the Hopotenuſal*
As Radius :	*To Coſecant given ſide* ::	*So ſine of the given angle* :	*To Coſecant of the Hipotenuſal*
As the ſine of the given ſide :	*To Radius* ::	*So ſine of the given angle* :	*To Coſecant of the Hipotenuſal*
As Coſecant of given ſide :	*To Radius* ::	*So Coſecant of the given angle* :	*To ſine of the Hipotenuſal*
As Coſecant of the given angle :	*To Radius* ::	*So the Coſecant of the given ſide* :	*To Coſecant of the Hipotenuſal*

The Hipotenuſal found will be leſs then a Quadrant, if both the Oblique Angles be Acute or Obtuſe, or if both the ſides be greater or leſs then Quadrants.

It will alſo be greater then a Quadrant, if one of the Oblique Angles be Acute, and the other Obtuſe, or if one of the ſides be leſs, and the other greater then a Quadrant.

As

15. *To find the Hipotenusal,*

Given both the sides, distinguished by the names of first and second.

As Radius : To Cosine 1st *side* :: *So Cosine* 2d *side : To Cosine of the Hipotenusal*
As Radius : To Secant 1st *side* :: *So Secant* 2d *side : To Secant of the Hipotenusal*
As Secant 1st *side : To Radius* :: *So Cosine of the* 2d *side : To Cosine Hipotenusal*
As Secant 2d *side ; To Radius* :: *So Cosine* 1st *side . To Cosine of the Hipotenusal*
As Cosine 1st *side : To Radius* :: *So Secant* 2d *side : To Secant of the Hipotenusal*
As Cosine 2d *side : To Radius* :: *So Secant* 1 *side : To the Secant of the Hipotenusal*

The Hipotenusal found will be less then a Quadrant, if both the Sides are less or greater ; But otherwise , it will be greater , if one be less and the other greater.

16. *To find the Hipotenusal.*

Given both the Oblique Angles, distinguisht by the names of the first and second.

As Radius : To Cotangent 1st *angle* :: *So Cotangent* 2d *angle : To Cosine of the Hipot:*
As the tangent 1st *angle : To Radius* :: *So Cotangent* 2d *angle : To Cosine of Hipoten:*
As tangent of 2d *angle : To Radius* :: *So Cotangent* 1st *angle : To Cosine Hipotenusal*
As the Radius : To tangent 2d *angle* : : *So tangent* 1st *angle : To Secant of the Hipot:*
As Cotangent 2d *angle : To Radius : So tangent* 1st *angle : To Secant of the Hipotenus:*
As the Cotangent 1st *angle : To Radius* :: *So Tangent* 2d *angle : To Secant Hipotenus:*

The

The Hipotenusal found will be less then a Quadrant, if both the Oblique angles be Acute or Obtuse, but greater if one of them be Acute, and the other Obtuse.

I shall not spend time to shew Examples of all these Cases, but shall onely instance in an Example or two. In the Right angled Sphœrical Triangle P S N, let the side P N represent the Poles height, the side S P the Complement of the Suns declination, the side S N the Suns Amplitude of rising from the North Meridian, the *Angle* S P N the time of Suns rising from Midnight, and the angle P S N the angle of the Suns Position; and in it let there be given the side P N 51^{d} 32', and the side P S the Complement of the Suns Declination to find the angle S P N the time of the Suns rising; then in this Case there is given the Hipotenusal, and the side adjacent to the angle sought to find the said angle; and this is the 2^{d} Case, whence the Proportion taken is,

As Radius, To **Cotangent** *of the* **Hipotenusal**: *So the Tangent of the given side, To the Cosine of the angle sought*; and so the Proportion to find the time of Suns rising will be

As the Radius, To the tangent of the Suns declination:
So the tangent of the Latitude, To the Sine of the time of Suns rising before 6 in Summer, or after it in Winter, the Complement whereof is the time of its rising from Midnight.

Tangent 13^{d} Suns declination —— 936336
Tangent 51^{d} 32' the Latitude — 1, 009922

Sine 16^{d} 53' ——————, 946328 the Compl: of which Ark is 73, 6, which converted into time shews that the Sun riseth in our Latitude when he hath 13^{d} of North Declination at 52^{m} and a half past 4 in the morning *ferè*.

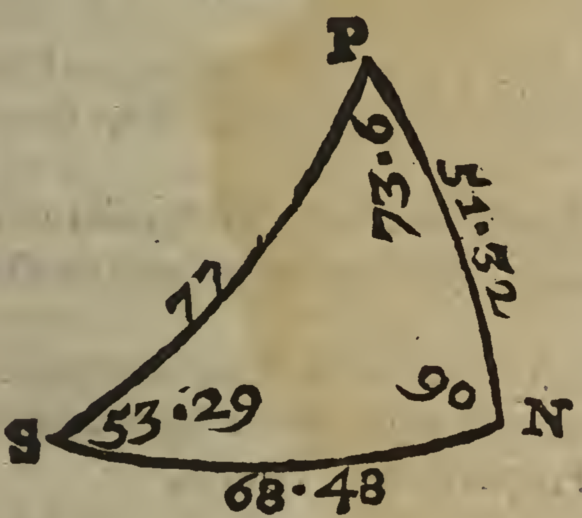
By the same *Data* we may find the Side S N the Suns Amplitude of rising or setting, and this will agree with the 7th Case, for here is given the Hipotenusal, and one of the Leggs to find the other Leg the Proportion will be,

As

As the Cofine of the given fide : To Ra ius : So the Cofine of the Hipotenufal, To the Cofine of the fide fought ; that is in this Cafs,
As the Cofine of the Latitude, To Radius : So is the Sine of the Suns Declination, To the Sine of his Amplitude from the Eaft or Weft.

Example,	*Logme.*
Sine 13d + Radius is	1,935208
Sine 38d 28'	979383
Sine 21d 12'	955825

theComplement whereof, *viz.* 68d 48' is the fide S N fought, and this Proportion is of good ufe to obtain the Variation of the Compafs at Sea by the Suns Coaft of rifing; More Examples need not be given, the Reader may try over all the Cafes by the Calculated Triangle annexed.



Some may fay here are more Proportions then needs, efpecially feeing there are no Logarithmical Tables of Secants; but *Alterna amant Camenæ*, they have not hitherto been publifhed in Englifh; the Inftruments to be treated of will have Secants; befides in fome Cannons there are Tables of the Arithmetical Complements of the Logarithmical Sines and Cofines, which augmented by Radius, are the Logarithmical Secants of the Complements of thofe Arks to which they do belong; and for Inftruments, efpecially Quadrants, a Proportion having Tangents or Secants many times cannot be Operated on the Quadrant without changing the Proportion, by reafon thofe Scales cannot be wholy brought on, being infinite;

Now

Now the chief Grounds for varying Proportions, are built upon a few Theorems.

1. That the Rectangle or Product of a Tangent, and its Complement is equal to the Square of the Radius, or which is all one, that the Radius is a mean Proportional between the Tangent of an Arch, and the Tangent of its Complement, that is,

As the Tangent of an Arch, To Radius: So Radius, To tangent of that Arks Complement,

And by Inversion.

As the Cotangent of an Arch, To Radius: So Radius, To tangent of that Arch, that is, *As the* 4^{h} *tearm to* 3^{d}, *So second to first.*

2. That the Radius is a mean Proportional between the Sine of an Arch, and the Secant of that Arks Complement.

That is, *As the Sine of an Arch, To Radius: So is the Radius, To Secant of that Arks Complement, and the Converse.*

3. That the Rectangles of all Tangents and their Complements, being respectively equal to the Square of the Radius, are Reciprocally Proportional, That is,

As the Tangent of an Arch or Angle: Is to the Tangent of another Arch or Angle:

So is the tangent of the Complement of the latter Arch: To the tangent of the Complement of the former,

And by varying the Second Tearm into the place of the Third, we may compare the Tangent of one Ark to the Cotangent of another, *&c.* that is,

As the tangent of an Ark or Angle: Is to the Cotangent of another Ark: So is the tangent of this latter Ark To the Cotangent of the former

4. That the Sines of Arches, and the Secants of their Complements are reciprocally proportional, that is,

As

As the Sine of an Arch : To the Sine of another Arch or Angle : So is the Cosecant of the latter Arch, To the Cosecant of the former,

And by changing the 2 and 3 Tearms, a Sine may be compared with a Secant.

Now hence to be directed to vary Proportions, observe that if 4 Tearms or Numbers are Proportional, it is not material which of the two middle Terms be in the second or third place; for instance if it be,

As 2 *to* 4 :: *So is* 3 *to* 6 : It will also hold, *As* 2 *to* 3 :: *So* 4 *to* 6.

Secondly, that when 4 Tearms are in direct Proportion, if a question be put concerning a fifth Tearm not ingredient in the Proportion, it is not material whether the two former, or the two latter Tearms be taken: As if it should be demanded; When 2 yards of Linnen cost 4sh. What shall 8 yards? Answer, 16.

It might as well be said, If 3 cost 6, What 8? Answer, 16.

Hence then in any Proportion, if the two first Tearms be, *As the Tangent of an ark, To Radius*, to bring the Radius into the first place, it may be said, *As the Radius, Is to the Cotangent of that Ark*, because there is the same Proportion between these two latter Tearms, as between two former; Now in all the former Theorems, the two latter Tearms consist either of the parts, or of the Complements of the parts of the two former, whence it will not be difficult to vary any Proportion propounded.

1. From whence it will follow, that a Proportion wholly in Tangents may be changed into their Complements without altering the Order of the Tearms, and the Converse.

If it were *As Tangent* 10^{d}, *To tang* 20^{d} : *So tang* 52^{d}, *To tan* 69^{d} 15′

It would also be, *As tang* 80^{d}, *To tan* 70^{d} : *So tan* 38^{d} : *To tan* 20^{d} 45′

2. That if the two latter Tearms of any Proportion being Tangents are only changed into their Complements, it infers a Transportation of the first Tearm into the second place.

That is in the first Example, *As Tang* 20^{d}, *To Tangent* 10^{d} : *So Tangent* 38^{d}, *To Tangent* 20^{d} 45′.

 3. That

3. That if the two former Tearms of a Proportion being Tangents are changed into their Complements, it likewise infers a changing of the third Tearm into the place of the fourth.

And then if the fourth Tearm be sought, it will hold,

As the second Tearm, To the first : So is the third Tearm, as at first propounded to the fourth.

In the first Example, *as tangent* 70d *to tangent* 80d :. *So tang* 52 *to tang* 69d 15′.

4 That a Proportion wholly in Secants may be changed into a Proportion wholly in Sines, without altering the Order of the places, only by taking their Complements, and the Converse.

If it were,

As Secant of 80 *To Secant* 70d :: *So Secant* 60d *To Secant* 10d

It would also hold in Sines,

As the Sine 10 *to Sine* 20d :: *So the Sine of* 30d *To Sine* 80d

5. That if the two latter Tearms being Secants, should be changed into Sines, and the Converse, if they were Sines to be turned into Secants, it will be done only by taking their Complements, but then must the second and first Tearms change places one with another.

If the Proportion were, *As Sine* 12d *to Sine* 42d :: *So is the Secant of* 36d *to Secant of* 75d 26′.

It would also hold, *As Sine* 42d *to Sine* 12d :: *So Sine of* 54d *to the Sine of* 14d 34′.

6. That if the two former Tearms of a Proportion in Secants, should be changed into Sines and the Converse; this would infer a changing of the fourth Term of that Proportion into the place of the Third : But the third Tearm not being that which is sought : The Rule to do it, would be to imagine the two first Tearms to change places, and then to take their Complements.

If the Proportion were, *As Secant of 39d to Secant of 75^{d} 26′*
So is the Sine of 12^{d} to the Sine of 42^{d}.
It would also hold, *As Sine 14^{d} 34′, to Sine of 52^{d}.*
So is the Sine of 12^{d}, to the Sine of 42^{d}.

7 Two Tearms whether the former or latter in any Proportion being as a Sine to a Tangent, may be varied.

For, *As the Tangent of an Arch, To the Sine of another Arch*
So is the Cosecant of the latter Arch,
To the Cotangent of the former.
And by transposing the Order of the Tearm.
As a Sine, To a Tangent:
So the Cotangent of the latter Arch,
To the Cosecant of the former.

This will be afterwards used in working Proportions on the Instrument, and three Instances shall be given of it.

8. Lastly, Observe that if 4 Tearms or Numbers are Proportional, their Order may be so transposed, that each of those Tearms may be the last in Proportion; and so of any 4 Proportional Tearms, if there be given, the other that is unknown may be found, Thus,
As first to second :: So third to fourth.
As second to the first :: So the fourth to the third:
As the third to the fourth :, So the first to the second.
As the fourth, To the third :: So the second, To the first.

Cases of Oblique Sphœrical Triangles.

1. TWo Sides together less then a Semicircle with the Angle comprehended given to find one of the other Angles.

At two Operations they may be both found by a Proportion demonstrated in the late Trigonometry of the Learned Mr *Oughtred*.

As the Sine of half the sum of the sides,
To Cotangent of half the contained angle :
So the sine of half the difference of the sides,
To the Tangent of half the difference of the other angles.

Again,

As the Cosine of half the sum of the sides;
To Cotangent of half the contained angle :
So the Cosine of half the difference of the sides,
To the Tangent of half the sum of the other angles.

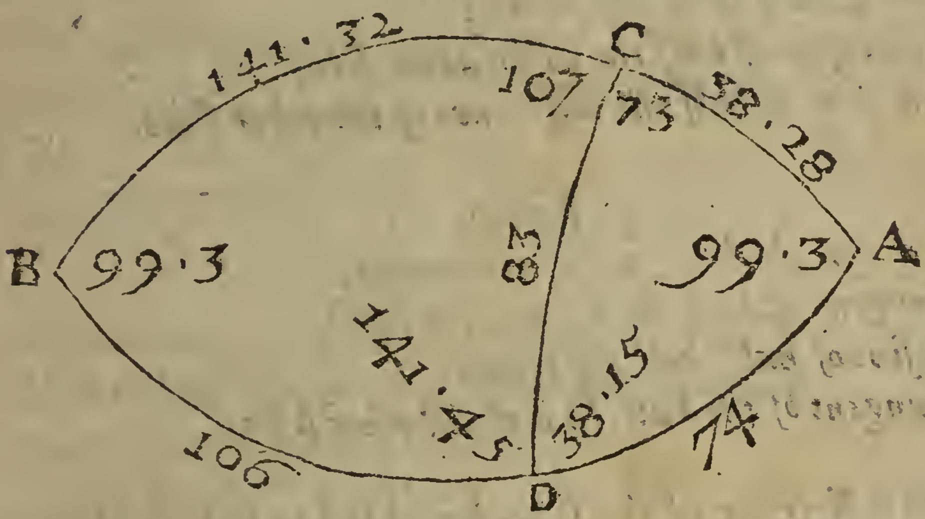
Add the half difference to the half sum, and you have the greater Angle; but substracted from it, and there remains the lesser angle.

If the sum of the two given Sides exceeds a Semicircle, the Opposite Triangle, must be resolved instead of that propounded.

Here note that evey Sphœrical Triangle hath opposite to each angular Point, another Triangle, having the side that subtends the said Angle common to both, and the angle opposite thereto equal, the other parts of it are the Complements of the several parts of the former to a Semicircle.

So if in the Triangle B C D there were given the sides B C, and C D with their contained Angle B C D to find the Angle C B D because these two sides are greater then a Semicircle, resolve the opposite Triangle C A D, in which there will be given C A, which may be the Complement of the Latitude 38' 28″, and C D the Comple-

Complement of the Altitude 83^{d} with the angle A C D, the Suns Azimuth from the North 73^{1} to find the angle C A D the hour from Noon.



CA 38^{1} : 28′ } Sides,
CD 83 : 00 }

121 : 28′ sum
60 : 44 half sum

		Logarithms		Logarith:
44 : 32 difference				
22 : 16 difference,	Sine	957854	Cosine	996634
36 : 30 half the Angle				
53 : 30 Complement,	Tang:	1013079	Idem	1013079
		1970933		2009713
Sine of 60^{1} 44′ half sum		994069	Cosine	968919
Tang: of 30^{1} 24′ $\frac{1}{2}$		976864	tan 68^{d} 39′	1040794

Sum 99 : 3 hour, in Time 36′ before 6 in the morning, or as much after it in the afternoon,
difference 38^{1} 15′ Angle of ☉ position.

2, *Two Angles together less then a Semicircle with the side between them,* alias, *the Interjacent side, To find one of the other sides.*

This is but the Converse of the former to be performed at two Ope-

Operations to get them both, and the Proportion thence applyed by changing the ſides into Angles.

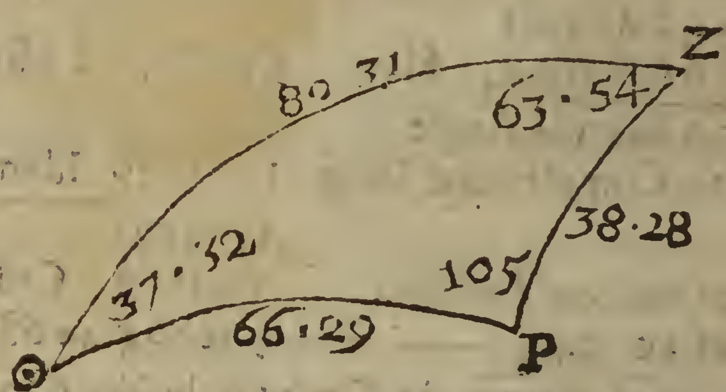
As the Sine of the half ſum of the angles,
To the Sine of half their difference:
So is the Tangent of half the interjacent ſide,
To the Tangent of half the difference of the other ſides.

Again.

As the Coſine of the half ſum of the angles,
To the Tangent of half the interjacent ſide:
So the Coſine of half their difference,
To the Tangent of the half ſum of the other ſides.

If half the difference of the ſides be added to half the ſum of the ſides, it makes the greater ſide; but ſubſtracted from it, leaves the leſſer.

If the Sum of the two given Angles exceeds a Semicircle, then, as in the former Caſe, reſolve the Oppoſite Triangle.



So in the Triangle Z P ☉ if there were given the angle ☉ Z P, the Suns Azimuth from the North 63ᵈ 54′, and the hour from Z P ☉ 105ᵈ in time 5 in the morning, or 7 in the evening, and the Complement of the Latitude Z P 38ᵈ 28′, to find the Complement of the Altitude Z ☉ 80ᵈ 31′, or the Complement of the Declination ☉ P 66 29′, two Operations finds both, and neither with leſs.

Angle

Angle Z P ☉—105^d, 00^m *Example.*
P Z ☉——63 : 54

difference———41 6′
half difference—— 20 : 33 Sine—954533 Cosine 997144
half the side Z P—19 : 14 Tang—954268 Idem 954268
Sum of the sides—168:54 1908801 1951412
half sum ——— 84 : 27 Sine—999796 Cosine 898549
Tangent——7 : 1′ 909005 ta 73^{d}30′ 1052863
73: 30
Sum———80 31 the greater side Z ☉
Difference—66 29 the lesser side ☉ P

3. Two sides with an Angle opposite to one of them given, To find the Angle opposite to the other, its Affection being fore-known.

As the Sine of the side opposite to the angle given
Is to the Sine of its Opposite angle :
So is the sine of the side opposite to the angle sought,
To the sine of its opposite angle.

Here note, that the same Sine is common to an Arch, and to its Complement to 180^d, if the Angle sought be foreknown to be Obtuse, substract the Arch found from 180 and there remains the angle sought.

Example.

So in the former Triangle, if there were given the side ☉ P 66^d 29′ the Complement of the Declination with its opposite angle P Z ☉ 63^d 54′, the Suns Azimuth from the North, and the side Z ☉, the Complement of the ☉ Altitude 80^d 31′, the Angle Z P ☉ the hour from Noon would be found to be 105.

Sine

Sine 63^d 54′——995329
80 31——999402

1994731

Sine 66^d 29′——9906234

Sine 75^d——998497

The Complement of 75^d is the angle sought, being 105^d, and so much is the hour from Noon.

In some Cases the Affection of the angle sought cannot be determined from what is given;

Such Cases are,

When the given Angle is Acute, and the opposite Side less then a Quadrant, and the adjacent or other Side greater then the opposite Side, and its Complement to a Semicircle also greater then the opposite Side.

Also when the given Angle is Obtuse, and the opposite Side greater then a Quadrant, and also greater then the other side, and greater then the Complement of the said other Side to a Semicircle.

In all other Cases the Affection of the Angle sought may be determined from what is given; in these it cannot without the help of the third side (or something else given)

Where Cases are thus doubtful, there can be but a double answer, and both true; wherefore find the Acute Angle and its Complement to 180^d and the like answer give in Case 4^{th}, 5^{th}, 6^{th}, 7^{th} and 8^{th} following.

4, Two Angles with a Side opposite to one of them being given, To find the Side opposite to the other, its Affection being foreknown.

As the Sine of the angle opposite to the given side,
Is to the Sine of the given Side:
So is the Sine of the angle opposite to the side sought,
To the Sine of the side sought.

If the side sought be foreknown to be Obtuse, the Complement of the Ark found to 180^d will be the side sought.

Example.

Example.

So in the former Triangle, if there were given the angle at Z the Suns Azimuth from the North $63^d\ 54'$, and the Complement of the Suns Declination ⊙ P $66^d\ 29'$ with the hour from Noon Z P ⊙ to find the Side Z ⊙ the Complement of the Suns Altitude, it would be found to be $80^d\ 31'$, and the Altitude it self $9^d\ 29'$.

Sine $66^d 29'$——996234
Sine 105 that is of 75^d is–998497

1994731
Sine $63^d\ 54'$ is—— 995329

Sine $80^d\ 31'$——999402

In some Cases the Affection of the Side sought cannot be determined from what is given;

Such Cases are,

When the given Angle is Acute, and the opposite Side less then a Quadrant, and the other Angle greater then the former Angle, and its Complement to a Semicircle also greater then the said former Angle.

Also when the given Angle is Obtuse, and the opposite Side greater then a Quadrant, the other Angle being less then this Angle, and its Complement to a Semicircle also less then this Angle:

What *Snellius* hath spoke concerning these doubts, is in some Cases false, in others impertinent, however I conceive not that Learned Author mistakes, but the Supervisors after his death.

In all other Cases the determination is certain, as may be hereafter shewed.

5. *Two sides with an Angle opposite to one of them being given, To find the third side, the kind of the angle opposite to the other side being foreknown.*

First find the Angle opposite to the other side by 3^d Case, and then you have two Sides and their opposite Angles.

To find the third side by the Inverse of either of the Proportions used in the 2^d Case, the former will be,

As the Sine of half the difference of the angles given,
To tangent of half the difference of the sides given:
So is the sine of half the sum of those angles,
To the tangent of half the side required.

In the latter Case, if the sum of the given Angles exceed a Semicircle, the opposite Triangle must be resolved.

Example.

If in the former Triangle there were given the side ⊙ P, the Complement of the Declination $66^d\,29'$ and angle ⊙Z P, the Azimuth from the North $63^d\,54'$ with the side Z P, the Complement of the Latitude $38^d\,28'$, to find the side ⊙ Z, the Complement of the Suns Altitude on the Proposed Azimuth: The first Operation will be to find the Suns angle of Position Z ⊙ P $37^d\,32'$, which is always Acute when the Sun or Stars do not come to the Meridian between the Zenith and the elevated Pole.

The said angle being found by the former Directions, we proceed to the second Operation.

Sides $\left\}\begin{matrix}66 & 29\\ 38 & 28\end{matrix}\right\{$ difference $28^d\,1'$ half $14^d\,00^d\,30''$ Tang—939705

Angles $\left\}\begin{matrix}63 & 54\\ 37 & 32\end{matrix}\right\{$ Sum $101^d\,26$, half $50^d\,43$ Sine—988875

1928580

26 22 difference, half $13^d\,11'$ Sine——935806

Tangent of $40^d\,15'\,30''$ ——992774
doubled is $80^d\,31'$ the side sought being the Complement of the Suns Altitude.

6. *Two*

6. Two sides with an angle opposite to one of them being given, To find the angle included, or between them, the species of the opposite to the other side being foreknown.

First find the angle Opposite to the other side by 3d Case, and then we have two angles and their opposite sides to find the other angle, by the Inverse of either of the Proportions used in the first Case, the former will be,

As the sine of halfe the difference of the sides,
To the Tangent of halfe the difference of the angles :
So is the sine of halfe the sum of the sides,
To the Cotangent of half the angle required; That is, to the Tangent of an Ark, whose Complement is half the angle inquired.

If the sum of the given sides be more then a Semicircle, in the resolution of this latter Case resolve the Opposite Triangle.

Example.

In the former Triangle given ⊙ P Comple: Declination 66d 29'
Z P Comple: Latitude——38 28
Angle ⊙ Z P the Azimuth—63 54
To find the hour Z P ⊙——105

The first operation wil find the angle of Position as before 37d 32'

The second Operation.

half difference of the given angles 13d 11m Tangent——936966
half sum of the side————52 28 30'' Sine——989931

1926897

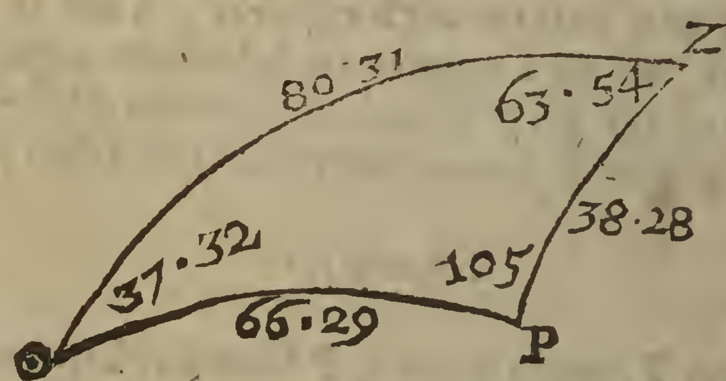
half difference of the sides 14d 00' 30'' Sine————938393

Tangent 37d 30' ————988504
Comple: is 52 30 doubled makes 105d, the Angle sought.

7. Two Angles with a side opposite to one of them being given. To find the third Angle, the kind of the side opposite to the other Angle being foreknown.

First find the side opposite to the other Angle by 4th Case, And then we have two angles, and their opposite sides to find the third angle; by transposing the order of either of the Proportions used in the first Case, the latter will be,

As the Cosine of halfe the difference of the sides,
To the Tangent of halfe the sum of the angles:
So the Cosine of halfe the sum of the sides,
To the Cotangent of half the contained angle.



Example.

In the Triangle Z ☉ P *Data* angle ☉—37d 32′
Angle P—105 00
Side ☉ Z—80 31
To find the angle Z—63 54
The first Operation will find Z P—38 28

The second Operation.

half sum of the angles——71*d* 16′ Tangent———1046963
half sum of the sides ——59*d* 29′ 30″ Sine Compl:——970558

2017521
half difference of the sides 21*d* 1′ 30″ Cosine———997007
Tangent 58d 3′———————————1020514
Compl: 31d 57′ doubled is 63d 54′ the angle sought.

8. Two

8. Two angles with a side Opposite to one of them being given, To find the Interjacent side, the kind of the side opposite to the other angle being fore known.

First find the side opposite to the other angle by 4 Case,

And then you have two sides, and their opposite angle given to find the 3 side by, tranposing the Order of either of the Proportions used in the 2^d Case, the latter will be,

As the Cosine of halfe the difference of the two angles,
To the tangent of halfe the sum of the two sides:
So the Cosine of halfe the sum of the two given angles,
To the Tangent of halfe the third side.

Example.

In the former Triangle given the Hour angle at P 105^d 00
Azimuth angle at Z 63 54
Compl Altitude Side Z ☉ 80 31
To find the Compl. of the
Latitude the side Z P 38 28

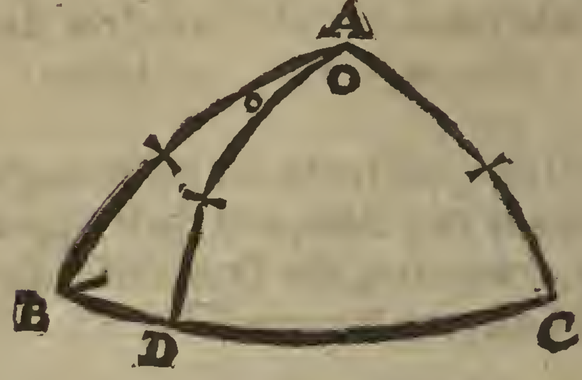
The first Operation will find the side P ☉ 66^d 29′

Second Operation.

half the sum of the two sides	73^d 30′	Tangent	1052839
half the sum of the two angles	84 27	Cosine	898549
			1951388
half the difference of the two angles	20^d 33′	Cosine	997144
Tangent of 19d 14′			954244
Doubled is	38 28 the side sought		

These 6 last precedent Cases may be called the Doubtful Cases, *because that three given terms are not sufficient* Data *to find one single answer without the quality of a fourth, which is demonstrated by* Clavius, *in* Theodosium, *and seeing it passes without due caution in our English Books, I shall insert it from him:*

Let



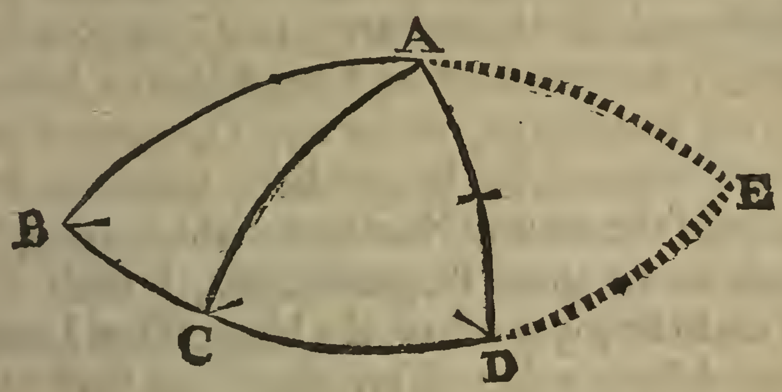
LEt A D and A C be two equal ſides including the angle D A C, and both of them leſs or greater then a Quadrant.

Draw through the Points C and D, the arch of a great Circle C D, continue it, and draw thereunto another Arch or Side from A, namely A B, neither through the Poles of the Arch C D, nor through the Poles of the Arch A D, ſo that the angles B and B A D may not be right angles, nor the angle A D B, if then each of theſe ſides A D A C be leſs then a Quadrant, the two angles C, and A D C will be Acute; and if theſe Arks be greater reſpectively then a Quadrant, the two angles C and A D C, will be Obtuſe, whence it comes to paſs that the angle A D B is Obtuſe, when the angle A D C is Acute, and the contrary: Now foraſmuch as the ſides A C and A D are equal to each other, the other *Data*, *viz.* the ſide A B, and the angle at B are common to both, for in each Triangle A B D, and A B C there is given two ſides with the angle at B oppoſite to one of them; Now this is not ſufficient *Data* to find the angle oppoſite to the other ſide, which may be either the acute angle at C, or the Obtuſe angle ADB the Complement thereof to a Semicircle: Nor to find the third ſide, which may be either B D, or the whole ſide B C, nor the angle included, which may be either B A D, or B A C, therefore in theſe 3 Caſes we have required the quality of the angle oppoſite to the other given ſide A B, and though it be not ſo much obſerved; in the other *Trigonometry*, by *Perpendiculars* let fall, without the knowledge of the ſaid angle it could not be determined whether the Perpendicular would fall with in or withont the Triangle, nor whether the angle found in the firſt Caſe be the thing ſought, or its Complement to 180^{d}, nor whether the angles or Segments found by 1ſt and 2^{d} Operation in the

other

other Cases are to be added together, or substracted from each other, to obtain the side or angle sought.

So also two angles with a side opposite to one of them, are not sufficient *Data* to obtain a fourth thing in the said Triangle, without the affection of the side opposite to the other given angle.



LEt A B and A C be two unequal sides containing the angle B A C both together equal to a Semicircle, one being greater, the other less then a Quadrant. Draw through the Points B and C, the arch of a great Circle B C, continue it, and draw thereto from A another side AD; but not through the Poles of AC, nor through the Poles of B C, so that the angles D and C A D may not be right angles, nor the angle A C D a right angle; for if it were a right angle, the angle A B C whereto it is equal, should be also a right angle, and so the two sides A B and A C, by reason of their right angles at B and C should be equal, and be Quadrants contrary to the Supposition; Now the angles A C D and A B C being equal, which is thus proved: Suppose the two sides A B and B D to be continued to a Semicircle at E, then will the said angle be equal to its opposite angle at B, the side A C by supposition is equal to the side A E, the Complement of the side A B to a Semicircle, but equal sides subtend equal angles, therefore the angle at C is equal to the angle at B or at E, which being admitted retaining the side A D and angle at D, we have another angle opposite thereto, either C or B, which are equal and common to both Triangles, and so if the side opposite to the given angle at D were sought, a double answer should be given, either the side A C, or the other side A B its Complement to 180°, and the interjacent side might be C D or B D, and the third angle the lesser angle C A D, or the greater B A D, which is not commonly animadverted.

Two Sides with the Angle comprehended, to find the third Side.

That the former Cases might be resolved without the help of Perpendiculars, hath been long since hinted by Mr *Gunter*, Mr *Speidel*, and Mr *Gellibrand*, but so obscurely that I suppose little notice was taken thereof; but this Case hath not hitherto been resolved by any man, to my knowledge, under two Operations with a Perpendicular let fall, working by Logarithms, unless by Multiplication and Division in the natural Numbers, which being the onely Case left wherein we are to use Perpendiculars, I shall shew how to shun both, with the joynt use of the Natural and Logarithmical Tables, by a novel Proportion of my own, and illustrate the usefulness thereof by some Examples.

Two Sides with the Angle comprehended, to find 3d Side.

As the Cube of the Radius,
To the Rectangle of the Sines of the comprehending sides:
So is the Square of the Sine of half the angle contained,
To half the difference of the Versed sines of the third side, and of the Ark of difference between the two including sides,

Which half difference doubled, and added to the Versed Sine of the difference of the Leggs or containing sides, gives the Versed Sine of the side sought.

And if you will make the third Tearm the Square of the Sine of half the Complement of the contained angle to 180^{d}, you will find the half difference of the Versed Sines of the third side, and of the sum of the two including sides to be doubled and substracted from the Versed Sine of the said sum.

But to apply the former to Logarithms.

Double the Logarithmical Sine of half the angle given, & thereto adde the Logarithms of the sines of the containing Sides, & from the left hand of the Sum, Substract 3 for the Cube of the Radius, so rests the Logarithm of half the difference of those two Versed Sines bove, And

And if instead of the second Tearm be taken into the Proportion, the double of the Rectangle of the Sines of the containing Sides; that is, if the Logarithm of the Number 2 be added to the Logm of the other middle Tearms, you will have the Logarithm of the whole Difference in the last place; having found it, take the Number that stands against it, either in the Natural Sines or Tangents, and accordingly add it to the Natural Versed Sine of the Difference of the Leggs, and the summe is the natural Versed Sine of the side sought.

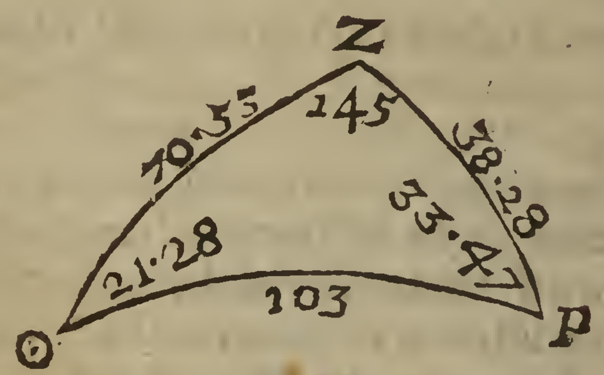
This is the Inverse of the 4th Axiom, used when 3 sides are given to find an angle, and will be of great use to Calculate the Distances of Stars by having their Declinations and right Ascensions, or Longitudes and Latitudes given, by means whereof the Altitudes of two of them, or of the Sun with the difference of time, or Azimuth, being observed at any time off the Meridian, the Latitude may be found, as also for Calculating the distances of places in the Arch of a great Circle, all of them Propositions of good use in Navigation; as for the latter it hath hitherto been delivered in our English Books doubtfully, erroneously, or not sufficiently for all Cases, the Rules delivered being only true in some Cases, and doubtful in most, not determining whether the side sought be greater or less then a Quadrant.

The Reader may observe how necessary it is to have such Tables, as have the natural Sines and Versed Sines, *&c.* standing against the Logarithmical Sines, for this and other following Proportions discovered by my self for the easie calculating a Table of hours and Azimuths to all Altitudes, as also a Tables of Altitudes to all hours; but as yet there are none such made as have the Versed Sines, but will in due time be added to Mr. *Gellibrands* Tables; in the interim it may be noted, that the Residue of the Natural Sine of an Ark from Radius called its Arithmetical Complement, is the Versed Sine of that Arks Complement; thus the natural Sine of 40d is 6427876 substracted from Radius, rests 3572124, the Versed Sine of 50*d*. And for Arks above 90*d* we need no natural Versed Sines, because the natural Sine of any Arks excess above 90*d* added to the Radius is equal to the Versed Sine of the said Ark, thus the Sine of 40*d* augmented by the Radius is equal to the Versed Sine of 130*d* and is 16427876

N *Example*

Example of this Case.

In the Triangle ⊙ Z P let there be given the side ⊙ Z, the Complement of the Altitude 70*d* 53′ and the side Z P the Complement of the Latitude 38*d* 28ᵐ with the angle ⊙ Z P 145*d* the Suns Azimuth from the North, to find the side ⊙ P, the Suns distance from the Elevated Pole.



Sine	38*d*	28ᵐ ————————	97938317
Sine	70	53 ————————	99753646
Sine	72	30 Logᵐ dobled	199588390

Natural Sine against ,97280353

it doubled is 10691964

Natural V Sine of 32*d* 25ᵐ the difference of the sides——} 1558280

The Versed Sine of 103*d* the——12250244 side sought, and therefore the Sun hath 13*d* of South declination.

Another Example of this Case for Calculating the Suns Altitude on all hours.

As the Cube of the Radius, To the double of the Rectangle of the Cosines, both of the Latitude, and of the Suns declination.
So is the Square of the Sine of half the hour from noon,
To the difference of the Sines of the Suns Meridian Altitude, and of the Altitude sought.

This Canon will finde two Altitudes at one Operation, and will have

have very little trouble in it, the double Rectangle, that is the second tearm of the Proportion, being fixed for that Declination.

Add the Logarithms of the Number two, and of the Cosines of the Declination and Latitude together the sum may be called the fixed Logarithm.

Double the Logarithm of the Sine of half the hour from noon, and add it to the fixed Logarithm the sum rejecting 3 towards the left hand, for the Cube of the Radius is the Logarithm of the difference: Take the natural Sine that stands against it, and substract it from the natural Sine of the Meridian Altitude, both for the Winter and Summer Declination, and there remains the natural Sines of the Altitudes sought.

If this difference cannot be substracted from the Sine of the Meridian Altitude, it argues the Sun hath no Altitude above the Horizon in this Case substract that from this, and there will remain the Natural Sine of the Suns Altitude for the like hour from midnight in Summer.

Example.

Let it be required to Calculate the Suns Altitude when he hath $23^d\ 31^m$ both of North and South Declination for our Latitude of *London* at 2 and 5 a Clock in the afternoon, or which is all one for the hours of 10 and 7 in the morning.

Sine $38^d\ 28^m$ Compl Latitude————979383l7
Sine of 66 29^m Compl Declination————99623428
Logarithm of Number 2 is————————03010300

Fixed Number————200572045
Logm of Sine of 15^d, doubled——188259924

Nearest natural Sine against it, 761900————88831969
$61^d\ 59^m$ Summer Meridian Altitude
Natural Sine———8828110
Substract————761900 the difference before found

Rests————8066210 the natural Sine of $53^d\ 46'$ the

Summer Altitude for the hours of 10 and 2

$14^d\ 57^m$ Winter Meridian Altitude Nat Sine—2579760

Substract the former difference——————761900

Rests the Natural Sine of $10^d\ 27^m$ the ——————1817860

Winter Altitude for the hours of two and ten.

The same day for the Altitude of 5 and 7.

Fixed number————200573045

Sine of $37^d\ 30^m$ Logm doubled———195688942

Natural Sine against it 4226183 ——— 96260987

Winter Meridian Altitude, as before Sine 2579760

Rests———1646423 the natural Sine of $9^d\ 29^m$ Summer Altitude for 5 in the morning, or 7 in the evening.

Natural Sine.

Summer Meridian Altitude, as before————8828110

The former difference——————————4226183

Rests the Natural Sine of $27^d\ 24^m$——————4601927

The Summer Altitude for 7 in the morning, or 5 in the afternoon.

The former Case may also be performed at two Operations by help of a Perpendicular supposed, without the help of Natural Tables.

1. If both Sides are equal,

As the Radius, To the sine of the Common side :

So the Sine of half the Angle, To the Sine of half the side sought.

2. If one of the sides be a Quadrant, this by contiuving the other side to a Quadran(as shall afterwards be shewed) wil become a Case of right angled Sphœricala Triangles, in which besides the right angle, instead of the quadrantal side, there will be given a Legg, and its adjacent angle to find the other angle by 4 Case of right angled Sphœrical Triangles; and so if the angle included were 90^d it would

would be a Case of right angled Sphœrical Triangles, in which besides the right angle, there would be given both the Leggs or Sides to find the Hipotenusal.

3. In all other Cases one or both of the including Sides being less then Quadrants, it will hold,

As the Radius, To the Cosine of the angle included:

So the tangent of the lesser side, To the tangent of a fourth Ark,

If the angle included, be less then 90^{d} substract the 4th Ark from the other side; but if it be more from the other sides Complement to 180^{d}, The remainder is called the Residual Ark.

Then, *As the Cosine of the* 4th *Ark, To the Cosine of the Ark remaining:*

So the Cosine of the lesser side, To the Cosine of the side sought.

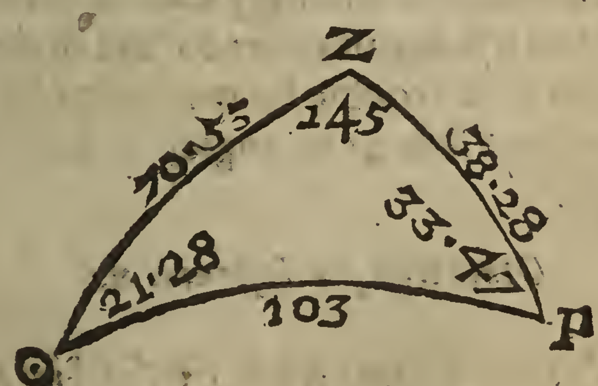
The side sought may be greater then a Quadrant, and so be doubtfull, but we may determine,

That when the Leggs are of the same kind, and the angle comprehended Acute, the side sought is less then a Quadrant.

And when the Leggs or containing Sides are of a different kind, and the angle comprehended Obtuse, the side sought is greater then a Quadrant.

Or it may be determined from the affection of the Residual Ark in all Cases.

When the contained angle is acute, and the residual Ark more then 90^{d}, or when the said angle is Obtuse, and the residual Ark less then a Quadrant, the side sought is greater then a Quadrant, in all other Cases less.



Example.

Example.

In the Triangle ⊙ Z P, let there be given Z P, and ⊙ Z with the angle at Z, to find the side ⊙ P, the Suns distance from the Elevated Pole.

angle included	145^d		Logm
Or,	35	Compl 55^d Sine	99133645
Tangent of	38 28'	lesser side	99000865
Tangent	33^d, 3'		98134510
Compl ⊙ P to 180^d is	109 7		
The Ark remaining or differ:	76^d 4^m	Cosine	93816434
Lesser side	51 32	Cosine	98937452
			192753886
Ark found	33^d 3^m	Cosine	99233450
Sine	13		93520436

The Complement hereof 77*d* should be the side sought, but because the angle was Obtuse, and the residual Ark less then a Quadrant, the side sought is greater, and therefore 103*d* the Complement hereof is the side sought.

This Case & the Converse of it being the next Case, I have thus setled to apply the to Logarithmical Tables only, in Case the natural ones were wanting, being all the other Cases are thereto fitted; and as the trouble about the Cadence of a Perpendicular is here shunned, without so much as the name of it; so may it be done in all the rest of the Oblique Cases, which I had so fitted up for my own use; but forbear to trouble the Reader with them, apprehending these to be better, and that he would not willingly Calculate for a portion of an angle, or a Segment of a Side, in order to the finding out the thing sought, when with as little tronble he may come by it, and yet Calculate always either for a side or an angle, one of the six principal parts of the Triangle.

Other Wise for Instruments.

As the Diameter, To the difference of the Versed Sines of the sum, and of the difference of any two sides, including an Angle.

Or

Or,

As the Cosecant of one of the including Sides, Is to the Sine of the other side;
So is the Versed Sine of the angle included.
To the difference of the Versed Sines of the Ark of difference between the two including Sides, and of the third side sought, Which difference added to the Versed Sine of the difference of the Leggs, makes the Versed Sine of the side sought.

And so is the Versed Sine of the contained angles Complement to 180d
To the difference of the Versed Sines of the sum of the Leggs, and of the side sought, which substracted from the Versed Sine of the said sum, there remains the Versed sine of the side sought,

Here note, that the same Versed Sine is common to an Ark greater then 180*d*, and to its Complement to 360*d*, So the Versed Sine of 200*d* is also the Versed Sine of 160*d*.

The Proportions delivered for Instruments having such Tables as before hinted, will not be so unsuitable to the Logarithms as commonly reputed.

Example for Calculating the distance of two places in the Arch of a great Circle, otherwise then according to the general Cannon before delivered.

As the Secant of one of the Latitudes, To the Cosine of the other;
So the Versed Sine of the difference of Longitude,
To the difference of the Versed Sines of these two Arks,

The one the Ark of distance sought, the other the Ark of difference between both Latitudes, when in the same Hemisphere, or the sum of both Latitudes when in different Hemispheres, which difference added to the Versed Sine of this latter Ark, the sum is the versed Sine of the distance,

By turning the Substraction to be made of the first Tearm into an Addition, the two first Tearms of the Proportion will be,
As the Square of the Radius,
To the Rectangle of the Cosines of both the Latitudes:

Then for the third Tearm being the difference of Longitude, take the natural Versed Sine thereof, and seek that Number in the natural

ral Tangents, and that Logarith Tangent that ſtands againſt it take into the Proportion inſtead of the Logarithm of the Verſed Sine propoſed.

Admit it were required to find the Diſtance between London *and* Bantam, *in the Arch of a great Circle.*

Logme
Bantam Longitude 140*d* Latitude 5*d* 40′ South Coſine 9,9978725
London Longitude 25,50 Latitude 51,32 North Coſine 9,7938317

difference of Long 114*d* 10′ Nat V Sine 14093923 equal
to the natural Tangent of 54*d* 38′ ½ neareſt Logm 10,1489900
Natural Sine 8723538 againſt it———2.9940694
Nat Verſed Sine of 57*d* 12′ } 4582918
the ſum of both Latitudes }
Sum————13306456 the natural Verſed Sine of 109*d* 18′ 30″ the Ark of diſtance ſought.
And if to the ſaid difference, namely——————8723538
Be added the natural Verſed Sine of the difference } ——3036695
of both Latitudes, namely the V Sine of 45*d* 52′ }
The ſum being the natural V Sine of 100*d* 8′ 30″ is——11760233
the diſtance of two places, having the ſame Latitudes, and difference of Longitude, but are both in the ſame Hemiſphere.

Here note, that no two places can have above 180*d* difference of Longitude, therefore in differencing the two Longitudes if the remainder be more take its Complement to 360*d*.

The Complements of theſe two diſtances, namely 70*d* 41′ 30″ and 79*d* 51′ 30″ are the diſtances of two places of the ſame Latitudes conſidered as in different Hemiſpheres, their difference of Longitude being 65*d* 50′ the Complement of the former, and two places in a ſuch Poſition compared with their former Poſitions may be apprehended to be Diametrically oppoſite upon the Globe, as thus, *Bantam* having 5*d* 40′ South Latitude, let another place have as much North Latitude, the difference of Longitude between them 180*d* and conſequently ſo much their diſtance; now whatever be the diſtance between *Bantam* and the third place, the Complement of it to 180*d* ſhall be the diſtance between the two other places.

10. *Two*

10. *Two angles with the Interjacent side given, To find the 3^d angle, the proportion derived from the former Case by changing the angles into sides, and holds without any such change supposed is,*

As the Cube of the Radius, To the double of the Rectangle of the Sines of the two given angles:
So is the Square of the Sine of half the given side, To the difference of the Versed Sines of these two Arks, the one is the angle sought, the other the Ark of difference between one of the including angles, and the Complement of the other to a Semicircle, which difference added to the Versed Sine of this Ark gives the Versed Sine of the angle sought.

How to work this by Tables need not be shewed after the Logm of the difference is got; if it be less then the Radius, it may be sought either in the Sines or Tangents, and the natural Sine or Tangent that stands against it and comes nearest taken; but when it exceeds the Radius always seek it in the Tangents, and take the natural Tangent that stands against it, which difference so found, is to be added to the Versed Sine of the difference of the Leggs to obtain the Versed Sine of the angle sought.

Otherwise for Tables the common way by a supposed Perpendicular

1. If both the angles are equal,

As the Radius To the Sine of the angle given: So the Cosine of half the given Side, To the Cosine of half the angle sought.

In all other Cases not belonging to right angled Triangles if one or both of the given angles be Acute, it holds,

As the Radius, To Cosine of the interjacent side:
So the Tangent of the lesser angle,
To the Tangent of a 4^h Ark.

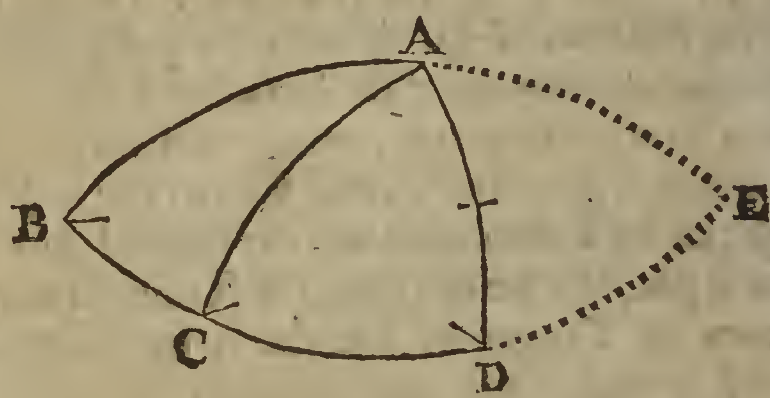
If the interjacent side be more then 90^d substract the 4^h Ark from the other angle; but if less then 90^d, substract the 4^h Ark from the other angles Complement to 180, noting the residual Ark.

Then, *As the Cosine of the 4^th Ark, To the Cosine of the Ark remainmaining:*

O So

So the Cosine of the lesser angle,
To the Cosine of the angle sought.

When the interjacent side is less then a Quadrant, and the residual Ark more, or when the interjacent side is greater then a Quadrant, and the residual Ark less, the angle sought is Obtuse, in all other Cases Acute.



In the Triangle ☉ Z P let there be given
The angle of Position at ☉—— 21^d 28'
The hour from noon angle at P— 33 47
And the side ☉ P the Suns distance from the elevated Pole } —103 00

To find his Azimuth the angle ☉ Z P

Sine 13*d* the Complement of the interjacent side———93520880
Tangent 21*d* 28' the lesser angle———————95946561

Tangent of 5*d* 3'————————89467441
The other angle—33 47

The difference being the residual ark	28 44	Cosine	99429335
Lesser angle	21 28	Cosine	99687773
			199117108
Ark first found	5^d 3'	Cosine	99983109

Sine 55*d*————————————99133999

The

The Complement whereof 35d in this Case is not the angle sought, but the residue hereof from a Semicircle 145d is the angle sought being Obtuse, because the interjacent side is greater then a Quadrant, and the residual Ark less; the residual Ark in Operation if greater then a Quadrant, take its Complement to 180d, because there are no Sines to Arks above a Quadrant, and then the Complement of this Ark to 90^d is the Complement of the residual Ark the Sine whereof must be taken for the Cosine of the residual Arke.

Otherwise for Instruments.

As the Diameter, To the difference of the Versed Sines of the sum and difference of the two including angles, Or,
As the Cosecant of one of those angles, Is to the Sine of the other,
So the Versed Sine of the interjacent side,
To the difference of the Versed Sine of an Ark left by substracting one of the including angles from the Complement of the other to a Semicircle, and of the angle sought, which difference added to the Versed Sine of the said Ark, gives the Versed Sine of the angle sought,

And so is the Versed Sine of the interjacent sides Complement to 180d, To the difference of the Versed Sines of an Ark made by adding one of the including angles to the Complement of the other to a Semicircle, and of the angle sought, which substracted from the Versed Sine of the said Ark, leaves the versed sine of the angle sought.

11. *Three Sides to find an Angle.*

The two sides including the angle sought are called Leggs, and the third side the Base.

As the Rectangle or Product of the Sines of the half sum of the three sides and of the difference of the Base therefrom.
Is to the Square of the Radius:
So is the Rectangle of the sines of the differences of the Leggs from the said half sum,
To the Square of the Tangent of half the angle sought.

And by changing the third Tearm into the place of the first.

As the Rectangle of the Sines of the differences of the Leggs from the half sum of the 2 sides,
Is to the Square of the Radius:
So the Rectangle of the Sines of the half sum of the three sides, and of the difference of the Base therefrom,
To the Square of the Tangent of an Ark, whose Complement doubled is the angle sought, or this Ark doubled is the Complement of the angle sought to 180*d*, or it might be expressed, *To the Square of the Cotangent of half the angle sought.*

Otherways in Sines.

As the Rectangle of the Sines of the containing Sides or Leggs,
Is to the Square of the Radius;
So the Rectangle of the Sines of the differences of the Leggs from the half sum of the three sides,
To the Square of the Sine of half the angle sought.

Or the Cosine may be found.

As the Rectangle of the Sines of the containing sides,
Is to the Square of the Radius:
So the Rectangle of the Sines of the half sum of the 3 sides, and of the difference of the Base therefrom,
To the Square of the Cosine of half the angle sought.

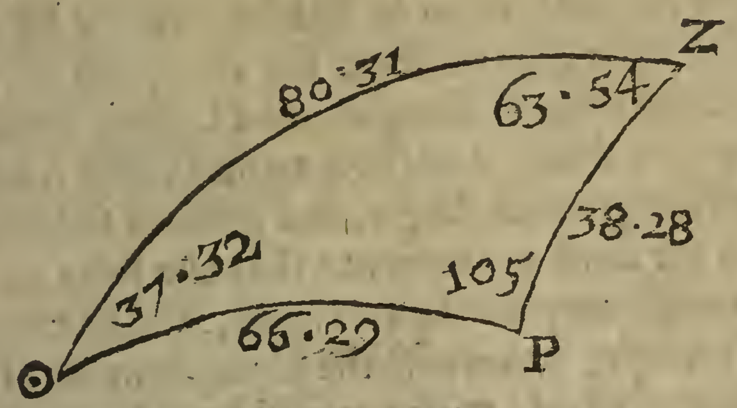
These two latter Proportions are demonstrated in the Treatises of the Lord *Napier*, Mr *Oughtred*, Mr *Norwood*, and are those from whence I shall educe the Demonstrations of the rest.

To work the third Proportion that finds the Square of the Sine of half the angle.

To the Arithmetical Complements of the Logarithms of the sines of the containing Sides or Leggs add the Logarithmical Sines of the differences of the said Leggs from the half sum of the three Sides, the half sum of these four Numbers will be the Logarithm of the sine of half the angle sought.

In

In the Triangle ☉ Z P, *Data*, the three Sides to find the angle a P the hour from noon.



80*d* 31′ Base
66 29 } Leggs ——— *Ar comp* ,0376572
38 28 } ——— *Ar comp* ,2061683
Sum—185,28 difference of the Leggs { 26d 15′ Sine 9,6457058
half—92 44 from half sum ——— { 54,16 Sine 9,9094190
Sum ——— 19,7989503
Sine of 52*d* 30′ half ——— 9, 8994751
doubled 105, the angle at P sought.

The Arithmetical Complement of a Logarithm, is the residue of that Logarithm from the next bigger Number, consisting of an Unite and Ciphers.

Otherwise for Instruments.

As the difference of the Versed Sines of the sum, and of the difference of any two sides including an angle,
Is to the Diameter,

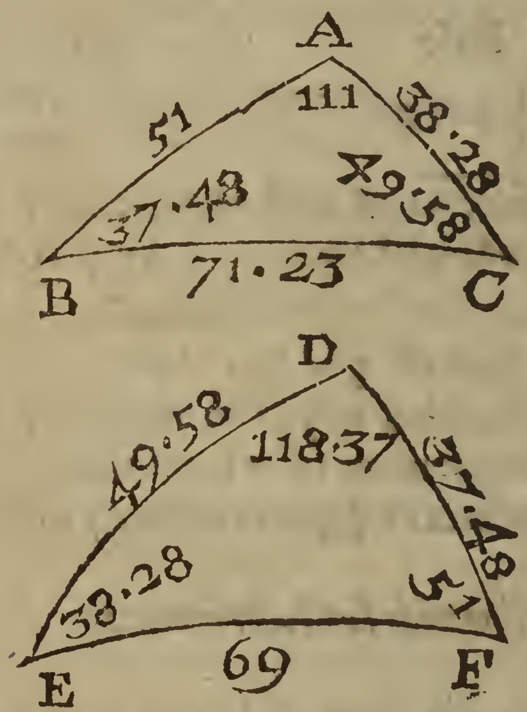
Or,

As the sine of one of the said sides, To the Secant of the Complement of the other.
So is the difference of the Versed Sines of the third side, and of the Ark of difference between the two including sides,
To the Versed Sine of the angle sought.
And so is the difference of the Versed Sines of the third, and of the sum of the two including sides,
To the versed Sine of the sought angles Complement to 180d.

12. *Three*

12. *Three Angles to find a Side.*

The work here for the Canon or Tables, will be by changing the Angles into Sides, the general Rule for changing all the parts of a Triangle, is to draw a new-Triangle, and let the angles be wrot against their Opposite sides, and these against those, only taking the Complements of the greatest Angle, and greatest side opposed thereto to 180*d*, this for most convenience that the sides or angles of the new framed Triangle may not be too large, and so cause recourse to the Opposite Triangle, otherwise the Complements of any side and its opposite angle to 180*d*, might as well have been taken.



But for this Case, seeing there are only angles to be changed into sides, take the Complement of the greatest angle to 180*d* and proceed as if there were three sides given to find an angle.

But the Proportion in Versed Sines, &c. without any such change will be,

As the difference of the Versed Sines of the sum, and of the difference of any two angles adjacent to the side sought.
Is to the Diameter,

Or,

As the Sine of one of the said angles,
Is to the Cosecant of the other :

I

So is the difference of the Versed Sines of the third or Opposite angle, and of an Ark left by substracting one of the including angles from the Complement of the other to a Semicircle,

To the Versed Sine of the side sought.

And so is the difference of the Versed Sines of the third angle, and of an Ark made by adding one of the including Angles to the Complement of the other to a Semicircle.

To the Versed Sine of the sought sides Complement to 180d.

Thus having finished the Cases, it is to be intimated that the Proportions here used in Versed Sines are variously demonstrated in diverse Writers, but in most the latter part for finding the Complement of an angle to 180^d, is quite omitted, those that have demonstrated the former part, do it in these tearms following.

As the Rectangle of the sines of the containing sides,

Is to the Square of the Radius:

So is the difference of the Versed Sines of the Base, or third Side, and of the Ark of difference between the two including sides,

To the Versed Sine of the angle sought, which the Reader may see in *Lansberg*, *Regiomontanus*, *Snellius*, *Pitiscus*, and the learned *Clavius*, who makes 15 Cases, and twice as many Scheams, to demonstrate this part of it, I shall only shew how it may be inferred from the common Proportions in use fitted to the Tables demonstrated by the Lord *Napier*, Mr *Oughtred*, Mr. *Norwood*.

We have two Proportions delivered in Rectangles and Squares the one for finding an angle, the other to find its Complement to 180^d. The two first tearms are the Proportion between the Rectangle of the Sines of the containing sides, and the Square of the Radius; these two tearms being divided by the Sine of one of those sides, the Quotient will be the Sine of the other, if the same Divisor divide the Square of the Radius, the Quotient will be the Secant of the Complement of the Ark belonging to the Divisor, because,

As the Sine of an Ark, To Radius, So is the Radius, To the Secant of that Arks Complement; But if any common Divisor divide eny two Tearms of a Proportion, the Dividends will be Acquimultiplex to the Quotients; and therefore by the Quotients will bear such Proportion each to other as the Dividends, and therefore it holds,

As

As the Rectangle of the Sines of the containing sides,
Is to the Square of the Radius:
So is the Sine of one of those sides,
To the Secant of the Complement of the other.

Again, for the third Tearm, to find an angle it is proposed.

So is the Rectangle of the Sines of the differences of the Leggs from the half sum of the three sides.

Or which is all one,

So is the Rectangle of the Sines of the half sum, and half difference of the Base or third side, and of the Ark of difference between the two including sides,

To the Square of the Sine of half the angle sought,

And so to find the Complement of an angle to 180*d*.

So is the Rectangle of the Sines of the half sum of the three sides, and of the difference of the third side or Base therefrom,

Or which is equivalent thereto,

So is the Rectangle of the Sines of the half sum, and half difference of th Base or third side, and of the sum of the two including sides,

To the Square of the Sine of an Ark, which doubled is the Complement of the angle sought to 180*d*, or the Complement of that Arch to a Quadrant doubled, is the angle sought.

The former of these two expressions of the third Tearm of the Proportion, as being the more facil for memory is now retained; but the latter, (formerly used, and now rejected) agrees best with the Proportion, as applyed to Versed Sines, for the inferring whereof note, that such Proportion, As the difference of two Versed Sines beareth to another Versed Sine, the same Proportion doth the half difference of those Versed Sines, bear to half the Versed Sine of that other Arch: But that is the same that the Rectangle of the Sines of the half sum and half difference of any two Arks doth bear to the Square of the Sine of half that other Arch, which will be thus inferred, because if the said Rectangle and Square be both divided by Radius, the two Quotients will be the half difference of the versed Sines of the two Arks proposed; and half the versed Sine of the 4th Arch.

That

That the Sines of the half sum and half difference of any two Arks are mean Proportionals between the Radius and the half difference of the Versed Sines of those Arks is demonstrated in Mr *Gellibrands* Trigonometry in Octavo, that is,

As the Radius, To the Sine of half the sum of any two arks:
So is the sine of half the difference of those two arks,
To half the difference of the versed sines of those two arks, and therefore the said Rectangle divided by Radius, the Quotient is half the difference of the versed sines of the two Arks.

And that the Sine of any Arch is a mean Proportional between the Radius and half the versed Sine of twice that Arch,

That is,

As the Radius, Is to the sine of an Arch:
So the sine of that Arch, To half the versed sine of twice that Arch, and therefore the Square of the sine of any Arch divided by Radius, the Quotient is the half versed sine of twice that Arch; whence the Rule to make a Cannon of whole Logarithmical versed sines is to take half the arch proposed, and to the Logarithm thereof doubled, or twice wrot down, to add the Logarithm of the number two, and from the sum to substract the Radius.

We have before inferred, that

As the Rectangle of the sines of the containing sides,
Is to the Square of the Radius:
So is the sine of one of those sides,
To the Secant of the Complement of the other, and that by dividing those two Plains by one of those sides; but if we divide the said two Plains, *viz.* the Rectangle of the sines of the containing sides, and the Square of the Radius, by the Radius as a common Divisor, the latter Quotient will be the Radius, and the former the half difference of the versed sines of those Arks whereof the two containing sides are the half sum and the half difference; but those Arks are found by adding the half difference to the half sum to get the greater, and substracting it therefrom to get the lesser; Which is no other then to get the sum and difference of the two containing sides, it therefore holds,

As the Rectangle of the sines of the containing sides,
Is to the Square of the Radius,

Or, *As the sine of one of those sides,*
To the Secant of the Complement of the other:
So is the half difference of the versed sines of the sum and difference of those two sides to the Radius,

And by consequence so is the whole difference to the Diameter, and this being admitted the whole Proportion in all its parts may be inferred from Mr *Daries* Book of the Uses of a Quadrant, where he demonstrates,

That, *As the difference of the versed Sines of the sum and difference of any two sides including an angle,*
Is to the Diameter:
So is the difference of the versed sines of the third side, and of the Ark of difference between the two including sides,
To the versed sine of the angle sought, in that Scheme it lyes,

As M S, *To* G H: *So is* M P, *To* H C.

And I further add,

As M S, *To* G H: as before, *So is* P S, *To* G C.

that is, retaining the two first Tearms of the Proportion, the same it holds for the third and fourth Tearm.
So is the difference of the versed sines of the third side, and of the sum of the two including sides, To the versed sine of the sought angles Complement to 180^{d}.

Now from these Proportions thus Demonstrated, are inferred those others that give the answer in the Squares of Tangents, in order whereto observe,

That if 4 Numbers are Proportional, their Squares are also Proportional (*quamvis non in eadem ratione*) so that any three of those Squares being given, the Square of the 4th will be found by direct Proportion; and the Proportion for making a Table of Natural Tangents from the Tables of natural sines is,

As the Cosine of an Ark, To the sine of the said Ark:
So is the Radius, To the Tangent of the said Ark.

It will therefore hold by 22 *Prop.* of 6th Book of *Euclid,*
As the Square of the Cosine of an Ark, Is to the Square of its sine:
So is the Square of the Radius, To the Square of its Tangent,

Now

Now from the two Demonstrated Proportions for the Tables, the two first Tearms are common to both, and therefore there is the like Proportion between the two latter Tearms of the first Proportion, and the two latter in the second, as between the two latter, and the two former in each Proportion: Now because the latter Proportion finds the Square of the Cosine, and the former the Square of the Sine of the same Ark, it is inferred that the third tearm in the latter Proportion, bears such Proportion to the third Tearm in the former Proportion, as the Square of the Cosine of an Ark, doth to the Square of its Sine, which is the same that the Square of the Radius bears to the Square of the Tangent of the said Ark, it therefore holds when three sides of a Spherical Triangle, are given to find an angle.

As the Rectangle of the Sines of the half sum of the three sides, and of the difference of the Base therefrom,
Is to the Rectangle of the Sines of the differences of the Leggs therefrom:
So is the Square of the Radius,
To the Square of the Tangent of half the angle sought, and by changing the 2d Tearm into the place of the first.
As the Rectangle of the sines of the differences of the Leggs from the half sum of the 3 sides,
Is to the Rectangle of the sines of the half sum of the three sides, and of the difference of the Base therefrom:
So is the Square of the Radius, To the Square of the Cotangent of half the angle sought.

These Proportions are published in order to their Application to the Serpentine Line, which will be accomodated for the sudden operating of any of them; the Axioms to be remembred are not many, the Reader will meet with their Demonstration and Application in Mr *Newtons Trigonometry* now in the Press, and said to be near finished: The four Proportions in plain Triangles, when three sides are given to find an angle without the Cadence of Perpendiculars are demonstrated in the 27 Section of the late *Miscellanies* of *Francis van Schooten.*

The Construction of diverse Instruments will require a Table of the Suns Altitudes to the Hour and Azimuth assigned; And for the Accurate bounding in of the Lines, it may be a Table of Hours and Azimuths

muths to any Altitude assigned; for the easie Calculating whereof, I am desired for the ease and benefit of the Trade, to render this part of Calculation as facil as I can, and therefore shall handle it the more largely.

To Calculate a Table of Hours to all Altitudes in all Latitudes.

The 1. Proportion shall be to find the Suns Altitude in Summer, or Depression in Winter at the hour of 6.

As the Radius, To the sine of the Latitude:
So the sine of the Declination, To the sine of the Altitude or Depression sought

This remains fixed for all that day the Suns Declination supposed not to vary, and then it holds,
As the Cosine of the Declination, To the Secant of the Latitude:
So in Summer is the difference in Winter the sum of the sines of the Suns Altitude proposed, and of his Altitude or Depression at 6
To the sine of the hour from 6 towards noon in Winter, and in Summer also, when the given Altitude is greater then the Altitude of 6, but when it is less towards midnight.

This Proportion also holds for Calculating the Horary distance of any Star from the Meridian.

In like manner to Calculate the Azimuth.

As the sine of the Latitude, To sine of the Declination:
So is the Radius, To the sine of the Suns Altitude or Depression in the prime Vertical, that is, being East or West.

This remains fixed for one day.

Then, *As the Cosine of the Altitude, To the Tangent of the Latitude:*
So in Summer is the difference, and in Winter the sum of the sines of the Suns Altitude proposed, and of his Vertical Altitude or Depression, To the sine of the Azimuth towards noon Meridian in Winter and in Summer also, when the given Altitude is greater then the Vertical Altitude or Depression, but when it is less towards Midnight Meridian.

This

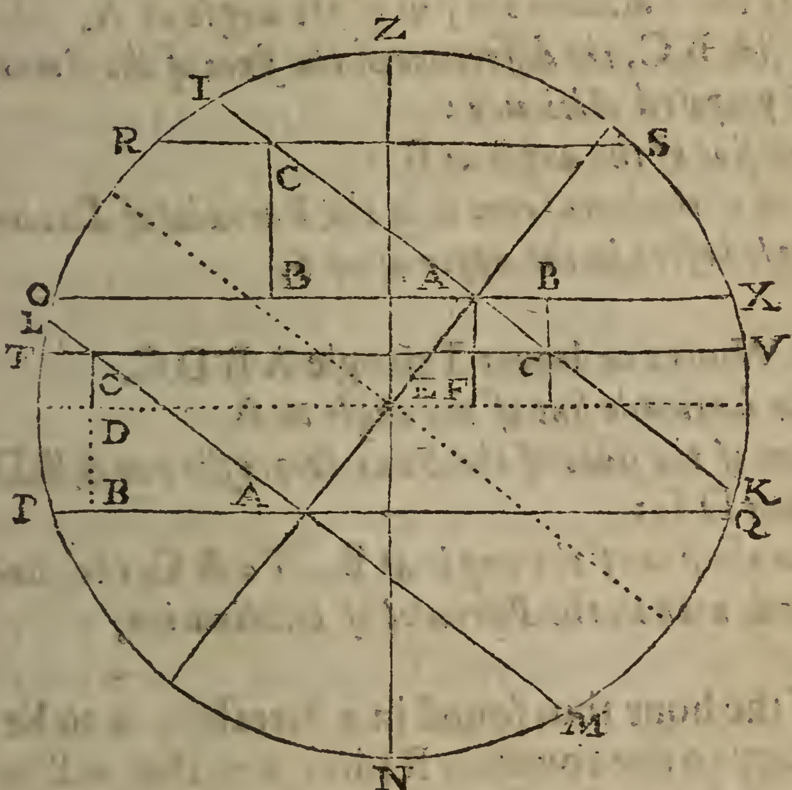
This Proportion is general either for Sun or Stars; when the Declination is less then the Latitude of the place; But when it is more, say as before,

As the sine of the Latitude, To the sine of the Declination:
So is the Radius, To a fourth we may call it a Secant.

Again.

As Cosine Altitude; To the Tangent of the Latitude: So in declinations towards the Depress'd Pole is the sum; but towards the Elevated Pole the difference of this Secant, and of the sine of the Sun or Stars Altitude, To the sine of the Azimuth from the Vertical towards the noon Meridian.

Before Application be made, the latter part of these Proportions being of my own peculiar Invention, and of very great use both for Calculation, and Instrumentally, it will be necessary to demonstrate the same.



For the Hour from the Analemma.

Having in the Scheme annexed drawn the Equator and Horizon, the two prickt Lines passing through the Center, as also the Prime Vertical and Axis, the two streight Lines passing through the same.

Let

Let I X and L M represent two Parralells of Declination on each side the Equator, and O X a Parralel of the Suns Altitude in Summer, and P Q of his Depression in Winter, at the hour of 6, because these Parralells pass through the Intersection of the Parralells of Declination with the Axis. Let R S be a Parralel of Altitude after 6, and T V a Parralel of Altitude before it; from the Intersections of these Parralells of Altitude with the Parralels of Declination let fall Perpendiculars on the Parralells of the Suns Altitude or Depression at 6, and then we shall have divers right Lined right angled Triangles Constituted, in which we shall make use of the Proportion of the sines of angles to their opposite sides an Axiom of common demonstration.

In the Triangle A F E, *As the sine of the angle at* F *the Radius,*
To its Opposite side A E, *the sine of the Declination: So the sine of the Latitude the angle at* E *to* A F, *the sine of the Suns Altitude at 6.*

Again in the two Opposite Triangles A B C, the smaller before the greater after 6.

As the Cosine of the Latitude the sine of the angle at A,
To its Opposite side B C, *the difference of the sines of the Suns Altitude at 6, and of his proposed Altitude:*
So is the Radius sine of the angle at B,
To C A, *the sine of the hour from 6 in the Parralel of Declination in the lower Triangle before, in the upper after 6.*

So in the Winter or lower Triangle A B D C.

As Cosine of the Latitude sine of the angle at A,
To B C, *the sum of the sines of the Suns Depression at 6* B D, *and of his given Altitude* D C:
So is the Radius the sine of the angle at B, *To* A C, *the sine of the hour from 6 towards noon in the Parralel of Declination,*

The sine of the hour thus found in a Parralel, is to be reduced by another Analogy to the common Radius, and that will be,

As the Radius of the Parralel I A, *the Cosine of the Declination,*
Is to the common Radius E Æ:
So is any other sine in that Parralel.
To the sine of the said Arch to the common Radius.

Now it rests to be proved that both these Analogies may be reduced

ced into one, and that will be done by bringing the Rectangle of the two middle Tearms of the first Proportion with the first Tearm under them as an improper Fraction to be placed as a single Tearm in the second Proportion, being in value the answer found in the Parralel, and then we have the Rule of three to Operate as it were in whole Numbers and mixt. The Proportion will run,

As the Cosine of the Declination, To Radius: So the said Improper Fraction, To the Answer.

and so proceeding according to the Rules of Arithmetick.

The Divisor will be the Rectangle of the Cosine of the Declination, and of the Cosine of the Latitude, one of the middle Tearms would be the Square of the Radius, and the other the former sum or difference.

Now if any two Tearms of a Proportion be divided by a common Divisor, the Dividends being Equimultiplex to the Quotients, the Quotients bear the same mutual Proportion as the Dividends by 18th *Propos. 7 Euclid.*

So in this instance if the Rectangle of the Cosines both of the Latitude and of the Declination be divided by one of those Tearms, the Quotient will be the other, and if the Square of the Radius be divided by the Sine of an Arch, the Quotient will be the Secant of that Arks Complement; So in the present Example, if the former Rectangle be divided by the Cosine of the Latitude, the Quotient is the Cosine of the Declination, if the Square of the Radius be divided by the same Divisor, the Quotient is the Secant of the Latitude, likewise if both those Plains were divided by the Cosine of the Declination, the Quotients would be the Cosine of the Latitude, and the Secant of the Declination, it therefore holds,

As the Cosine of the Declination, To the Secant of the Latitude,
Or, *As the Cosine of the Latitude, To the Secant of the Declination:*
So is the former sum or difference of sines, To the sine of the hour from 6, which was to be proved.

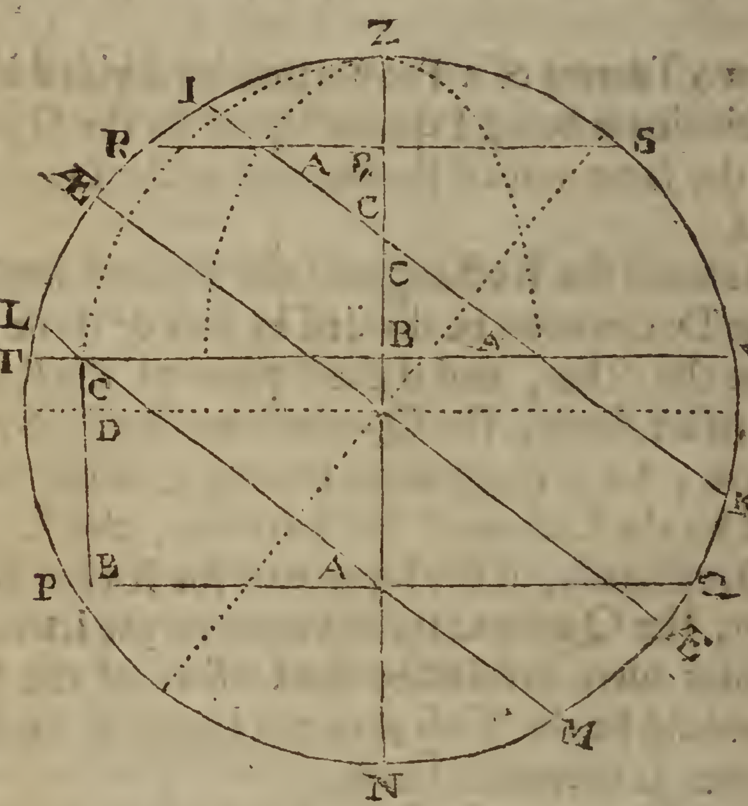
Corrollarie.

As the Radius, To the sine of an Arch in a lesser Circle or Pararlell: So is the Secant of that Parralell, To the sine of the said Arch, to the common Radius.

Hence may be observed a general Canon for the double or compound

Pound Rule of three, divide the Tearms into two single Rules, by placing two Tearms of like Denomination in each Rule, and the other remaining Tearm may in most Cases be put among either of these two Tearms of like Denomination, and then by arguing whether like require like, or unlike, the Divisor in each single Rule, may be discovered, and then it will hold in all Cases,

As the Rectangle or Product of the two Divisors,
Is to the product of any two of the other Tearms:
So is the other Tearm left,
To the Number sought,



For the Azimuth.

Having drawn the Horizon and Axis, the two prickt Lines, the Vertical Circle Z N, and the Equinoctial Æ Æ, the Parralels of Declination I K and L M, draw T V a Parralel of lesser Altitude then that in the Vertical, and R S a Parralel of greater Altitude; Draw also P Q a Parralel of Depression equal to the Vertical Altitude, in the point C aboue the Center the Point A being as much below it b,eing the point where the Parralel of Declination intersects the Vertical

cal Circle, and from the point C in the leſſer parralel of Altitude, let fall the perpendicular C B on the parralel of Depreſſion P Q, by this means there will be Conſtituted divers right lined, right angled Triangles, and through thoſe Points where the parralel of Declination, and parralels of Altitude interſect, are drawn Elipſes prickt from the Zenith to repreſent the Azimuths, and in the three ſeveral Triangles thus Conſtituted, the ſide A B meaſureth the quantity of the Azimuth in the parralel of Altitnde, and B C in the two upper Triangles is the difference of the ſines of the Suns propoſed Altitude, and of his Altitude in the prime Vertical: But in the lower Triangle the ſum of them, it then holds by the Proportion of the ſines of Angles to their oppoſite ſides.

In the two upper Triangles,

As the Coſine of the Latitude, the ſine of the angle at A,
To its oppoſite ſide B C, *the difference of the ſines of the Suns Vertical, and of his propoſed Altitude:*
So is the ſine of the Latitude, that is the ſine of the angle at C,
To its oppoſite ſide B A, *the ſine of the Azimuth from the Eaſt and Weſt,*

And the like in the lower Triangle, onely there the third Tearm, B C, is the ſum of the ſines of the Suns Vertical Depreſſion, and of his given Altitude:

Such Proportion as as the Coſine of an Ark doth bear to the ſine of an Ark, doth the Radius bear to the Tangent of the ſaid Ark, this being the Canon by which the natural Tangents are made from the natural ſines, and therefore we may change the former Proportion, and inſtead thereof ſay,

As the Radius, To the Tangent of the Latitude:
So the ſaid ſum or difference of Sines,
To the Sine of the Azimuth in the Parralel of Altitude:

The anſwer falling in a Parralel or leſſer Circle is to be reduced to the common Radius by another Analogy, and that is

As the Coſine of the Altitude (the Radius of the parralel)
To the Radius:
So any ſine in the ſaid parralel,
To the like ſine in the common Radius.

Now it is to be proved that both theſe Proportions may be brought

into one, and that will be as before, by making an improper Fraction whose Numerator shall be the Rectangle of the two middle Tearms of the former Proportion, the first Tearm, *viz.* the Radius being the Denominator, and placing this as the third Tearm in the second Proportion, and then those that understand how to operate the Rule of three in whole Numbers and mixt, will find their Divisor to be the Rectangle or Product of the Cosine of the Altitude, and of the Radius; and the Dividend the Product of the three other Tearms, namely, of the Tangent of the Latitude, the Radius, and the former sum or difference of sines, whence it holds,

As the Rectangle of the Cosine of the Altitude, and of the Radius,
Is to the Rectangle of the Tangent of the Latitude, and of the Radius:
So is the former sum or difference of sines,
To the sine of the Azimuth.

The Reader may presently espy that the two former Tearms of this Proportion may be freed from the Radius by dividing them both thereby, and the Quotients will be the Cosine of the Altitude, and the Tangent of the Latitude,

It therefore holds,

As the Cosine of the Altitude, To the Tangent of the Latitude:
So in Summer is the difference, in Winter the sum of the sines of the Suns Vertical and proposed Altitude, To the sine of the Azimuth from the Vertical.

This is general either for Sun or Stars, when their Declination is less then the Latitude of the place; but when it is more, the Case doth but little vary.

In the Scheme annexed fitted to the Latitude of the *Barbados* having drawn H H the Horizon, P P the Axis, Æ Æ the Equator, Z A the Vertical draw two parralells of Declination F R, K A continued till they intersect the Vertical prolonged, draw the parralel of Altitude B ☉, and parralel thereto from the Point A draw A E, Then doth the latter part of the Proportion lye as evident as before,

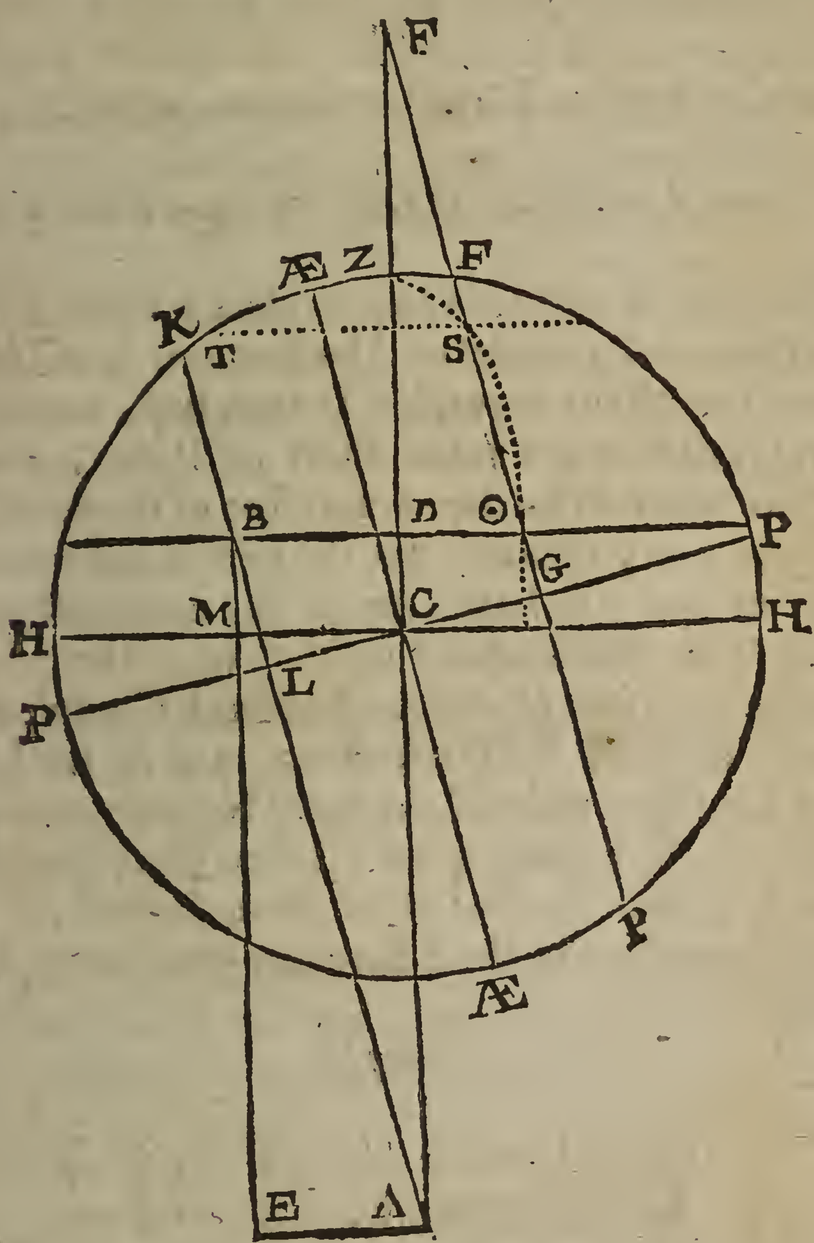
In the right angled Triangle C G F right angled at G,

As the sine of the Latitude the angle at F,
To its Opposite side C G *the sine of the Declination,*
So the Radius the angle at G, *To the Secant* C Z F.

Again,

Again in Summer.

As the Cosine of the Latitude the angle at ⊙,
To its opposite side D Z F, *the difference between the former Secant and the sine of the Altitude* :
So is the sine of the Latitude, the angle at F,
To its opposite side D ⊙ , *the sine of the Azimuth from the Vertical in the Parralel of Altitude.*



In Winter,

As Cosine Latitude angle at A,

To B E *the sum of the former Secant equal to* E M, *and of the sine of the Altitude* M B:

So is the sine of the Latitude the angle B,

To A E, *equal to* B D *the sine of the Azimuth in a Parralel as before, to be reduced to the common Radius.*

From this Schem may be observed the reason why the Sun in those Latitudes upon some Azimuths hath two Altitudes, because the Parralel of his Declination F R intersects, and passeth through the Azimuth, namely, the prickt Ellipsis in the two points S, ☉.

I now proceed to the Use in Calculating a Table of Hours.

For those that have occasion to Calculate a Table of Hours to any assigned Altitude and parralel of Declination, it will be the readiest way to write down all the moveable Tearms first, as the natural sines of the several Altitudes in a ruled sheet of Paper, and then upon a peice of Card to write down the natural sine of the Suns Altitude at 6 and removing to every Altitude, get the sum or difference accordingly, which being had, seek the same in the natural sines, and write down the Log^m that stands against it, then upon the other end of the piece of Card get the sum of the Arithmetical Complements of the Logarithmical Cosine of the Declination, and of the Logarithmica Cosine of the Latitude, and add this fixed Number to the Logme before wrote down; by removing the Card to every one of them, and the sum is the Logme of the sine of the Hour from 6, if the Logmes be well proportioned out to the differences which may be sufficiently done by guess.

Example.

Comp Latitude 38^d 28' *Ar Comp* — 0, 2061683
Comp Delinat 66, 29 *Ar Comp* — 0,0376572
fixed Number———,2438255
Let the Altitude be 36^d 42' Nat Sine 5976251
Natural sine Altitude at 6———3124174
difference ——2852077 *Log against* 9 4550441
Sine of 30^d the hour from 6 towards noon——9, 6988696

Another

Another Example.

N S Altitude at 6———3124174
Let the Altitude be 13^{d}46' N S 2379684
difference———744490 Logm———— 8,8715646
The former fixed Number———————————0,2438255
Sine of 7^d 30' the hour from 6 towards midnight 9,1153901
because the Altitude is less then the Altitude of 6.

This method of Calculation will dispatch much faster then the common Canon, when three sides are given to find an angle; the Azimuth may in like manner be Calculated but will be more troublesome not having so many fixt Tearms in it, and having got the hour, the Azimuth will be easily found; in this Case we have two sides and an angle opposite to one of them given, to find the angle opposite to the other, and the Proportion, will hold,

As the Cosine of the Altitude,
To the sine of the hour from the Meridian:
So the Cosine of the Declination,
To the sine of the Azimuth from the Meridian.

And in this Case the three sides being given, we may determine the affection of any of the angles.

If the Sun, or Stars have declination towards the depressed Pole, the Azimuth is always Obtuse, and the hour and angle of position Acute.

If the Sun, *&c.* have declination towards the Elevated Pole, but less then the Latitude of the place the angle of Position is always acute, the hour before 6 obtuse, the hour and Azimuth between the Altitude of 6, and the Vertical Altitude both acute, afterwards the hour acute, and the Azimuth obtuse.

But when the Sun or Stars come to the Meridian between the Zenith and the Elevated Pole, as when their declination is greater then the Latitude of the place, the Azimuth is always acute, the hour before 6 obtuse, afterwards acute.

The angle of position from the time of rising to the remotest Azimuth from the Meridian is acute, afterwards obtuse.

Another

Another General Proportion for the Hour.

As the Radius,
To the Tangent of the Latitude:
So the Tangent of the Suns declination,
To the sine of the hour of rising from six:

Again.

To the Rectangle of the Cosine of the Latitude, and of the Cosine of the declination, Is to the Square of the Radius: So is the sine of the Altitude, To the difference of the Versed sines of the Semidiurnal Ark, and of the hour sought.

Having got the Logarithm of this difference, take the natural number out of the Sines or Tangents that stands against it, accordingly as the Logme is sought, and in Winter add it to the natural sine of the hour of rising from 6, the sum is the natural sine of the hour from 6 towards noon.

In Summer get the difference between this fourth and the sine of rising from 6, the said difference is the natural sine of the hour from 6 towards noon, when the Number found by the Proportion is greater then the sine of rising, towards midnight when less.

The Canon is the same without Variation as well for South declinations as for North, and therefore we may by help thereof find two hours to the same Altitude.

Example.

Comp Lat 38^d 28′ *Ar Comp* Sine——————,2061683
Comp declin 77^d *Ar Comp*——————,0112761
fixed number——————,2174444
Let the Altitude be 14^d 38′ Sine——————9,4024889
Natural sine against it——41660,00——————9,6199333
Nat sine of rising from 6—29058,79
Sum————70718,79 N sine of 45^d the hour from six in Winter.
Difference——12601,21 Sine of 7^d 14′ the hour from 6 in Summer towards noon
to the former Altitude, and like declination towards Elevated Pole.

Another

Another Example for the same Latitude and Declination.

			Logme
Let the Altitude be $20^{d}\ 25'$ Sine			9,5426321
The former fixed Logme			2174444
Natural sine against it	5754811	Sum	9,7600765
Natural sine of rising	2905879		
Sum	8660690 sine 60^{d} the ho from 6 in Wint		
Difference	2848932 sine $16^{d}\ 33'$ the hour from 6 in Summer towards noon;		

And thus may two hours be found at one operation for all Altitudes less then the Winter Meridian Altitude, to be converted into usual Time by allowing 15^{d} to an hour, aud 4^{m} to a degree.

To Calculate a Table of the Suns Altitudes on all Hours.

As the Secant of the Latitude,
To the Cosine of the Declination,
Or which is all one,

As the Square of the Radius: To the Rectangle of the Cosines, both of the Latitude and of the Declination: So is the sine of the hour from 6. To a fourth, namely, in Summer the difference of the sines of the Suns Altitude at 6, and of the Altitude sought, in Winter the sum of the sines of the Suns Depression at 6, and of the Altitude sought.

Having wrote down the Logarithmical sines of the hour from 6 on the Paper, at one end of a piece of Card may be wrote down the sum of the Logarithmical Cosines of the Latitude and Declination, and add the same to the sine of the hour rejecting the double Radius, and take the natural sine that stands against the sum sought in the Logarithmical sines; having this natural sine, get the sum and difference of it, and of the natural of the Suns Altitude at 6, the sum is the natural sine of the Altitude for Summer declinations, and the difference for Winter Declinations when the sine of the Suns Altitude is the lesser: But when it is the greater, the said difference is the natural sine of the Altitude for hours beyond 6 towards midnight.

Example.

			Log^m
Complement Latitude	38d 28′	Sine	9,7938317
Compl Declination	66 29	Sine	9,9623428
fixed Logme			19,7561745
Let the hour be 30d that is 2 hours before and after six in Summer sine			9,6989700
Sum			9,4551445

Natural sine against it — 2851308

Nat sine of the Altitude at 6 — 3124174

Sum — 5975482 Sine of 36d 42′ }
Difference — 272866 Sine of 1 34 } being the two Altitudes for 4 and 8 in the morning or afternoon, in Summer.

Another Example.

Let the hour be 45d from six	Sine	9,8494850
the former fixed Logme		19,7561745
N Sine against it — 4032791 } N Sine of Altitude at 6 — 3124174 }	Sum	9,6056595

Sum — 7156965 Sine of 45d 42′ }
D.fference — 908617 Sine — 5 13 } being the two Altitudes for the hours of 9 or 3 in Summer or Winter for Declination 23d 31′ both towards the Elevated and Depressed Pole.

By the former Canon was the following Table of Altitudes calculated, and that with much celerity beyond any other way, it will not be amiss to Calculate the Suns Altitude at 6 by the natural Tables only, however the Logarithms will accurately discover the natural sine of it, if duly Proportioned by the differences.

A Table of the Suns Altitudes for each Hour and quarter for the Latitude of London.

	North.			South.		
Declination.	23d, 31	13	*Equator.*	13d	23 31	
XII.	61d, 59	51, 28	38, 28.	25, 28	14, 57	XII.
·	61, 49	51, 21	38, 22	25, 20	14, 52	
*	61, 23	50, 59	38, 4	25, 8	14, 39	
·	60, 40	50, 24	37, 36	24, 43	14, 18	
I.	59, 42	49, 36	36 56	24, 10	13, 48	XI.
	58, 29	48, 35	36, 5	23, 26	13, 9	
	57, 4	47, 24	35, 4	22, 34	12, 23	
	55, 29	46, 1	33, 55	21, 33	11, 29	
II.	53, 45	44, 3[illegible]	32, 36	20, 25	10, 28	X.
	51, 53	42, 51	31, 8	19, 8	9, 19	
	49, 54	41, 4	29, 34	17, 44	8, 3	
	47, 51	39, 10	27, 53	16, 14	6 41	
III.	45, 42	37 13	26, 6	14, 38	5 13	IX.
	41, 31	35, 9	24, 12	12, 55	3, 39	
	41, 16	33, 2	22, 15	11, 7	1, 59	
	38, 59	30, 52	20, 13	9, 15	0, 15	
IIII.	36, 42	28, 37	18. 7	7, 17		VIII.
	34, 23	26, 22	15, 58	5, 17		
	32, 4	24, 5	13, 46	3, 12		
	29, 43	21, 46	11, 32	1, 4		
V.	27, 23	19, 27	9, 16		VII.	
	25, 4	17, 7	6, 58			
	22, 46	14, 47	4, 39			
	20, 28	12, 27	2, 20			
VI.	18, 12	10, 9	00, 00	VI.		
	15, 58	7, 51				
	13, 46	5, 34				
	11, 37	3, 20				
VII.	9, 30	1, 7	V.			
	7, 25		*Declination.*	*Ascensionall difference.*		
	5, 24					
	3, 27		13d	16d, 54′		
VIII.	1, 34	IIII.	23, 31	33, 12	R	*A*

A Table of the Suns Altitudes for every 5 degrees of Azimuth *from the Meridian for the Latitude of* London.

North.			South.		
Declination.	2,^{d}. 31	13^{d}	*Equator.*	13^{d}.	23^{d}, 31′
Mer Alt	61, 59	51, 28	38^{d} 28′	25^{d}, 28′	14^{d}, 57
5	61, 55	51, 23	38, 21	25, 21	14, 49
10	61, 42	51, 7	38, 2	24. 57	14, 22
15	61, 21	50, 40	37. 30	24, 20	13, 39
20	60, 51	50, 3	36, 44	23, 27	12, 39
25	60, 11	49, 14	35, 45	22, 16	11, 19
30	59 21	48, 13	34. 32	20, 51	9, 43
35	58, 20	46, 59	33, 3	19, 7	7, 46
40	57, 7	45, 31	31, 19	17, 7	5, 31
45	55, 43	43, 50	29, 19	14, 50	2, 57
50	54, 3	41, 53	27, 3	12, 13	0, 03
55	52, 7	39, 39	24, 30	9, 21	
60	49, 56	37, 9	21, 40	6, 11	
65	47, 27	34. 22	18, 34	2, 46	
70	44, 39	31, 18	15, 12		
75	41, 34	27, 58	11, 37		
80	38, 10	24, 23	7, 51		
85	34, 32	20, 38	3, 59		
90	30 39	16, 42			
95	26, 34	12, 40			
100	22, 28	8, 41			
105	18, 20	4, 44	*Declinat*	*Amplitude.*	
110	14, 15	0, 54	13^{d},	21^{d}, 12′	
115	10, 19		23, 31′	39^{d}, 54′	
120	6, 36				
125	3, 7				

Many

Many Tables may want the naturall Tables standing against the Logarithmicall; therefore the method of Calculation by the Logarithmicall Tables onely, is not to be omitted, albeit we wave the common Proportions, when three Sides are given to find an Angle.

A general Proportion derived from the book of the honorable Baron of *Marchiston*, which may bee wrought on a Serpentine Line without the use of Versed Sines, or finding the half distance between the 7th Tearme and the Radius, not encumbred with Rectangles, Squares, or Differences of Sines, or Versed Sines.

Three Sides to find an Angle; two of them or all three being lesse then Quadrants.

By a supposed perpendicular, which need not to be named.

As the Tangent of half the greater of the containing sides,
To the Tangent of the half sum of the other sides:
So is the Tangent of half their difference,
To the tangent of a fourth Arke.

If this Arke be greater then the half of the first assumed side; namely, then the Arke of the first Tearme, in the Proportion, the (Supposed perpendicular falls without) Angles opposite to the two other Sides are of a different Affection, the greatest side subtending the Obtuse angle, and the lesser the Accute.

If the Angle Opposite to the greater of the other Sides be sought, Take the difference; if to the lesser, the Sum of the 4th Arke, and of half the containing Side, which half is the first Tearme in the Proportion,

Then,

As the Radius, To the Cotangent of the other Containing Side:
So the Tangent of the said Sum or difference,
To the Cosine of the Angle sought.

The first Tearme above needs no Restraint, but when one of the Containing Sides is greater then a quadrant:

If the 4th Arke be less then the half of the first assumed Side, the

Perpendicular falls within, in this Case, the two Angles, opposite to the two other sides may be found, being both Acute.

Get the Sum and difference of half the first assumed Side, and of the 4th Arke, the Sum is the greater Segment, and the Difference or residue, the lesser Segment; the Perpendicular alwayes falling on the side assumed, first into the Proportion; Then,

As the Radius, To the Cotangent of the { *greater* / *lesser* } *of the other Containing Sides, So is the Tangent of the* { *greater* / *lesser* } *Segment. To the Cosine of the Angle sought.*

From these general Directions is derived this Canon for Calculating the Azimuth,

As the Tangent of half the Complement of the Altitude,
To the Tangent of the half sum of the Sun or Stars distance from the elevated pole, and of the Complement of the Latitude:
So is the Tangent of half their difference,
To the Tangent of a 4th Arke.

If this Ark be less then half the Complement of the Altitude, the Azimuth is Acute; if more obtuse, in both Cases, get the difference of these two Arkes, if there be no difference, the Azimuth is 90^{d} from the Meridian; Then,

As the Radius, To the Tangent of the Latitude;
So the Tangent of the said Arke of difference,
To the Sine of the Azimuth from the prime Verticall.

This when the Sun or Stars do not come to the Meridian between the Zenith and the elevated pole: but when they do, Let the sum of the 4th Ark, and of half the Complement of the Altitude, be the third Tearme in the latter Proportion.

This is a ready Way to Calculate a Table of Azimuths; two Tearms in each Proportion being fixed for one Declination; and the Azimuth being known, the Hour may be found by a single Operation,

As

As the Cosine of the Declination,
Is to the Sine of the Azimuth, from the Meridian:
So is the Cosine of the Altitude,
To the Sine of the Hour, from the Meridian.

Example, for 13ᵈ *of North Declination.*

77ᵈ Complement of Declination the Polar distance.
38. 28 Complement of Latitude at *London.*
115, 28 Sum: half sum 57ᵈ, 44′ Tangent 10, 1997231
38, 32 difference: half diff. 19, 16 Tangent 9 5434994
fixed for that declination —— 19, 7432225
Altitude 4ᵈ, 44′ Comp. 85ᵈ 16′ half 42ᵈ 38′ Tang. 9, 9640811
Tangent of —— 31, 1 —— 9, 7791414
difference —— 11, 37 Tangent 9, 3129675
Tangent of 51ᵈ 32′ the Latitude —— 10, 0999135
Sine of 15ᵈ, the Azimuth 9, 4128810
from East or West Northwards, because the Ark found by 1st Operation was less then half the Complement of the Altitude.

Another Example for the Altitude 34ᵈ 22′

Logⁿ
The fixed Number —— 19, 7432225
Compl. Altitude 55ᵈ 38′ half } 27, 49ᵈ′ Tangent 9, 7223147
Tangent of —— } 46, 23 —— 10, 0209078
Difference —— 18, 34 Tangent 9, 5261966
Tangent of the Latitude —— 10, 0969135
The Sine of 25ᵈ the Azimuth from —— 9, 6261101
East to West Southwards because the first Ark was more then half the Coaltitude.

The hours to these two Azimuths will be found by the latter Proportion to be —— { 98ᵈ, 54′ / 50, 9 } from Noon.

To Calculate the Suns Altitude on all Hours and Azimuths.

The first operation shall be to find such an Ark as may remain fixed in one Latitude to serve to all Declinations in both Cases: So that but one Operation more need be required.

The Proportion to find it is
As the Radius to the Cotangent of the Latitude,
So is the Sine of any Hour from 6, or Azimuth from the Vertical
To the Tangent of a fourth Ark.

This 4th Arke (if the Azimuth be accounted from the Vertical, that is, from the points of East or West towards noon Meridian) Is the Altitude that the Sun shall have, being in the Equinoctial, upon that Azumuth, and so one of the Quæsita.

If the hour from 6 be accounted upward on the Equinoctial, this 4th Ark is the ark or portion of the hour Circle, between the Equinoctial and Horizon.

This Ark for Hours and Azimuths beyond 6, or the Vertical, towards the midnight Meridian, is the Depression under the Horizon, according to the Denominations already given it.

For the Altitudes on all Hours.

When the Sun is in the Equinoctial,
As the Radius, is to the Cosine of the Latitude:
So is the Sine of the Hour from 6
To the Sine of his Altitude.

In all other Cases,

If the Hour from noon be $\left\{\begin{array}{l}\text{more}\\\text{less}\end{array}\right\}$ then 6 Substract the Equinoctial Arks that Correspond to such parts of time as you would Calculate Altitudes for; out of the Suns distance from the $\left.\begin{array}{l}\text{Depressed}\\\text{Elevated}\end{array}\right\}$ Pole, and then it will hold,

As

As the Cosine of the Ark found by the first common Proportion,
Is to the Cosine of the Ark remaining;
So is the Sine of the Latitude,
To the Sine of the Altitude sought:

For the more speedy Calculating a Table of the Suns Altitudes for this Latitude to any Declination, there is added a Table of these Equinoctial Arks for every hour and quarter, as also for every 5^d of Azimuth; The use whereof shall be illustrated by an Example or two, observing by the way, that the same Ark belongs to two hours alike remote on each side from six, as also like Arks to two Azimuths equally remote on each side the Vertical.

Hours on each side six,			*Azimuths on each side the Vertical.*	
VI	0, 00	fixed Arks		d ′
	5, 55		5d	3, 59
	8, 49		10	7, 51
VII	11, 37	V	15	11, 37
	14, 19			
	16, 55		0	15, 12
	19, 22		25	18, 34
VIII	21, 40	IIII	30	21, 40
	23, 49			
	25, 48		35	24, 30
	27, 39		40	27, 3
IX	29, 19	III	45	29, 19
	30, 52			
	32, 13		50	31, 19
	33, 27		55	33, 3
X	34, 32	II	60	34. 22
	35, 28			
	36, 17		65	35, 45
	36, 57		70	36, 44
XI	37, 30	I	75	37, 30
	37, 55 $\frac{1}{2}$			
	38, 13 $\frac{1}{2}$		80	38, 2
	38, 24 $\frac{1}{2}$		85	38, 21
XII	38, 28		90	38, 28

Equinoctial Altitudes or Depressions.

Example.

Example.

Admit it were required to Calculate the Suns Altitude for 2 or 10 of the Clock, when his declination is 23^d 31′ North.

Suns distance from the Elevated Pole — 66 : 29
fixed Ark for 2 or 10 is —————— 34 : 32

Residue ——————— 31 : 57

Complement of the Residue	58^d	3′	Sine ——	9,9286571
Sine of ————————	51	32	the Latitude —	9,8937452
				19,8224023
Sine of ————————	55	28	Com first ark	9,9158200
Sine of ————————	53	45	the Altitude sought ——	9,9065823

So if it were required to Calculate the Suns Altitude for the hour of 5 in the morning when his declination is 20^d North

☉ distance from depressed Pole — 110^d
fixed Ark ——————— 11 : 37

The Residue is 81^d, 33′ Or ——— 98 23

Sine of 8^d, 23′ the Compl of the Residue ——	9, 1637434
Sine of the Latitude ————————	9,8937452
	19, 0574886
Sine of 78^d 23′ the Compl fixed ark ——	9, 9910119
Sine of 6^d 42′ the Altitude sought ——	9, 0664767

For the Altitudes on all Azimuths.

As the Sine of the Latitude,
Is to the Cosine of the Equinoctial Altitude:
So is the Sine of the Declination,
To the Sine of a fourth Ark.

Get the sum and difference of the Equinoctial Altitude, and of this fourth Ark the sum is the Summer Altitude for Azimuths from the Vertical towards noon Meridian.

The difference when this 4^th Ark is { lesser / greater } then the Equinoctial Altitude is the { Winter / Summer } Altitude for Azimuths from the Vertical towards { Noon / Midnight } Meridian.

Example.

Example.

Let it be required to Calculate Altitudes for the Suns Azimuth 30[d] from the Meridian, that is 60[d] from the Vertical for Declination 23[d] 31′ both North and South.

This will be speedily done, add the Logme of the Sine of the Declination to the Arithmetical Complement of the Logme of the Sine of the Latitude, this number varies not for that declination, and to the Amount add the Logme of the Cosine of the Equinoctial Altitude, the sum rejecting the Radius is the Logarithmical Sine of the 4[th] Ark.

Sine Latitude *Ar Comp* — 0, 1062548
Sine declination ——— 9, 6009901

Sine of 55[d] 28′ the Complement } 9, 7072449
of Equinoctial Altitude ——— } 9, 9158200

Sine ——— 24[d] 49′ ——— 9, 6230649

Equinoct Altitude — 24 22

Sum ——— 59 21 } Summer } Altitude for that
Difference ——— 9 43 } Winter } Azimuth.

Another Example:

The former Number ——— 9, 7072449
Sine 78[d] 23′ Comp of Equinoct 9, 9910119
Altitude for 15[d] of Azimuth ———
Sine of 29[d] 57′ ——— 9, 6982568

Eq Altitude 11 37

Difference — 18 20 } The Summer Altitudes for 15[d] Azimuth
Sum ——— 41 34 } each way from the Vertical to that declination.

The Suns Declination 20[d] North to find his Altitudes for 15[d] Azimuth on each side the Vertical.

Sine of 51[d] 32′ the Latitude *Ar Com* ——— 0, 1062548
Sine of 20[d] the declination ——— 9, 5340517
Sine of 78[d] 23′ Comp Eq Altitude ——— 9, 9910119
Sine of 25[d] 20′ ——— 9, 6313184

S   Sine

Sine of	25d	20′	
Eq Altitude	11	37	
Difference	13	43	The Summer Altitudes for 15′ Azimuth on each side the Vertical to that Declina ion.
Sum	36	57	

By the Arithmetical Complement of a Number is meant a residue which makes that first Number equal to the other :

And so if from a number or numbers given another Number is to be substracted, and instead thereof a third number added, the totall shall be so much encreased more then it should by the sum made of the number to be substracted, and of that was added :

That is to say, in this last Example instead of substracting the Sine of the Latitude from another sum we added the residue thereof, being taken from Radius thereto, and so increased the Total too much each time by the Radius, which is easily rejected.

If the Sun or Stars come to the Meridian between the Zenith and the Elevated Pole, as when their Declination is more then the Latitude of the place, the former Rule of Calculation varies not only the sum of the Equinoctial Altitude, *alias*, the fixed Ark, and of the Ark found by the second Proportion will be more then 90 In this Case the Complement of it to 180*d* is the Altitude sought.

A double Advertisement.

The Declination towards the Elevated Pole supposed more then the Latitude of the place.

If the Complement of the Declination be more then the Latitude of the place also, as in this case it always is for the Sun ; the Sun or such Stars shall have two Altitudes on every Azimuth between the Coast of rising or setting, and the remotest Azimuth from the Meridian ; To find what Azimuths those shall be,

As the Cosine of the Latitude, To Radius ;
So is the Sine of the Declination, To the Sine of the Amplitude,
And *So is the Cosine of the Declination, To the Sine of the remotest Azimuth from the Meridian.*

Between the Azimuth of rising, and the remotest Azimuth, the angle of Position is Acute, afterwards Obtuse.

The

The Sun upon the remoteſt Azimuth, the angle of Poſition being a right angle, will have but one Altitude to find it.
As the Sine of the Declination. To Radius;
So is the Sine of the Latitude, To the Sine of that Altitude.

Example.

In North Latitude 13^d of *Barbados*.

Declination 20*d* North, the Suns Amplitude or Coaſt of riſing 69*d* 23′ from the North, or 20*d* 33′ from the Eaſt Northwards, and his remoteſt Azimuth from the North Meridian 74*d* 4′ His two Altitudes upon the Azimuth of 74*d* from the Meridian 27*d* 27′ the leſſer and 52*d* 27′ the greater, the fixed Ark found by the firſt Operation, being ———————————————— 50*d* 3′
And by the ſecond Operation the Ark found is ———————77 30
Difference being the leſſer Altitude is———27 27
The ſum 127^d 33′ the Comp to 180*d* being the greater Alt is 52 27
Altitude on the remoteſt Azimuth ——————————— 41 07
Upon Azmuths nearer the Meridian then the Coaſt of riſing or ſetting, it need not be hinted that there will be but one Altitude.

The Proportion from the 5th Caſe of Oblique Sphærical Triangles to find the Suns Altitude on all Azimuths, would be

As the Coſine of the Declination, To the Sine of the Azimuth from the Meridian: So is the Coſine of the Latitude, To the Sine of the angle of Poſition.

In ſuch Caſes when it will be acute or obtuſe is already defined, and where two Altitudes are required it will be both, and being accordingly ſo made, the Proportion to find the Altitudes would be,
As the ſine of half the difference of the Azimuth and angle of Poſition,
To the Tangent of half the difference of the Polar diſtance and Colatitude
So the Sine of half the ſum of the Azimuth and angle of Poſition,
To the Tangent of half the Complement of the Altitude.

The Azimuth being an angle always accounted from the Midnight Meridian; but the former Proportion derived from the other Trigonometry in this Caſe is more ſpeedy.

Such Stars as have more Declination then the Complement of the

Latitude never rise nor set ; if their declination be also more then the Latitude of the place, they will have two Altitudes upon every Azimuth, except the remotest from the Meridian, and the Calculation the same as before.

Example for the Latitude of London.

The middlemost in the great Bears Rump, declination 56d 45′, The remotest Azimuth will be *61d 49′* from the Meridian, and the Altitude thereto *69d 26′*.

If that Star have *30d* of Azimuth from the Meridian,

The first ark will be 34*d* 22′ }
The second ark— 61 39 } difference being the lesser Alt 27′ 7′
Sum———96 11 Comp the greater Altitude— 83 49

This will be very evident on a Globe for having rectified it to the Latitude extend a Thread from the Zenith over the Azimuth in the Horizon, then turn the Globe round, and such Stars as have a more utmost remote Azimuth from the Meridian, and do not rise or set will pass twice under the Thread, the Azimuth Latitude and Declination being assigned if it were required to know the time when the Star shall be twice on the same Azimuth it may be found without finding the Altitudes first by 6 h Case of Oblique Spherical Triangles,

As before get the angle of Position.

As the Sine of half the difference of the Complement of the Latitude, and of the Complement of the declination.

To the Sine of half their sum :

So the Tangent of half the difference of the Azimuth from the Meridian, and of the angle of Position,

To the Cotangent of half the hour from the Meridian, to be converted into common time if it relate to Stars : In each Proportion there is two fixed Tearms.

The

The Illustration how these two fixed Arks are obtained, is evident from the Analemma in the Scheme annexed.

Æ E represents the Equator.

H K the Horison, P T the Axis, Z N the Prime, Vertical S D a Parralel of North declination, ⊙ another of South, declination, P S ⊙ and P A N the Arks of two hours Circles between six and noon, and P D T another before 6. In the Triangle E L B right angled, there is given the angle at E, the Complement of the Latitude, the side E L the hour from 6, with the right angle at L to find the side L B the ark of the hour Circle contained between the Equinoctial and the Horzion by 11th Case of right angled Sphærical Triangles, the Proportion will be,

As Æ E *Radius*,
To Æ H *Cotangent Latitude*:
So E L *Sine of the hour from 6*,
To L B *The Tangent of the said Ark*.

From Z draw the Arches Z S and Z ⊙ (being Ellipses) through the Points where the Hour Circle, and Parralels of Declination intersect, and they represent the Complements of the Altitudes sought, and let fall

fall a Perpendicular from Z to R, then will P R be equal to L B, because the Proportion above is the same that would Calculate R P; Which substracted from P S or P ☉ the Suns distance in Summer or Winter from the Elevated Pole rests R S or R ☉: Now in an Oblique Spherical Triangle reduced to two right angled Triangles by the Demission of a Perpendicular, the Cosines of the Bases are in direct Proportion to the Cosines of the Hipotenusals a consequence derived from the general Axiom of the Lord *Napier*, and therefore it holds,

As the Cosine of the Ark R P, *or* B L, that we call the common fixed ark,
Is to the Sine of the Latitude, the Complement of Z P:
So the Cosine of the Ark remaining, that is the Complement of R S or S O,
To the Sine of the Altitude, the Complement of Z S or Z ☉:

This for all hours under 90' from the Meridian, but for those before or after 6 in the Summer it may be observed in the opposite Triangles E A N and E A I, counting the hour E A each way from 6 that the Ark of the hour Circle A N equal to A I, as much as it is above the Horizon in Winter, so much is it below the same in Summer, and the Suns distance from the Elevated Pole then equal to his distance from the Depressed Pole now: and the Zenith distance then equal to the Nader distance now, as is evident in the Triangle T N D,

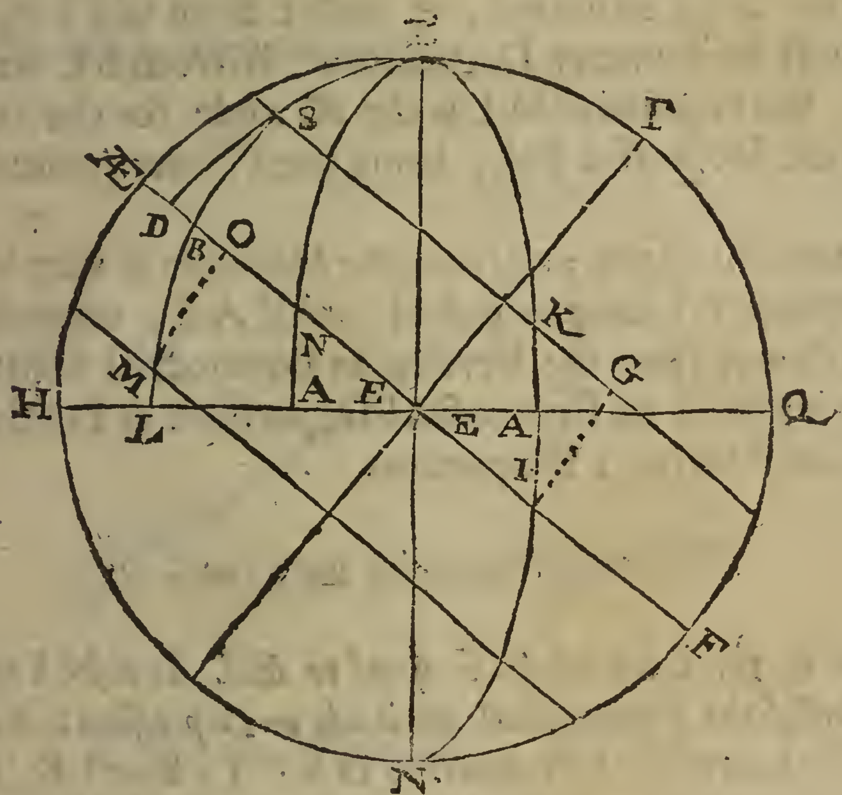
So that in this Case the Sun is only supposed to have Winter instead of Summer Declination, and the Rule for Calculating his Altitude the same as for Winter Altitudes.

In like manner for the Azimuth.

In the Schem following H Q represents the Horizon, Æ F the Equator, S G a Parralel of Summer declination, and another passing through M of Winter declination, Z S L and Z N A two Azimuths between the Vertical and Noon Meridian, Z K I N another between it and the Midnight Meridian, from the Points S M I let fall the Perpendiculars S D, M O, I G representing the ☉ Declination in the Elipses of several hour Circles; So will L S, M L, A K, represent the Altitudes of these 3 Azimuths respectively, according to the proper De-

Declination; for the finding whereof there is given in the Triangle E L B the side E L the Azimuth from the Vertical the angle B E L, the Complement of the Latitude and the right angle, and the Proportion by 11th Case of right angled Sphœrical Triangle is

As Æ H *the Radius,*
To H Æ *the Cotangent of the Latitude:*
So E L *the Sine of the Azimuth from the Vertical,*
To L B *the Tangent of the Equinoctial Altitude to that Azimuth.*



Thus we see the first Proportions common to both, this Case issuing from the 2d Axiom of *Pitiscus*, that in many right angled Sphœrical Triangles having the same Acute angle at the Base, the Sines of the Bases and Tangents of the Perpendiculars are proportional.

For the second Operation to find B S.

Though the Analogy is derivable from the general Proposition of the Lord *Napier*, yet here I shall take it from *Pitiscus* the 1 Axiom.

That in many right angled Sphœrical Triangles, having the same acute angle at the Base the Sines of the Perpendiculars and Hipothenusals are in direct Proportion. There-

Therefore in the Triangle Æ Z B, and D S B it will hold,

As Æ Z *the Sine of the Latitude*, *To* Z B *the Cosine of the Equinoctial Altitude*,

So is D S, *the Sine of the Declination*,

To the Sine of B S, *which is equal to* B M : See 29th *Prop.* of 3d book *Regiomontanus*, I prove it thus, The opposite angles at B are equal, and the angles at D and O are equal, and the side D S equal to M O, it will then be evinced by Proportion, *As the Sine of the angle at* B, *To its opposite side* M O *or* D S : *So is the Radius, that is the angle at* O *or* D, *To its opposite side* S B, *or* M B.

This equality being admitted, if unto L B we add B S, the sum is L S the Altitude for Summer Declination, if from B L we take B M equal to B S, the remainder M L is the Altitude for the like Declination towards the Depressed Pole, being the Winter Altitude of that Azimuth.

But for Azimuths above 90d from the Meridian it may be observed in the two Opposite Triangles E A N, and E A I, counting the Azimuth E A each way from the Vertical its Equinoctial Altitude A N in the Winter is equal to its Equinoctial Depression A I in the Summer, and is to be found by the 1 Proportion.

The second Proportion varies not.

As the Sine of the Latitude N F *equal to* Æ Z, *Is to* N I *equal to* Z B *the Cosine of the Equinoctial Altitude or Depression* : *So is the Sine of the Declination*, I G *equal to* D S : *To Sine* I K, *from which taking* A I, *the Equinoctial Depression rests* A K, *the Altitude sought.*

To Calculate a Table of the Suns Altitude for all Azimuths and hours under the Equinoctial.

This will be two Cases of a Quadrantal Sphœrical Triangle.

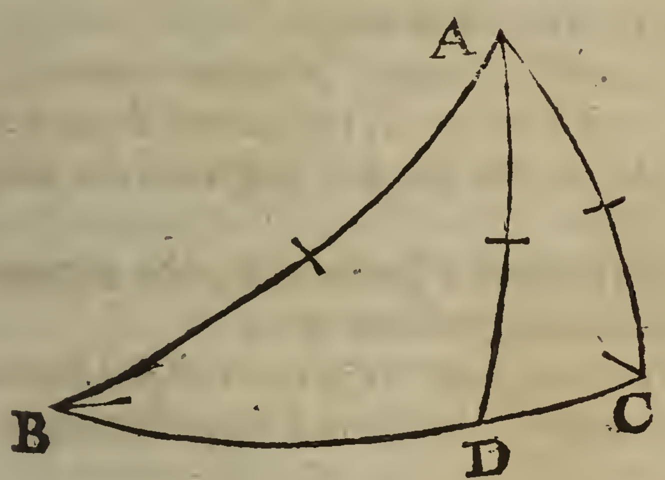
1. *For the Altitudes on all Azimuths.*

There would be given the side A B a Quadrant, the angle at B the Azimuth from the Meridian, and the side A D the Complement of the Suns Declination.

If

If the side B D be continued to a Quadrant, the angle at C will be a right angled, besides which in the Triangle A D C, there would be given A D as before the Complement of the Suns Declination, and A C the measure of the angle at B to find D C the Suns Altitude being the Complement of B D, and so having the Hipotenusal, and one of the Leggs of a right angled Sphœrical Triangle, by the 7th Case we may find the other Legg, the Proportion sutable to this question would be

As the Sine of the Azimuth from East or West,
Is to the Radius:
So is the Sine of the Declination,
To the Cosine of the Altitude sought.



2. For the Altitudes on all Hours.

There would be given the side A B a Quadrant, A D the Complement of the Suns declination with the contained angle B A D the hour from noon, to find the side B D the Complement of the Suns Altitude.

Here again if B D be continued to a Quadrant, the angle at C is a right angle, the side A D remains common, the angle D A C is the Complement of the Angle B A D, See Page 57 where it is delivered, That if a Sphœrical Triangle have one right angle, and one side a Quadrant, it hath two right angles, and two Quadrantal sides, and therefore the angle B A C is a right angle; this is coincident with the 8th Case of right angled Spherical Triangles, the Proportion thereof is,

As the Radius, Is to the Cosine of the Declination,
So is the Sine of the hour from six,
To the Sine of the Altitude sought.

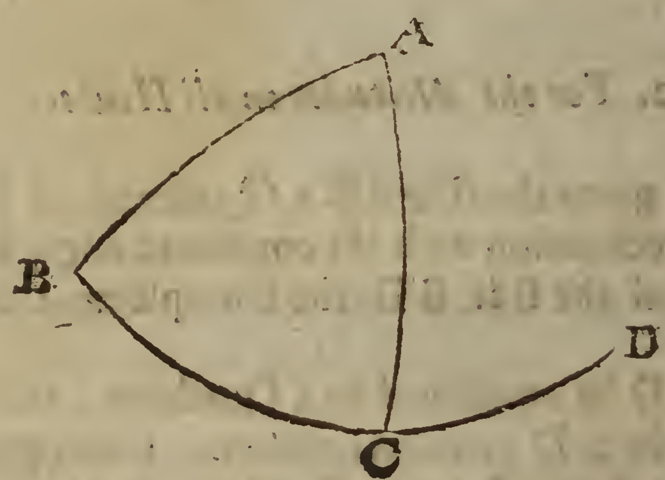
Affections of Sphærical Triangles.

BEcause the last Affection in page 57 is not Braced in the beginning, and a mistake of lesser for greater, in the last Brace but one, I thought fit to recite it at large.

Any side of a Sphærical Triangle being continued, if the other sides together are equal to a Semicircle, the outward angle on the side continued shall be equal to the inward angle on the said side opposite thereto.

If the sides are less then a Semicircle, the outward angle will be greater then the inward opposite angle;

But if the said sides are together greater then a Semicircle, the outward angle will be less then the inward opposite angle.



In the Triangle annexed, if the sides A B and A C together are equal to a Semicircle, then is the angle A C D equal to the angle A B C.

If

If leſs then a Semicircle then is the ſaid angle greater then the angle at B.

But if they be greater, then is the ſaid angle A C D leſs then the angle at B.

By reaſon of the firſt Affection in *page* 58 (which wants a Brace in the firſt Line) after the words two ſides) We require in the firſt, ſecond, and other Caſes of Oblique angled Sphœrical Triangles, the ſum of the two ſides or angles given, to be leſs then a Semicircle.

Before I finiſh the Trigonometrical part, I think it not amiſs to give a Determination of the certain Caſes about Oppoſite ſides and Angles in Sphœrical Triangles, having before ſhewn which are the doubtful, and the rather becauſe that this was never yet ſpoke to.

Two ſides with an Angle oppoſite to one of them, to determine the Affection of the Angle oppoſite to the other.

1. If the given angle be Acute, and the oppoſite ſide leſs then a Quadrant, and the adjacent ſide leſs then the former ſide.

The angle it ſubtends is acute becauſe ſubtended by a leſſer ſide, for in all Sphœrical Triangles the leſſer ſide ſubtends the leſſer angle and the Converſe.

2. If the given angle be acute, and the oppoſite ſide leſs then a Quadrant, and the other ſide greater then the former ſide:

This is a doubtfull Caſe, if it be leſs then a Quadrant it may ſubtend either an Acute or an Obtuſe angle, and ſo it may alſo do if it be greater then a Quadrant, yet we may determine,

That when the given angle is acute, and the oppoſite ſide leſs then a Quadrant, but greater then the Complement of the adjacent ſide to a Semicircle (which it cannot be unleſs the adjacent ſide be greater then a Quadrant) the angle oppoſite thereto will be obtuſe.

3. The given angle Acute, and the oppoſite ſide greater then a Quadrant, and the other ſide greater.

If two ſides be greater then Quadrants, if one of them ſubtends an Acute angle, the other muſt ſubtend an Obtuſe angle; by the 1 ſt. Affection in *pag* 58.

4. The given angle Acute, and the oppoſite ſide greater then a Quadrant; The other ſide cannot be leſſer then the former, by what was now ſpoken.

5. If the given angle be Obtuse, and the opposite side less then Quadrant, the other side less subtends an Acute angle.

6. If the given angle be Obtuse, and the opposite side less then a Quadrant; The other side greater then the former side must of necessity be also greater then a Quadrant, otherwise two sides less then Quadrants should subtend two Obtuse angles, contrary to the first Affection in *p* 58.

7. If the given angle be Obtuse, and the Opposite side greater then a Quadrant; If the other side be greater then the former it will subtend a more Obtuse angle.

8. If the given angle be Obtuse, and the Opposite side greater then a Quadrant, If the other side be less then the former, whether it be lesser or greater then a Quadrant it may either subtend an Acute or Obtuse angle.

But we may determine

That when the given angle is Obtuse, and the Opposite side greater then a Quadrant; If the Complement of the adjacent side to a Semicircle be greater then the said opposite side, the angle subtended by the said adjacent side is Acute.

Two Angles with a side Opposite to one of them, to determine the Affection of the side opposite to the other.

1. If the given angle be Acute, and the opposite side less then a Quadrant; If the other angle be less, the side opposite thereto shall be less then a Quadrant because it subtends a lesser angle.

2. If the given angle be Acute, and the Opposite side less then a Quadrant, If the other angle be greater the Case is ambiguous, yet we may determine,

If the given angle be Acute, and the opposite side lesser then a Quadrant, if the Complement of the other angle to a Semicircle be less then the Acute angle (which it cannot be but when the latter angle is Obtuse) the side subtending it shall be greater then a Quadrant.

3. If the given angle be Acute, and the opposite side greater then a Quadrant, if the other angle be lesser.

Then by 12 of 4 book of *Regiomontanus*, if two Acute angles be un-

unequal, the side opposite to the lesser of them shall be less then a Quadrant.

4. If the given angle be Acute, and the opposite side greater then a Quadrant, If the other angle be greater,

It must of necessity be Obtuse, because otherways two Acute unequal angles, the side opposite to the lesser of them should not be lesser then a Quadrant, contrary the former place of *Regiomontanus.*

5. If the given angle be Obtuse, and the opposite side lesser then a Quadrant, If the other angle be lesser,

It must of necessity be Acute, and the side subtending it less then a Quadrant, otherways two sides less then Quadrants should subtend two Obtuse angles, contrary to 1st Affection in *page* 58.

6. If the given angle be Obtuse, and the opposite side less then a Quadrant, If the other angle be greater,

By 13 *Prop* of 4th of *Regiomontanus*, if a Triangle have two Obtuse unequal angles, the side opposite to the greater of them shall be greater then a Quadrant.

7. If the given angle be Obtuse, and the opposite side greater then a Quadrant, If the other angle be greater or more obtuse then the former; it is subtended by a greater side.

8. If the given angle be Obtuse, and the opposite side greater then a Quadrant; If the other angle be less then the former, the Case is ambiguous; yet we may determine,

That when the given angle is Obtuse, and the opposite side greater then a Quadrant, if the other angle be less then the Complement of the said Obtuse angle to a Semicircle, the side subtending it shall be less then a Quadrant.

The former Cases that are still, and alwais will be doubtful, may be determined when three sides are given.

A Sphœrical Triangle having two sides less then Quadrants and one greater, will always have one Obtuse angle opposite to that greater side, and both the other angles Acute.

A Sphœrical Triangle having three sides less then Quadrants, can have but one obtuse angle (and many times none) and that obtuse angle shall be subtended by the greatest side;

But whether the greatest side subtend an Acute or Obtuse angle can-

cannot be known, unless given or found by Calculation, and that may be found several ways.

First by help of the Leggs or Sides including the angle sought by 15th Case of right angled Sphœrical Triangles.

As the Radius, To the Cosine of one of those Leggs;
So is the Cosine of the other Legg, To the Sine of a fourth Arch.

If the third side be greater then the Complement of the fourth Arch to 90^{d}, the angle included is Obtuse, if equal to it a right angle, if less an Acute angle.

Secondly, By help of one of the Leggs and the Base or Side subtending the angle sought by 7th Case of right angled Sphœrical Triangles.

As the Cosine of the adjacent side, being one of the lesser sides,
Is to the Radius:
So the Cosine of the opposite side, To the sine of a fourth Arch.

If the third side be greater then the Complement of the 4th Arch to 90^{d}, the angle subtended is Acute, if equal to it a right angle, if less an Obtuse angle.

All other Cases need no determination, if a Triangle have two sides given bigger then Quadrants, make recourse to the opposite Triangle and it will agree to these Cases.

If three angles were given to determine the Affection of the sides if they were all Acute the three sides subtending them will be all less then Quadrants.

But observe that though a Triangle that hath but one side greater then a Quadrant, can and shall always have but one Obtuse angle, yet a Triangle that hath but one Obtuse angle may frequently have two sides greater then Quadrants.

In this and all other Cases let the angles be changed into sides, and the former Rules will serue,

I should have added a brief Application of all the Axioms that are necessary to be remembred, and have reduced the Oblique Cases to setled Proportions (with the Cadence of Perpendiculars only to shew how they arise) whereby they will be rendred very facil; as also the Demonstration of the Affections, which may be hereafter added to some other Treatise to be bound with this Book.

Of

Of working Proportions by the Lines on the Quadrant.

BEfore I come to shew how all Proportions may in some measure be performed upon the Lines of this Quadrant, it is to be intimated in general, That the working of a Proportion upon a single natural Line, was the useful invention of the late learned *Mathematician*, Mr *Samuel Foster*, and published after his decease as his; In the use of his Scale, a Book called *Posthuma Fosteri*, as also by Mr *Stirrup* in a Treatise of *Dyalling*; in which Books, though it be there prescribed, and from thence may be learned, yet I acknowledge I received some light concerning it. from some Manuscripts lent me by Mr *Foster*, in his life time to Transcribe for his and my own use touching Instrumental Applications; Yet withal be it here intimated, that there are no ways used upon this Quadrant for the obtaining the Hour and Azimuth with Compasses, and the Converse of the 4th. Axiom, but what are wholly my own, and altogether novel, though not worth the owning, for Instrumental Conclusions not being so exact as the Tables are of small esteem with the learned as in Mr *Wingates* Preface to the *Posthuma*; besides the taking off of any Line from the Limb to any Radius; the Explanation of the reason of Proportions so wrought, the supply of many Defects, and the inscribing of Lines in the Limb, I have not seen any thing of Mr *Fosters*, or of any other mans, tending thereto.

Of the Line of equal Parts.

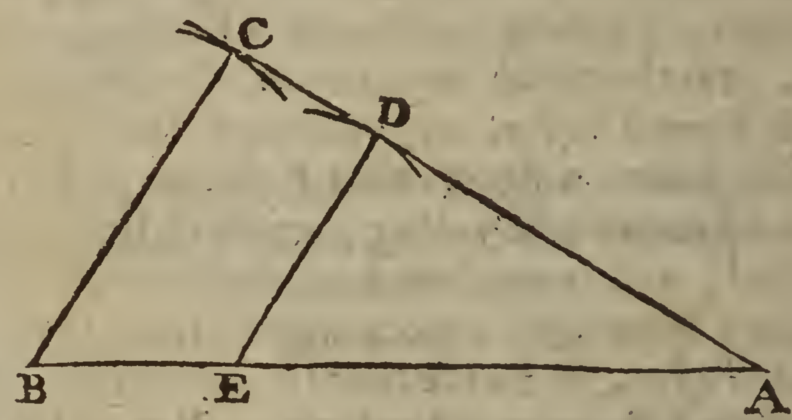
THis Line issueth from the Center of the Quadrant on the right edge of the foreside, and will serve for Mensurations, Protractions and Proportional work.

The ground of working Proportions by single natural Lines, is built upon the following grounds.

That Equiangled Plain Triangles have the sides about their equal angles Proportional, and this work hath its whole dependance on the

the likeneſs of two equiangled Plain Right angled Triangles; as in the figure annexed, let A B repreſent a Line of equal parts, Sines or natural Tangents iſſuing from the Center of the Quadrant ſuppoſed at A, and let A C repreſent the Thread, and the Lines B C, E D making right angles with the Line A C, or with the Thread, the neareſt diſtances to it from the Points B and E.

I ſay then that this Scheme doth repreſent a Proportion of the greater to the leſs, and the Converſe of the leſs to the greater.



Firſt of the greater to the leſs, and then it lies, *As* A B *to* B C: *So* A E *to* A D, whence obſerve that the length of the ſecond Tearm B C muſt be taken out of the common Scale A B, and one foot of that extent entred at B the firſt Tearm, the Thread muſt be laid to the other foot at C, according to the neareſt diſtance then the neareſt diſtance, from the Point E to the Thread that is from the third Tearm called *Lateral* entrance, being meaſured in the Scale A B, gives the quantity of the 4th Proportional.

Secondly of the leſs to the greater.

And then it lies, *As* B C *to* A B: *So* E D *to* A E, Or, *As* E D *to* A E: *So* B C *to* AB, by which it appears that the firſt Tearm B C muſt be taken out of the common Scale, and entred one foot at the ſecond Tearm at B, and the Thread laid to the other at C according to neareſt diſtance then the third Tearm E D muſt be taken out of the common Scale and entred between the Thread and the Scale, ſo that one foot may reſt upon the Line, as at E, and the other turned about may but juſt touch the Thread, as at D, ſo is the diſtance from the Center to E the quantity of the 4th Proportional; and this

is

is called Parralel entrance, becaufe the extent E D is entred Parralel to the extent B C: To avoid *Circumlocution*, it is here fuggefted, that in the following Treatife, we ufe thefe expreffions to lay the Thread to the other foot, whereby is meant to lay it fo according to neareft diftance, that the faid foot turned about may but juft touch the Thread, and fo to enter an extent between the Thread and the Scale is to enter it fo that one foot refting upon the Scale, the other turned about may but juft touch the Thread.

Another chief ground in order to working Proportions by help of Lines in the Limb is,

That in any Proportion wherin the Radius is not ingredient the Radius may be introduced by working of two Proportions in each of which the Radius fhall be included, and that is done by finding two fuch midle tearms (one whereof fhall always be the Radius) as fhall make a Rectangle or Product equal to the Rectangle or Product of the two middle Tearms propofed, to find which the Proportion will be.

As the Radius, To one of the middle Tearms:
So the other middle Tearm, To a fourth,

I fay then, that the Radius and this fourth Tearm making a Product or Rectangle equal to the Product of the two middle Tearms, thefe may be affumed into the Proportion inftead of thofe, and the anfwer or fourth Tearm will be the fame without Variation, and therefore holds,

As the firft Tearm of the Proportion, To the Radius:
So the fourth found as above, To the Tearm fought.

Or,

As the firft Tearm of the Proportion, Is to the fourth found as above: *So is the Radius, To the Tearm fought*; and here obferve, that by changing the places of the fecond and third Tearm, many times a Parralel entrance may be changed into a Lateral, which is more expedite and certain then the other, having thus laid the foundation of working any Proportion, I now come to Examples.

1. *To*

1. *To work Proportions in equal parts alone.*

If the firſt Tearm be greater then the ſecond, take the ſecond Tearm out of the Scale, and enter one foot of that extent at the firſt Tearm, laying the Thread to the other foot, then the neareſt diſtance from the third Tearm to the Thread gives the 4th Proportional ſought, to be meaſured in the Scale from the Center.

If the firſt Tearm be leſs then the ſecond, ſtill as before keep the greateſt Tearm on the Scale, and enter the firſt Tearm upon it, laying the Thread to the other foot, then enter the third Tearm taken out of the Scale between the Thread and the Scale and it finds the 4th Proportional.

Example.

Admit the Sun ſhining, I ſhould meaſure the length of the Shaddow of a Perpendicular Staff and find it to be 5 yards, the length of the Staff being 4 yards, and at the ſame time the length of the Shaddow of a Chimny, the Altitude whereof is demanded, and find it to be 22 ½ yards, the Proportion then to acquire the Altititude would be,

As the length of the ſhaddow of the Staff,
To the length of the Staff:
So the length of the ſhadow of the Chimney,
To the height thereof, that is

As 5 *to* 4 : *So* 22 | 5 *to* 18 yards the Altitude or height of the Chimney ſought, Enter 4 of the great diviſions upon 5, laying the Thread to the other foot, then the neareſt diſtance from 22 | 5 to the Thread meaſured will be 18; and in this latter part each greater diviſion muſt be underſtood to be divided into ten parts.

And ſo if the Sun do not ſhine, the Altitude might be obtained by removing till the Top of a Staff of known height above the eye upon a level ground be brought into the ſame Viſual Line with the Top of the Chimney, and then it holds,

As

As the diſtance between the Eye and the Staff, To the height of the Staff above the eye :
So the diſtance between the Eye and the Chimney.
To the height of the Chimney above the Eye.

Some do this by a Looking Glaſs, others by a Bowl of Water, by going back till they can ſee the top of the object therein, and then the former Proportion ſerves, *mutatis mutandis.*

But Proportions in equal parts will be eaſily wrought by the Pen, the chief uſe therefore of this Line will be for Protraction, Menſuration, and to divide a Line of leſſer length then the Radius of the Quadrant Proportionally into the like parts the Scale is divided, which may be readily done, and ſo any Proportional part taken off, to do it Enter the length of the Line propoſed at the end of the Scale at 10, and to the other foot lay the Thread the neareſt diſtances from the ſeveral parts of the graduated Scale to the Thread ſhall be the like Proportional parts to the length of the Line propoſed, the Proportion thus wrought is,

As the length of the graduated Scale, To any leſſer length :
So the parts of the Scale, To the Proportional like parts to that other length.

Of the Line of Tangents on the left edge of the Quadrant.

THe chief Uſes of this Scale will be to operate Proportions either in Tangents alone or jointly, either with Sines or equal parts, to prick down Dyals, and to proportion out a Tangent to any leſſer Radius,

To work Proportions in Tangents alone.

1. Of the greater to the leſs.

Enter the ſecond Tearm taken out of the Scale upon the firſt, laying the Thread to the other foot, then the neareſt diſtance from the third Tearm to the Thread being taken out and meaſured from the Center ſhews the 4th Proportional.

But if the Proportion be of the leſs to the greater,

Enter the firſt Tearm taken out of the Scale upon the ſecond, and lay the Thread to the other foot, then enter the third Tearm taken out of the Scale between the Thread and the Scale, and the foot of Compaſſes will ſhew the 4 Proportional.

Example.

Of the greater to the leſs,

As the Tangent of 50^{d}, *To the Tangent of* 20^{d}: *So the Tangent of* 30^{d}, *To the Tangent of* 10^{d}.

To work this take the Tangent of 20^{d} in the Compaſſes, and entring one foot of that extent at 50^{d}, lay the Thread to the other, according to the neareſt diſtance, then will the neareſt diſtance from the Tangent of 30^{d} to the Thread being meaſured on the Line of Tangents from the Center be the Tangent of 10^{d} the fourth Proportional.

By inverting the Order of the Tearms, it will be, Of the leſs to the greater.

As the Tangent of 20^{d}, *To the Tangent of* 50^{d}: *So the Tangent of* 10^{d}, *To the Tangent of* 30^{d}, to be wrought by a Parralel entrance.

This Scale of Tangents is continued but to two Radii, or 63^{d} 26′ whereas in many Caſes the Tearms given or ſought may out-reach the length of the Scale, in ſuch Caſes the Propprtion muſt be changed according to ſuch Directions as are given for varying of Proportions at the end of the 16 Caſes of right angled Sphœrical Triangles.

In two Caſes all the Rules delivered for varying of Proportions will not ſo vary a Proportion as that it may be wrought on this Line of Tangents.

Firſt when the firſt Tearm is greater then 63^{d} 26′ the length of the Scale, and the two middle Tearms each leſs then 26^{d} 34′ the Comple-

Complement of the Scale wanting; In this Case if any two Tearms of the Proportion be varied according to the Rules for varying of Proportions, there will be either in the given Tearms or Anſwer ſuch a Tangent as ſhall exceed the length of the Scale, but it may be remedied by a double Proportion by the reaſon before delivered for introducing the Radius into a Proportion wherein it is not ingredient.

As the Radius, To the Tangent of one of the middle Tearms:
So the Tangent of the other middle Tearm, To a fourth Tangent:

Again.

As the Radius, To that fourth Tangent:
So is the Cotangent of the firſt Tearm,
To the Tangent of the fourth Ark ſought.

The Radius may be otherways introduced into a Proportion then here is done, but not conducing to this preſent purpoſe, and therefore not mentioned till there be uſe of it, which will be upon the backſide of a great Quadrant of a different contrivance from this, upon which this trouble with the Tangents will be ſhunned.

An Example for this Caſe.

As the Tangent of 65^{d}, To Tangent of 24^{d}:
So the Tangent of 20^{d}, To what Tangent? the Proportion will find 5^{d}, 26′.

Divided into two Proportions will be,
As Radius, To Tangent 24^{d}: So Tangent of 20^{d}, To a fourth, the quantity whereof need not be meaſured.

Again.

As Radius, To that fourth: So the Tangent of 25^{d}, the Complement of the firſt Tearm, *To the Tangent of 5^{d} 26′*, the fourth Tangent ſought.

Operation.

Firſt enter the Tangent of 24^{d} on the Radius or Tangent of 45^{d} laying the Thread to the other foot, then take the neareſt diſtance to it from 20^{d}, and enter tha t extent at 45^{d}, laying the Thread to the

the other foot, then will the nearest distance from 25^d, to the Thread if measured from the Center be the Tangent of 5^d 26′ sought.

The second Case is when the first Tearm of the Proportion is less then the Complement of the Scale wanting, and the two midle Tearms greater then the length of the Scale.

This ariseth from the former, for if the Tearms given were the Complements of those in the former Example, they would be agreeable to this Case, and so no further direction is needful about them, for the Tangent sought would be the Complement of that there found, namely 84^d 34′.

Hence it may be observed, that a Table of natural Tangents only to 45^d, or a Line of natural Tangents only to 45^d may serve to operate any Proportion in Tangents whatsoever.

To Proportion out a Tangent to any Radius.

Enter the length of the Radius proposed upon the Tangent of 45^d and to the other foot of the Compasses lay the Thread according to the nearest distance, then if the respective nearest distances from each degree of the Tangents to the Thread be taken out they shal be Tangents to the assigned Radius:

Because the Tangents run but to 63^d 26′ whereas there may be occasion in some declining Dyalls to use them to 75^d though seldom further; to supply this defect, they may be supposed to break off at 60 and be supplied in a Line by themselves not issuing from the Center, or only pricks or full points made at each quarter of an hour, for the 5th hour, that is, from 60^d to 75^d, and so these distances prickt agian from the Center as here is done, either one way or other, the Proportion will hold,

As the common Radius of the Tangents,
Is to any other Assigned Radius:
So is the difference of any two Tangents to the common Radius.
To their Proportional difference in that Assigned Radius,

And so having Proportioned out the first four hours, the 5th hour may be likewise Proportioned out and pricked forward in one continued streight Line from the end of the 4th hour.

To work Proportions in Sines and Tangents by help of the Limb and Line of Tangents issuing from the Center.

THough this work may be better done on the backside where the Tangents lye in the Limb, and the Sines issue from the Center, and where also there is a Secant meet for the varying of some Proportions that may excur, yet they may be also performed here supposing the Radius introduced into any Proportion wherein it is not ingredient, the two middle Tearms not being of the same kind as both Tangents or both Sines.

To find the 4th Proportional if it be a Sine.

Lay the Thread to the Sine in the Limb being one of the middle Tearms, and from the Tangent being the other middle Tearm take the nearest distance to it, then entring this extent upon the first Tearm being a Tangent lay the Thread to the other foot, and in the equal Limb it shews the Sine sought.

So if the Example were,
As the Tangent of 50d, *To the Sine of* 40d: *So is the Tangent of* 36d, *To a Sine, the* 4th *Proportion would be found to be the Sine of* 23d, 4′.

If a Tangent be sought.

Lay the Thread to the Sine in the Limb being one of the middle Tearms, and from the Tangent being the other middle Tearm, take the nearest distance to it, then lay the Thread to the first Tearm in the Limb, and the former extent entred between the Scale and the Thread finds the Tangent, being the 4th Proportional sought.

If the Example were,
As the Sine of 40d, *To the Tangent of* 50d: *So is the Sine of* 23d 4′ *To a fourth a Tangent, it would be found to be the Tangent of* 36d.

These Directions presuppose the varying of the Proportion, as to the two Tangents, when either of them will excur the length of the Scale

Scale, of which more when I come to treat of the joint use of the Sines and Tangents on the backside.

If both the middle Tearms be Sines, the 1st Operation will be wholly on the Line of Sines on the backside, by introducing the Radius, and the second upon the Line of Tangents on the foreside, likewise, if both the middle Tearms were Tangents, the first Operation would be on the Line of Tangents on the foreside, and the second on the Line of Sines on the backside; but this is likewise pretermitted for the present, for such Cases will seldom be reducible to practise.

To work Proportions in equal Parts and Tangents.

Because the Lines to perform this work do both issue from the Center, the Radius need not be introduced in this Case; but here it must be known whether the first or second Tearm of the Proportion taken out of its proper Scale be the longer of the two, and accordingly the work to be performed on the Scale of the longer Tearm, which shall be illustrated only by a few Examples, the ground of what can be said being already laid down.

Example. *To find the Suns Altitude, the length of the Gnomon or Perpendicular being assigned, and the length of it's Shadow measured.*

As the length of the Shadow.
Is to the Radius : So is the length of the Gnomon,
To the Tangent of the Suns Altitude.

Example.

If the length of the Shaddow were 8 foot, and the length of the Gnomon but 5 foot, because 8 of the greater divisions of the equal parts are longer then the Tangent of 45^d take the said Tangent or Radius, and enter it at 8, laying the Thread to the other foot, then the nearest distance from 5 of ~~the~~ equal parts to the Thread measured on the Tangents sheweth 32^d for the Suns Altitude sought.

So the distance from a Tower and its Altitude being observed the Proportion, to get the height of the Tower is,

As

As the Radius, To the meaſured diſtance: So the Tangent of the Altitude. To the height of the Tower.

So in the Example in *Page* 38. the meaſured diſtance K B was 100 yards, and the Altitude 43^{d} 50′ to find the height of the Tower take the Tangent of 45*d*, and enter it on 10 at the end of the equal Scale, laying the Thread to the other foot, then take the Tangent of 43*d* 50′, and enter it between the Scale and the Thread, and the Compaſſes will reſt at 96 the height of the Tower in yards, ſometimes each grand diviſion of the equal Scale muſt repreſent but one ſometimes 10, and ſometimes 100, as in Caſe L B, and the Altitude thereto were given to find the height aſſigned.

Another Example.

As the Tangent of 60^{d}, *Is to* 50: *So is the Tangent of* 40*d*, *To* 24 | 2 as before.

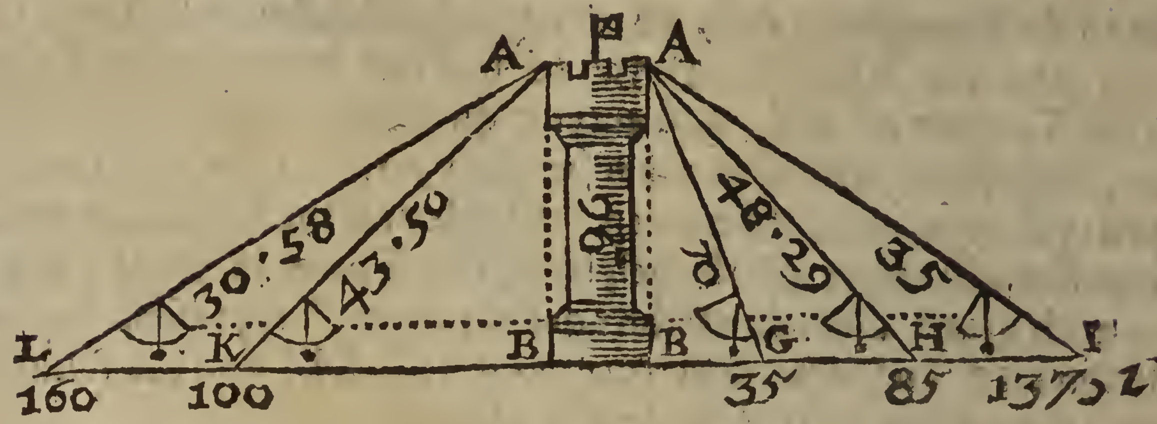
Take 50 equal parts, and enter it upon the Tangent of 60*d* laying the Thread to the other foot, then the neareſt diſtance from the Tangent of 40*d* to the Thread meaſured on the equal parts from the Center will be 24 | 2 as before.

Otherwiſe.

Enter the Tangent of 40*d* upon the Tangent of 60*d* laying the Thread to the other foot, then enter 50 equal parts down the Line of Tangents from the Center, and the neareſt diſtance from the termination to the Thread meaſured in the equal parts will be 24 | 2 as before.

If both the Tangents in any Proportion be too long, they may be changed into their Complements if one of them may and the other may not be ſo changed without excurſion, then the Proportion may be wrought by the Pen, taking the Tangents out of the Quadrat and Shaddows, or it may be made two Proportions by introducing the Radius as before ſhewed; it will not be needful to ſpeak more to this, only one Example for obtaining the Altitude of a Tower at two Stations.

As the difference of the Cotangents of the Arks cut at either Stations: Is to Radius: So the diſtance between thoſe Stations: To the Altitude of the Tower.



In the Diagram for this Case the Complements of the angles observed at the two Stations, *viz.* at G were 20*d*, at H 41*d* 31′.
Take the distance on the Line of Tangents between these two Arks, and because equal parts are sought, and the said Extent less then 50, the measured distance changing the second Tearm of the Proportion into the place of the third, Enter the said Extent upon 50 in the equal parts, laying the Thread to the other foot, then if the Tangent of 45*d* be entred between the Scale and the Thread, the Compasses will rest upon 96 for the Altitude sought.

To work Proportions in equal Parts and Sines by help of the Limb.

TO suppose both the middle Tearms to be either equal Parts or Sines, will not be practical, yet may be performed as before hinted, without introducing the Radius, if it be not ingredient, because both these Lines issue from the Center, and may also be performed by the Pen by measuring the Sines one the Line of equal parts, as was instanced in *page* 41.

But supposing the middle Tearms of a different kind.

1. *If a Sine be sought, Operate by introducing the Radius.*
Lay the Thread to the Sine in the Limb being one of the middle Tearms, and from the other middle Tearm being equal parts, take the nearest distance to it, one foot of this extent enter at the first

Tearm

Tearm, and the Thread laid to the other foot cuts the Limb at the Ark sought.

If the Ark sought be above *70d* this work may better be performed with the Line of equal parts and Sines jointly, as issuing from the Center.

2. *If a Number be sought,*

Lay the Thread to the Sine in the Limb being one of the middle Tearms, and from the other middle Tearm being equal parts take the nearest distance to it, Then lay the Thread to the first Tearm in the Limb being a sine, and enter the former extent between the Scale and the Thread, and the foot of the Compasses will on the Line of equal parts shew the fourth Proportional.

The Proportion for finding the Altitude of a Tower at one Station by the measured distance, may also be wrought in in equal parts and Sines.

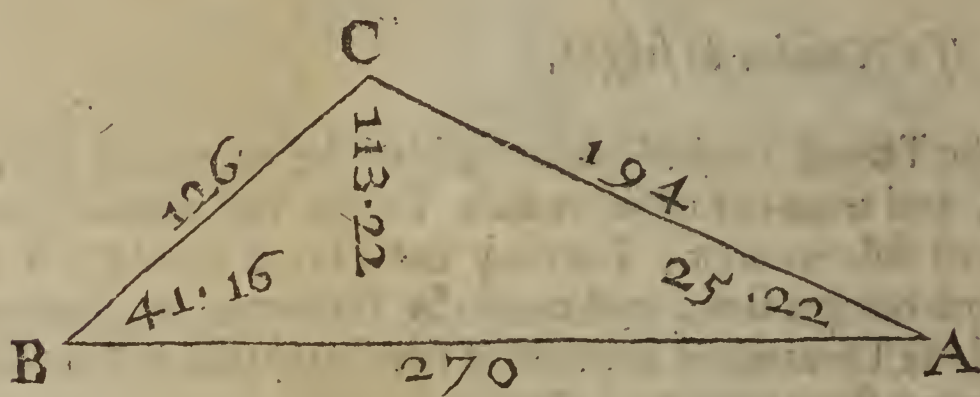
For, *As the Cosine of the Ark at first Station,*
To the measured distance thereof from the Tower:
So is the Sine of the said Ark,
To the Altitude of the Tower.

In that former Scheme, the measured distance B H is 85, and the angle observed at H 48*d* 29′

Wherefore I lay the Thread to the Sine of the said Ark in the Limb, counted from the right edge, and from the measured distance in the equal parts take the nearest extent to the Thread, then laying the Thread to the Cosine of the said Ark in the Limb, and entring the former extent between the Thread and the Scale, I shall find the foot of the Compasses to fall upon 96 the Altitude sought.

So also in the Triangle A C B, if there were given the side A C 194, the measured distance between two Stations on the Wall of a Town besieged, and the observed angles at A 25*d* 22′, at C 113*d* 22′, if B were a Battery we might by this work find the distance of it from either A or C, for having two angles given all the three are given, it therefore holds,

As the Sine of the angle at B 41d 16′,
To its opposite side A C 194,
So the Sine of the angle at C 66d, 38′ *the Complement,*
To its Opposite side B A 270, *the distance of the Battery from* A



Such Proportions as have the Radius in them will be more easily wrought; we shall give some few Examples in Use in Navigation.

1. *To find how many Miles or Leagues in each Parrallel of Latitude answer to one degree of Longitude.*

As the Radius, To the Cosine of the Latitude.
So the number of Miles in a degree in the Equinoctial,
To the Number of Miles in the Parralel.

So in 51*d* 32′ of Latitude if 60 Miles answer to a degree in the Equinoctial 37 | 3 Miles shall answer to one degree in this Parralel.

This is wrought by laying the Thread to 51*d* 32′ in the Limb from the left edge towards the right, then take the nearest distance to it from 60 in the equal parts which measured from the Center will be found to reach to 37 | 3 as before.

The reason of this facil Operation is because the nearest distance from the end of the Line of equal parts to the Thread is equal to the Cosine of the Latitude, the Scale it self being equal to the Radius; and therefore needs not be taken out of a Scale of Sines and entred upon the first Tearm the Radius as in other Proportions in Sines of of the greater to the less, when wrought upon a single Line only issuing from the Center, where the second Tearm must be taken out of a Scale, and entred upon the first Tearm.

2. *The*

2. *The Course and Distance given to find the difference of Latitude in Leagues or Miles.*

As the Radius, To the Cosine of the Rumb from the Meridian :
So the Distance sailed, To the difference of Latitude in like parts.

Example.

A Ship sailed S W by W, that is on a Rumb 56*d* 15′ from the Meridian 60 Miles, the difference of Latitude in Miles will be found to be 33 | 3 the Operation being all one with the former, Lay the Thread to the Rumb in the Limb, and from 60 take the nearest distance to it, which measured in the Scale of equal parts will be found as before,

3. *The Course and Distance given to find the Departure from the Meridian,* alias *the Variation.*

As the Radius, To the Sine of the Rumb from the Meridian :
So the distance Sailed, To the Departure from the Meridian.

In the former Example to find the Departure from the Meridian, Lay the Thread to the Rumb counted from the right edge towards the left, that is to 56*d* 15′ so counted, and from 60 in the equal parts being the Miles Sailed, take the nearest distance to it; this extent measured in the said Scale will be found to be 49 | 9 Miles, and so if the converse of this were to be wrought, it is evident that the Miles of Departure must be taken out of the Scale of equal parts and entred Parralelly between the Scale and the Thread lying over the Rumb.

Many more Examples and Propositions might be illustrated, but these are sufficient, those that would use a Quadrant for this purpose may have the Rumbs traced out or prickt upon the Limb : Now we repair to the backside of the Quadraut.

Of

Of the Line of on the right Edge of the Backside.

THe Uses of this Line are manifold in Dyalling in drawing Pro jections in working Proportions, &c.

1. *To take of a Prcportional Sine to any lesser Radius then the sii of the Quadrant, or which is all one, to divide any Line shorti in length then the whole Line of Sines in such manner as the sami is divided.*

Enter the length of the Line proposed at 90*d* the end of the Scale of Sines, and to the other foot lay the Thread according to nearest Distance, or measure the length of the Line proposed on the Line of Sines from the Center, and observe to what Sine it is equal, then lay the Thread over the like Arch in the Limb, and the nearest distances to it from each degree of the Line of Sines shall be the Proportional parts sought.

And if the Thread be laid over 30*d* of the Limbe the nearest distances to it will be Sines to half the Common Radius.

2. *From a Line of Sines to take off a Tangent, the Proportion to do it is,*

As the Cosine of an Arch, To the Radius of the Line proposed:
So the Sine of the said Arch, To the Tangent of the said Arch.

Enter the Radius of the Tangent proposed at the Cosine of the given Arch, and to the other foot lay the Thread then from the Sine of that Arch take the nearest distance to the Thread, this extent is the length of the Tangent sought; thus to get the Tangent of 20*d* enter the Radius proposed at the Sine of 70*d*, then take the nearest distance to the Thread from the Sine of 20*d*, this extent is the Tangent of the said Arch in reference to the limited Radius.

Other-

Otherways by the Limb.

Lay the Thread to the Sine of that Arch counted from the right edge whereto you would take out a Tangent, and enter the Radius proposed down the Line of Sines from the Center and take the nearest distance to the Thread then lay the Thread to the like Arch from the left edge, and enter the extent between the Scale and the Thread, the distance of the Foot of the Compasses from the Center shall be the length of the Tangent required.

3. *From the Sines to take off a Secant.*

The Proportion to do it is,

As the Cosine of the Arch proposed, To Radius of the Line proposed So the Sine of 90*d*, the common Radius, *To the length of the Secant of that Arch, to the limitted Radius.*

By the Limb,

Lay the Thread to that Arch in the Limb counted from left edge whereto you would take out a Secant, then enter the Limitted Radius between the Scale and the Thread and the distance of the foot of the Compasses from the Center shall be the length of the Secant sought, and the Converse if a Secant and its Radius be given to find the Ark thereto enter the Secant of 90*d* then enter the Radius of it between the Thread and the Sines, and the Compasses shews the Ark thereto; if counted from 90*d* towards the Center.

Otherways.

Enter the Radius of the Line you would devide into Secants at the Cosine of that Arch whereto you would take out a Secant, and to the other foot lay the Thread then the nearest distance to the Thread from the Sine of 90*d* is the length of the Secant sought: Thus to get the Secant of 20*d* enter the Radius limited in the Sine of 70*d* then the nearest distance from 90*d* to the Thread, is the length of the Secant sought.

And here it may be noted, that if you would have the whole length of the Line of Sines to represent the Secant sought, then the Cosine of

of that Arch which it repreſents ſhall be the Radius to it; ſo the whole Line of Sines repreſenting a Secant of 70*d*, the length of the Sine of 20*d* ſhall be the Radius thereto.

It may alſo be obſerved, that no Tangent or Secant can be taken of at once larger then the Radius of the Quadrant, nor no Radius entred longer then that is, and that if the Radius entred be in

Length	of the Sines	Tangents to		Secants to	
$\frac{1}{2}$		63^{d}	$26'$	60^{d}	
$\frac{1}{3}$		71	34	70	32
$\frac{1}{4}$		75	58	75	32

may be taken off by help of the Line of Sines.

And here it may be obſerved, That if the Tangent and Secant of any Arch be added in one ſtreight Line or otherwiſe in Numbers, the Amount ſhall be equal to the Tangent of ſuch Ark as ſhall biſect the remaining part of the Quadrant, as is demonſtrated in *Pitiſcus* & *Snellius*.

Whence it follows, That if we have a Tangent and Secant no further then to 60*d* each, yet a Tangent by the joint uſe of both Lines may from them be prickt down to 75*d*: Wherefore at anty time to lengthen the Tangents double the Arch propoſed; and out of the Amount reject 90*d*; The Tangent and Secant of the remainder connected in one ſtreight Sine ſhall be the Tangent of the Arch ſought.

Thus to get the Tangent of 70*d*, the double is 140*d* whence 90*d* rejected reſts 50*d*; the Tangent and Secant of 50*d* joined in one ſtreight Line ſhall be the Tangent of ſuch an Arch as biſects the remaining part of the Quadrant, namely of 70*d*.

It may alſo be obſerved, That the Tangent of an Ark, and the Tangent of half its Complement is equal to the Secant of that Arch as is obvious in drawing of any Projection.

A Chord may alſo be taken off from the Line of Sines, but more facilly by the Line of Chords on the left edge of the Quadrant, and is therefore pretermitted.

To work Proportions in Sines alone.

Frſt, Without the help of the Limb or leſſer Sines without introducing the Radius, but upon this Line alone independently.

There

There will be two Cases, 1. If the first Tearm be greater then the second, the entrance is lateral; Enter the second Tearm upon the first, laying the Thread to the other foot.

Then from the third Tearm in the Scale take the nearest distance to the Thread, and measure that Extent from the Center, and it shews the Tearm sought, and so if it were,

As Sine 30^{d}, *To Sine* 10^{d}: *So Sine* 80^{d}, the fourth Proportional would be found to be 20^{d}.

In giving Examples to illustrate the matter, I shall make use of that noted Canon for making the Tables,

As the Semiradius, or Sine of 30^{d}, *To the Sine of any Arch: So the Cosine of that Arch, To the Sine of that Arch doubled.*

But when the first Tearm is less then the second,

Enter the length of the first Tearm upon the second, laying the Thread to the other foot of that Extent, then enter the third Tearm Parralelly between the Scale and the Thread, and it shews the fourth Proportional sought.

So if it were, *As the Sine of* 10^{d}, *To the Sine of* 30^{d}: *So the Sine of* 20^{d}, *To a fourth,* the 4th Proportional would be found to be 80*d.*

Another general way will be to do it by help of the Limb, by introducing the Radius in such Proportions wherein it is not

Lay the Thread to one of the middle Tearms in the Limbe, and from the other middle Tearm on the Line of Sines take the nearest distance to it, then enter one foot of that extent at the first Tearm on the Line of Sines, and lay the Thread to the other foot, and in the Limbe it shews the 4th Proportional sought.

Example.

If the three Proportionals were, *As Sine* 55*d*, *To Sine* 70*d*: *So Sine* 30*d*, *To a fourth,* the fourth Proportional would be found to be 35*d.*

But if the first Tearm of the Proportion be either a small Ark or the answer above 70*d*, the latter part of this general direction for more certainty may be turned into a Parralel entrance, that is to say instead of entring the Extent taken from one of the middle

Tearms in the Sines to the Thread laid over the other middle Tearm in the Limb, and entring it at the first Tearm in the Sines finding the Answer in the Limb lay, the Thread to the first Tearm in the Limb, and find the Answer in the Line of Sines by, entring the former extent parralelly between the Scale and the Thread.

What hath been spoken concerning the Limb may also be performed by the Line of lesser Sines in the Limb by the same Directions.

So if it were, *As the Sine of* 5^{d}, *To Sine* 30^{d} : *So the Sine of* 10^{d} *To a fourth*, the 4^{th} Proportional would be found to be the Sine of 85^{d}, and the Operation best performed by the joint use of the Line of Sines, and the lesser Sines by making the latter entrance a Parralel entrance.

When the Radius is in the third place of a Proportion in Sines of a greater to a less, the Operation is but half so much as when it is not ingredient.

Example.

As the Cosine of the Latitude, To the Sine of the Declination,
So the Radius, To the Sine of the Suns Amplitude.

If the Suns declination were 13^{d}, to find his Amplitude in our Latitude for *London*, take the Sine of 13^{d} and enter one foot of it on the Sine of $38^{d}\,28'$ and to the other foot lay the Thread, and in the Limb it shews the Amplitude sought to be $21^{d}\,12'$.

By changing the places of the two middle Tearms, this Example will be turned into a Parralel entrance.

Lay the Thread to the Complement of the Latitude in the Limb, and enter the Sine of the Declination between it and the Scale, and you will find the same Ark in the Sines for the Amplitude sought, as was before found in the Limb.

Such Proportions of the greater to the less wherein the Radius is not ingredient, that have two fixed or constant Tearms, may be most readily performed by the single Line of Sines without the help of the Limb.

An

An Example for finding the Suns Amplitude.

As the Cosine of the Latitude, To the Sine of the Suns greatest declination :
So the Sine of the Suns distance from the next Equinoctial Point,
To the Sine of the Suns Amplitude.

Because the two first Tearms of this Proportion are fixed, the Amplitude answerable to every degree of the Suns place may be found without removing the Thread ; To do it enter the Sine of the Suns greatest Declination 23d 31′, at the Sine of the Latitudes Complement, and to the other foot lay the Thread, where keep it without alteration, then for every degree of the Suns place counted in the Sines take the nearest distance to the Thread, and measure those extents down the Line of Sines from the Center, and you will find the correspondent Amplitudes.

Example.

So when the Sun enters ♉ ♍ ♏ ♓, his Equinoctial distance being 30*d*, the Amplitude will be 18*d* 41′, and when he enters ♊ ♌ ♐ ♒ Equinox distance 60*d*, the Amplitude will be 33*d* 42′; and when he enters ♋ ♑ the greatest Amplitude will be 39d 50′, his distance from the nearest Equinoctial Point being 90*d*.

But for such Proportions in which there is not two fixed Tearms, the best way to Operate them will be by the joint help of the Limbe and Line of Sines.

An Example for finding the Time of the day the Suns Azimuth Declination and Altitude being given.

By the Suns Azimuth is meant the angle thereof from the midnight part of the Meridian, the Proportion is

As the Cosine of the Declination, To the Sine of the Azimuth :
So the Cosine of the Suns Altitude. To the Sine of the hour from the Meridian.

Example.

So when the Sun hath 18*d* 37′ North Declination, if his Azimuth be 69*d* from the Meridian, and the Altitude 39*d*, the hour will be found to be 49*d* 58′ from Noon.

So if there were given the Hour, the Declination and Altitude by transposing the Order of the former Proportion, it will hold to find the Azimuth,

As the Cosine of the Suns Altitude, To the Sine of the hour from the Meridian:

So the Cosine of the Suns Declination, To the Sine of the Azimuth from the Meridian.

Commonly in both these Cases the Latitude is also known, and the Affection is to be determined according to Rules formerly given.

A Proportion wholly in Secants we have shewed before may be changed wholly into Sines; but the like mutual conversion of the Sines into Tangents is not yet known, however it may be done in any of the 16 Cases wherein the Radius is ingredient, for instance, let the Proportion be to find the time of Sun rising.

As Radius, To Tangent of Latitude: So the Tangent of the Declination, To the Sine of the hour from 6.

Instead of the two first Tearms it may be,

As the Cosine of the Latitude, To the Sine of the Latitude, then instead of the Tangent of the Declination say, *So is the Sine hereof to a fourth.*

Again,

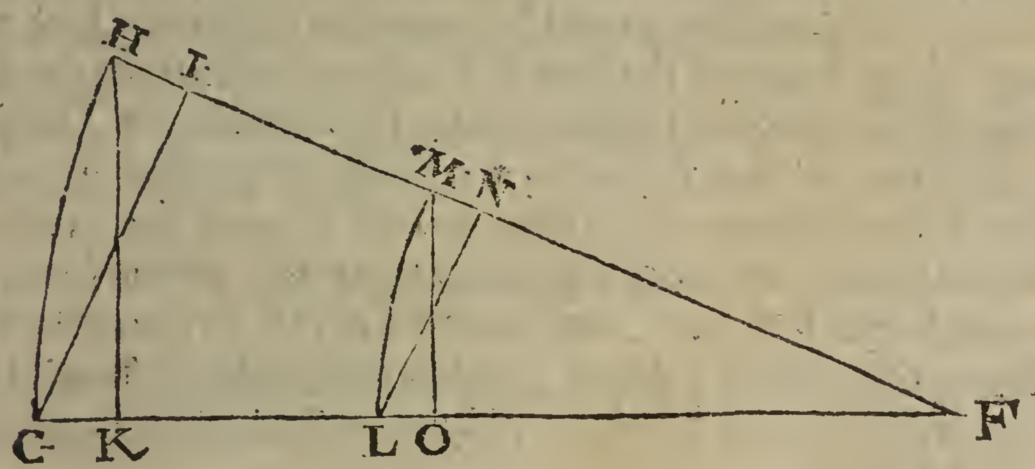
As the Cosine of the Declination, To that fourth: So Radius, To the Sine of the hour from six: This being derived from the Analemma by resolving a Triangle, one side whereof is the Arch of a lesser Circle.

If a Quadrant want Tangents or Secants in the Limb, but may admit of a Sine from the Center, the Tangent and Secant of the Latitude, *&c*, may be taken out by what hath been asserted, to half the common Radius, and marked on the Limb, and the Quadrant thereby fitted to perform most of the Propositions of the Sphære in one Latitude, and how to supply the Defect of a Line of Versed Sines in the Limb shall afterwards be shewne.

What

What hath been ſpoken concerning a Line of Sines graduated on a Quadrant from the Center, may by help of the equal Limb be performed without it.

1. *A Proportional Sine may be taken off to any diminutive Radius.*



By the Definition of Sines the right Sine of an Arch is a Line falling from the end of that Arch Perpendicularly to the Radius drawn to the other end of the ſaid Arch;

So the Line H K falling Perpendicularly on the Radius F G ſhall be the Sine of the Arch H G, and by the ſame Definition the Line G I falling perpendicularly on the Radius F H ſhall alſo be the Sine of the ſaid Arch, and whether the Radius be bigger or leſſer, this Definition is common; but the Line G I on a Quadrant repreſents the neareſt diſtance from the Radius to the Thread, therefore a Sine may be taken off from the Limb to any Diminutive Radius, to perform which,

Enter the length or Radius propoſed down the ſtreight Line that comes from the Center of the Quadrant, and limits the Limb; obſerve where the Compaſſes reſts, this I call the fixed Point, becauſe the Compaſſes muſt be ſet down at it, at every taking off, then to take off the Sine of any Arch to that Radius, lay the Thread over the Arch counted in the Limb from the ſaid edge of the Quadrant, and take the neareſt diſtance to it for the length of the Sine ſought: But to take out Sines to the Radius of the graduated Limb ſet down one foot at the Ark in the Limb, and take the neareſt diſtance to the two edge Lines of the Limb, the one ſhall be the Sine, the other Coſine of the ſaid Ark.

2. *A Proportion in Sines alone may be wrought by help of the Limbe.*

Take out one of the middle Tearms by the former *Prop.* and entring it down the right edge from the Center, take the neareſt diſtance to the Thread laid over the other middle Tearm in the Limbe, counted from right edge, then lay the Thread to the firſt Tearm in the Limb, and enter that extent between the right edge Line and the Thread, the diſtance of the foot of the Compaſſes from the Center, is the length of the Sine ſought, to be meaſured in the Limb by entring one foot of that Extent in it: So that the other turned about may but juſt touch one of the edge or ſide Lines of the Limb iſſuing from the Center, or enter that Extent at the concurrence of the Limbe with the ſaid Line, and lay the Thread to the other foot according to the neareſt diſtance, and in the Limbe it ſhews the Ark ſought: Whence may be obſerved how to prick of an angle by Sines inſtead of Chords.

From this and ſome other following Propoſitions I aſſert the Hour and Azimuth may be found generally by the ſole help of the Limb of a Quadrant without Protraction.

How from the Lines inſcribed in the Limbe to take off a Sine, Tangent, Secant and Verſed Sine to any Radius, if leſs then half the common Radius of the Quadrant.

IT hath been aſſerted, that a Sine may be taken off from the Limb, and by conſequence any other Line there put on; for by being carried thither they are converted into Sines, and put on in the ſame manner, for by the Definition of Sines, if Lines were carryed Parralel to the right edge of the Quadrant from the equal degrees of the Limb to the left edge they would there conſtitute a Line of Sines and the Converſe.

To find the fixed Point enter the Radius propoſed twice down the Line of Sines from the Center, or which is all one, Lay the

the Thread over 30*d* of the Limb counted from the right edge towards the left, and enter the limitted Radius between the Thread and the Scale; so that one foot turned about may just touch the Thread, and the other resting on the Line of Sines, shews the fixed Point, at which if the Compasses be always set down, and the Thread laid over any Ark in the Tangent, Secant or lesser Sines, the nearest distances from the said Point to the Thread shall be the Sine, Tangent, Secant, of the said Ark to the limitted Radius.

But for such Lines as are put on to the common Radius, as the Tangent of 45*d*, *&c.* the Radius is to be entred but once down from the Center to find the fixed Point.

Of the Line of Secants.

This Line singly considered is of small use, but junctim with other Lines of great use for the general finding the Hour and Azimuth: Mr *Foster* makes use of it in his *Posthuma* to graduate the Meridian Line of a Mercators Chart, which is done by the perpetual addition of Secants, and the like may be done from this Line lying in the Limb but a better way wil be to do it from a well graduated Meridian Line by doubling or folding the edge of the Chard thereto, and so graduate it by the Pen.

Of the Line of Tangents.

The joint use of this Line with the Line of Sines is to work Proportions in Sines and Tangents, in any Proportion wrought by help of Lines in the Limb wherein the Radius is not ingredient, the Radius must be introduced according to the general Direction.

If the two middle Tearms be Sines there must be one Proportion wrought wholly on the Line of Sines on the Backside, and another on the Line of Tangents on the foreside; but such Cases are not usual:

But if the two middle Tearms be Tangents, the first Operation must be on the line of Tangents on the foreside, and the latter on the line of Sines on this backside, unless the Radius be ingredient:

A general Direction to work Proportions when the middle Tearms are of a different Species:

If a Sine be sought,

Lay

Lay the Thread to the Tangent in the Limb being one of the middle Terms, and from the Sine being another of the middle Terms take the neareſt diſtance to it, then lay the Thread to the other Tangent in the Limb, being the firſt Tearm, and enter the former extent between the Scale and the Thread, and the foot of the Compaſſes on the Line of Sines will ſhew the fourth Proportional.

Example.

If the Proportion were, *As the Tangent of* 30*d*, *To the Sine of* 25*d* *So is the Tangent of* 20*d*, *To the Sine of* 15*d* 27′.

Lay the Thread over the Tangent of 20*d* in the Limb, and from the Sine of 25*d* take the neareſt diſtance to it, then lay the Thread to the Tangent of 30*d*, and the former extent ſo entred that one foot reſting on the Sines, the other foot turned about may but juſt touch the Thread, and the reſting foot will ſhew 15*d* 27′ for the Sine ſought.

2. If a Tangent be ſought.

Lay the Thread to the Tangent being one of the middle Tearms, and from the other middle Tearm being a Sine take the neareſt diſtance thereto, then Enter one foot of that extent at the firſt Tearm being a Sine, and the Thread laid to the other foot ſhews the fourth Proportional in the Line of Tangents in the Limb.

Example.

So if the Proportion were, *As the Sine of* 25*d*, *To the Tangent of* 30*d*: *So is the Sine of* 32*d*, *To a Tangent*, the fourth Proportional would be found to be the Tangent of 35*d* 54′.

If the anſwer fall near the end of the Scale of Tangents, the latter entrance may be made by laying the Thread to the firſt Tearm in the Limb, and by a Parralel entrance an Ark found on the Line of Sines, then if the Thread be laid over the like Ark in the Limb it will interſect the Tangent ſought.

Theſe Directions preſuppoſe the varying of the Proportion when the Tangens excur the length of the Scale, according to the Directions in the *Trigonometrical* part; but as before ſuggeſted, thoſe Di-

rections

rections are insufficient when one of the Tearms or Tangents are less then the Complement of the Scale wanting, and the other greater then the length of the Scale, for two such Arks cannot be changed into their Complements without still incurring the same inconvenience; in this Case only change the greater Tearm, which may be done by help of the Line of Secants, for,

As the Tangent of an Arch, To the Sine of another Arch;
So is the Cosecant of the latter Arch, To the Cotangent of the former.

And by Transposing the Order of the Tearms.

As a Sine, To a Tangent:
So the Cotangent of the latter Arch, To the Cosecant of the former.

Example.

If the Proportion were, *As the Sine of* 8^d, *To the Tangent of* 25^d *So is the Sine of* 60^d, *To the Tangent of* 71^d*:* Here we might foreknow by the nature of the Tearms that the Tangent sought would be large or finde by tryal that it cannot be wrought upon the Quadrant: We may therefore vary it thus, *As the Tangent of* 25^d *To the Sine of* 8^d*: So the Secant of* 30^d, *To the Tangent of* 19^d, the Complement of 71^d, the Arch sought.

Lay the Thread over 8 in the lesser Sines, and set down one foot of the Compasses at the Sine of the same Arch the Thread lyes over in the Limb; and take the nearest distance to the Thread laid over the Secant of 30, then lay the Thread to the Tangent of 25^d, and enter the former extent between the Thread and the Line of Sines, and the distance of the foot of the Compasses from the Center measured on the Tangents on the foreside sheweth 19^d.

But a more general Caution in this Case without the help of the Secants, would be by alring the larger Tangent into its Complement by introducing the Radius, and operating the Proportion on the greater Tangent of 45^d.

If the Proportion were,
As the Tangent of 70d, To the Sine of 60d:
So the Tangent of 25d, To the Sine of 8d 27′.

Z By

By introducing the Radius at two Operations it would be easily wrought, *As Radius, To Tangent 25d, So Sine 60d, To a fourth,*

Again, *As the Radius, To the Tangent of 20d: So that fourth, To the Sine sought.*

So the former Example wherein a Tangent is sought may be likewise varied.

As Radius, To Tangent 25d: So Sine 60d, To a fourth,

Again, *As that fourth, To the Radius: So is the Sine of 8d, To the Cotangent of the Arch sought*, namely to the Tangent of 19' as before.

Two Proportions with the Radius in each are as suddenly done as one without the Radius.

Operation.

Lay the Thread over the Tangent of 25' in the greater Tangents, and from the Sine of 60d take the nearest distance to it, enter that extent at 90, or the end of the Line of Sines, laying the Thread to the other foot according to the nearest distance; then enter the Sine of 8' parralelly between the Scale and the Thread and the distance of the foot of the Compasses from the Center is the Tangent of the Complement of the Ark sought to be measured in the greater Tangents by setting down one foot at 90d, and the Thread laid to the other, according the nearest will lye over the Tangent of 19d.

An Example with the Radius ingredient and a Sine sought, *Data*, Latitude, and Declination, to find the time when the Sun shall be East or West.

As the Radius, To the Cotangent of the Latitude:
So the Tangent of the Declination,
To the Sine of the hour from 6.

To be wrought by the help of the lesser Tangents.

When the Radius comes first and two Tangents in the middle, change the largest Ark into its Complement to bring it into the first place, and the Radius into the second; then take out the Tangent of the other middle Ark, either from the foreside from the Scale, or out of the Limb by setting one foot at the Sine of 90d, and taking the nearest

neareſt diſtance to the Thread laid over the Tangent given, then laying the Thread to the Tangent of the firſt Ark, enter the former extent between the Scale and the Thread, and the foot of the Compaſſes will ſhew the Sine ſought.

Otherways the two middle Tearms being Tangents, as alſo when the firſt Tearm and one of the middle Tearms is a Tangent, change the Radius and one of thoſe Tangents into Sines.

For, *As the Radius, To the Tangent of any Ark:*
So is the Coſine of the ſaid Ark, To the Sine thereof.
And, *As the Tangent of any Ark, To Radius:*
So is the Sine of that Ark, To the Coſine thereof.

And ſo the former Proportion changed will be,
As the Sine of the Latitude, To the Coſine of the Latitude:
So the Tangent of the Declination, To the Sine of the hour from ſix,
When the Sun ſhall be Eaſt or Weſt.

Example.

If the Declination were 23^{d} 30′ North, in our Latititude of *London* 51^{d} 32′ to find the Sine ſought, Lay the Thread to the Tangent of the Declination in the Limb, and from the Complement of the Latitude in the Sines take the neareſt diſtance to it, then lay the Thread to the Sine of the Latitude in the leſſer Sines and enter the former extent between the Thread and the Scale and the foot of the Compaſſes ſheweth the anſwer in degrees, if the Thread be laid to the Ark found in the Limb it there ſheweth it in Time; So in this Example the time ſought is 20^{d} 14′, or in Time 1^{h} 17^{m} qefore 6 in the morning or after it in in the Evening.

If the Latitude and Declination were given, To find the Suns Azimuth at the Hour of 6.

As the Radius, To Coſine of the Latitude: So the Tangent of the Suns Declination, To the Tangent of his Azimuth from the Vertical.

In this Caſe a Tangent being the 4th Tearm ſought, the Operation is very facil.

Lay the Thread to the Tangent of the Declination in the leſſer Tangents, and from the Coſine of the Latitude take the neareſt diſtance to it, and either meaſure that extent on the Tangents on the foreſide, or ſet one foot of that extent upon the Sine of 90, and to the other lay the Thread and it will interſect the Tangent ſought in the Limb: So in our Latitude when the Sun hath 23 30′ of declination, his Azimuth at the hour of 6 will be 15*d* 9′ from the Eaſt or Weſt.

Another Example, So if the Suns diſtance from the neareſt Equinoctial Point were 60*d*, his right Aſcenſion would be found to be 57d 48′.

The Proportion to perform this Propoſition is,

As the Radius, To the Coſine of the Suns greateſt Declination:

So the Tangent of the Suns diſtance from the next Equinoctial Point,

To the Tangent of the Suns right Aſcenſion, or when the Tangents are large,

As the Coſine of the Suns greateſt declination, To Radius:

So the Cotangent of the Suns diſtance from the Equinoctial Point.

To the Cotangent of his right Aſcenſion.

By what hath been ſaid it appears that the working Proportions by the natural Lines is more troubleſome then by the Logarithmical, however this trouble wil be ſhunned in the uſe of the great Quadrant by help of the Circle on the backſide.

I now come to ſhew how the Hour of the Day, and the Azimuth of the Sun may be found univerſally by the Lines on the Quadrant, which is the principal thing intended.

The firſt Operation for the Hour will be to find what Altitude or Depreſſion the Sun ſhall have at the hour of 6.

The Proportion to find it is,

As the Radius, To the Sine of the Latitude:

So the ſine of the Suns Declination,

To the ſine of the Altitude ſought.

Example.

Example.

So in Latitude 51*d* 32′, the Suns declination being 23*d* 31′, To find his Altitude or Depreſſion at 6, Lay the Thread to the Sine of the Latitude in the Limb, and from the ſine of the Suns Declination take the neareſt diſtance to it, which extent meaſured from the Center will be found to be 18*d* 12′.

This remains fixed for one Day, and therefore muſt be recorded, or have a mark ſet to it.

Afterwards the Proportion is,

As the Coſine of the Declination, To the Secant of the Latitude,

Or, *As the Coſine of the Latitude, To the Secant of the Declination:*

So in Summer is the difference, but in Winter the Sum of the ſines of the Suns propoſed or obſerved Altitude, and of his Altitude or Depreſſion at 6;

To the Sine of the hour from 6 *towards Noon in Winter*, as alſo in Summer when the Altitude is more then the Altitude of 6, otherways towards Midnight.

To Operate this.

In Winter to the ſine of the Suns Depreſſion at 6, add the ſine of the Altitude propoſed, by ſetting down the extent hereof outward at the end of the former extent; in Summer take the diſtance between the ſine of the Suns Altitude, and the ſine of his Altitude at 6, and enter either of theſe extents twice down the Line of ſines from the Center, then lay the Thread to the Secant being one of the middle Tearms, and take the neareſt diſtance to it. Laſtly, enter one foot of this extent at the firſt Tearm, being a Sine, and to the other foot lay the Thread, and in the equal Limb it ſhews the hour from 6, which is accordingly numbred with hours.

But when the Hour is neer Noon, the anſwer may be found in the Line of Sines with more certainty by laying the Thread to the firſt Tearm in the Limb, and entring the latter extent Parralelly between the Scale and the Thread.

Otherways.

Enter the aforeſaid ſum or difference of ſines once down the Line of Sines from the Center, and laying the Thread to the Secant, being one of the middle Tearms, take the neareſt diſtance to it, then lay the Thread to the firſt Ark in the leſſer ſines, and enter the former

extent between the Thread and the Scale, and the foot on the Compasses on the Line sheweth the Sine of the Hour.

Example.

If the Altitude were 45^d 42′, take the distance between it and the sine of 18*d* 12′ before found enter this extent twice down the Line of sines from the Center, and laying the Thread over the Secant of 51*d* 32′ take the nearest to it, then entring one foot of this extent at 66*d* 29′ in the Line of Sines the Thread being laid to the other according to nearest distance will lye over 45*d* in the Limb, shewing the hour to be either 9 in the morning, or 3 in the afternoon, and so it will be found also in the latter Operation by entring the first extent once down the sines, and taking the distance to the Thread lying over the Secant of the Latitude, and then laying the Thread to 66*d* 29′ in the Limbe, and entring that extent between the Scale and the Thread.

To find the Suns Azimuth

The first Operation will be to get the Suns Altitude in the Vertical Circle, that is, being East ar West.

As the sine of the Latitude, To Radius:
So is the Sine of the Declination,
To the sine of the Altitud.

So in our Latitude of *London*, when the Sun hath 23^d 31′ of declination, his Vertical Altitude in Summer will be found to be 30 39′ and so much is the Depression when he hath as much South declination.

This found either by a Parralel entrance on the Line of Sines by laying the Thread to the sine of the Latitude in the Limb, and entring the sine of the Declination between the Scale and the Thread, or by a Lateral entrance in the Limbe changing the Radius into the third place, and then enter the sine of the Declination on the Sine of the Latitude, laying the Thread to the other foot, and in the Limb it shewes the Altitude sought; having found this Ark let it be recorded

corded or have a mark set to it, because it remains fixed for one Day, afterwards the Proportion to be wrought is,

As the Cosine of the Altitude, To the Tangent of the Latitude:
So in Summer is the difference in Winter the sum of the Sines of the Suns Altitude, and of his Vertical Altitude or Depression:
To the Sine of the Azimuth from the East or West towards noon Meridian in Winter as also in Summer, when the given Altitude is more then the Vertical Altitude, but if less towards the Midnight Meridian.

This Proportion may be wrought divers ways on the Quadrant after the same manner as the former, I shall therefore illustrate it by some Examples.

Declination 13^d, Latititude 51^d 32′, Vertical Altitude 16^d 42′,

Proposed Altitude in { Summer———————41^d, 53′.
Winter———————12 · 13

Enter the aforesaid sum or difference of Sines twice down from the Center of the Quadrant, and take the nearest distance to the Thread being laid over the Tangent of the Latitude, this extent set down at the Cosine of the Altitude, and lay the Thread to the other foot and in the Limbe it shews the Azimuth sought. So in this Example the Azimuth will be found to be 40^d both in Summer and Winter from East or West towards Noon Meridian.

Otherways.

Enter the aforesaid sum or difference of sines but down from the Center, and take the nearest distance to the Thread laid over the Tangent of the Latitude, then lay the Thread to the Complement of the Altitude in the lesser sines, and enter the former extent between the Scale and the Thread, and the answer will be given in the Line of Sines, supposing the declination unchanged, if the Altitude were 9^d 21′ both for the Winter and the Summer Example, the Azimuth at *London* would be 9^d 22′ from the East or West Northwards in Summer, and 35*d* Southwards in Winter.

Hitherto

Hitherto we suppose the Latitude not to exceed the length of the Tangents, whether it doth or not this Proportion may be otherways wrought by changing the two first Tearms of it; Instead of the Cosine of the Altitude to the Tangent of the Latitude, we may say, *As the Cotangent of the Latitude, To the Secant of the Altitude:*

So when the Sun hath 23*d* 31′ of North Declination in our Latitude, and his Altitude 57*d* 7′, take the distance between the sine thereof and the sine of 30*d* 39′ the Altitude of East, and enter it once down from the Center, and take the nearest distance to the Thread laid over the Secant of the Altitude, *viz.* 57*d* 7′, then lay the Thread to 38*d* 28′ in the Tangents, and enter the former extent between the Scale and the Thread, and the Compasses on the Line of Sines will rest at 50*d* for the Azimuth from East or West Southwards, because the Altitude was more then the Vertical Altitude.

OtherWays Without the Secant in all Cases by help of the greater Tangent of 45d.

Enter the aforesaid Sum or difference of the Sines once down from the Center and lay the Thread to the Tangent or Cotangent of the Latitude in the greater Tangents, and take the nearest distance to it.

Then for Latitudes under 45*d* enter the former extent at the Complement of the Altitude in the Line of Sines, and find the answer in the Limb by laying the Thread to the other foot, or if it be more convenient make a Parralel entrance of it, and find the answer in the Sines as before hinted.

But for Latitudes above 45*d*, first find a fourth by entring the sum or difference of sines between the Scale and the Thread, and then it will hold, *As the first Tearm, To that fourth: So Radius, To the Sine of the Azimuth*, and may be either a Lateral or Parralel entrance, according as it falls out, and as the Radius is put either in the second or third place, in all these Directions the introducement of the Radius is supposed according to to the general Advertisement.

The finding of the Amplitude this way presupposeth the Vertical Altitude known, and then the Proportion derived from the Analemma, not from the 16 Cases is,

As

As the Radius, To the Tangent of the Latitude,
So the Sine of the Vertical Altitude, To the Sine of the Amplitu'e:

So alſo to find the time of Sun riſing.

As the Coſine of the Declination, To Secant of the Latitude:
So the ſine of the Suns Altitude at 6, To the Sine of the hour of riſing from ſix.

To find the Suns Azimuth at ſix of the clock otherways then by the 16 Caſes.

As the Coſine of the Suns Altitude at 6, To Tangent of the Latitude,
So is the difference of the ſines of the Suns Altitude at 6, and of his Vertical Altitude,
To the ſine of the Azimuth from the Vertical.

To find the time when the Sun ſhall be due Eaſt or Weſt.

As Coſine of the Declination, To Secant of the Latitude:
So the difference of the Sines *of the Suns Altitude at 6, and of his Vertical Altitude, To the Sine of the hour from 6, When the Sun ſhall be due Eaſt or Weſt.*

Theſe Proportions derived from the Analemma, are general both for the Sun and Stars in all Latitudes; but when the Declination either of Sun or Stars exceed the Latitude of the place, this Proportion for finding the Azimuth cannot be at ſome times conveniently performed on a Quadrant, but muſt be ſupplyed from another Proportion, whereof more hereafter.

Of the Hour and Azimuth Scales on the Edges of the Quadrant.

Theſe Scales are fitted for the more ready finding the Hour and Azimuth in one Latitude, being only to facilitate the former general Way.

The Labour ſaved hereby is twofold, firſt the Suns declination is graved againſt the Suns Altitude of 6 in the Hour ſcale, and the ſaid Declinations continued at the other end of the ſaid hour Scale to give

the quantity of the Suns Depreſſion in Winter equal to his Altitude in Summer; and ſecondly they are of a fitted length as was ſhewed in the Deſcription of the Quadrant, and thereby half the trouble by introducing the Radius ſhunned.

The Uſe of the Azimuth Scale.

The Altitude and Declination of the Sun given to find his Azimuth.

Take the diſtance between the Suns Altitude in the Scale, and his Declination in Summer time in that Scale that ſtands adjoyning to the ſide; in Winter in that Scale that is continued the other way beyond the beginning, and laying the Thread to the Complement of the Suns Altitude in the leſſer ſines, which is double numbred, enter this extent between the Scale and the Thread parralelly, and the foot of the Compaſſes ſheweth in the Line of Sines the Azimuth accordingly,

Declination 23*d* 31′, Altitude 47*d* 27′, the Azimuth thereto would be 25*d* from Eaſt or Weſt in Summer, and if the Altitude were 9*d* 43′ in Winter the Azimuth thereto would be 30*d* either way from the Meridian.

And ſo when the Sun hath no Altitude, lay the Thread over 90*d* in the leſſer Sines and enter the extent from the beginning of the Azimuth Scale to the Declination, and you will finde the Amplitude which to this Declination will be 39*d* 50′.

The Uſes of the Hour Scale.

To find the Hour of the Day.

TAke the diſtance between the Suns Altitude in the hour Scale, and his Declination proper to the ſeaſon of the year, then laying the Thread to the Complement of the Suns Declination in the leſſer

leſſer ſines enter the former extent between the Scale and the Thread and the foot of the Compaſſes ſheweth the ſine of the hour.

Example.

If the Declination were 13*d* North, and the Altitude 37^{d} 13′ take the diſtance between it in the Scale and 13*d* in the prickt Line, then laying the Thread to 77*d* in the leſſer ſine enter that extent between the Scale and the Thread, and the reſting foot will ſhew 45*d* for the hour from 6, that is either 9 in the forenoon, or 3 in the afternoon.

The Converſe of the former Propoſition will be to find the Suns Altitude on all Hours.

The Thread lying over the Complement of the Suns Declination in the leſſer ſines from the ſine of the hour, take the neareſt diſtance to it, then ſet down one foot of that extent in the hour Scale at the Declination, and the other will reach to the Altitude.

Example.

At *London*, for theſe Scales are fitted thereto, I would find the Suns Altitude at the hours of 5 and at 7 in the morning in Summer when the Sun hath 23*d* 31′ of Declination.

Here laying the Thread to 23*d* 31′ the Suns declination from the end of the leſſer Sines being double numbred, from the ſine of 15*d*, taking the neareſt diſtance to it, ſet down one foot of this extent at 23*d* 31′ the declination it reaches downwards to 9*d* 30′, and upwards to 27*d* 23′ the Suns Altitude at 5 and 7 a clock in the morning in Summer.

Another Example.

Let it be required to find the Suns Altitudes at the hours of 10 or 2 when his declination is 23^{d} 31′ both North and South.

The Thread lying as before over the leſſer ſines take the neareſt diſtance to it from 60^{d} in the ſines, the ſaid extent ſet down at 23^{d} 31′ in the prickt Line reaches to 53*d* 44′ for the Summer Altitude, and being ſet down at 23^{d} 31′ on the other or lower continued Line reaches to 10^{d} 28′ for the Winter Altitude.

The Hour may be also found in the Versed sines by help of this fitted hour Scale,

Take the distance between the Suns Altitude, admit 36d 42′, and his Meridian Altitude to that Declination *61d* 59′, and enter one foot of this extent at the sine of *66d* 29′ , and laying the Thread to the other foot according to nearest distance and it will lye over the hours of 8 in the morning, or 4 in the afternoon in the Versed sines in the Limb , and thereby also may the time of Suns rising be found by taking the distance from 0 to the Meridian Altitude and entring it at the Cosine of the Declination as before and the Converse will find the Suns Altitudes on all hours by taking the distance from the Cosine of the Declination to the Thread laid over the Versed sine of the hour from Noon , and the said Extent will reach from the Meridian Altitude in the fitted Scale to the Altitude sought.

To find the time of Sun rising or setting,

Lay the Thread over the Complement of the declination as before; in the lesser sines, and enter the extent between the ☉ Altitude, which is nothing that is from the beginning of the Hour Scale to the Declination between the Scale and the Thread and the foot of the Compasses shews it in the Line of sines , which may be converted into Time by help of the Limb.

If these Scales be continued further in length as also the Declinations they will after the same manner find the Stars hour for any Star whatsoever to be converted into common Time, as in the uses of the Projection, as also the Azimuth of any Star that hath less declination then the place hath Latitude, but of this more in the next Quadrant.

In Dyalling there will be often use of natural sines ; whereas these Scales are continued but to *62d* , if therefore it be desired to take out any sine to the same Radius, the rest of the Scale wanting may be easily supplyed, for the difference of the sines of any two Arks equidistant from *60d* is equal to the sine of their distance.

Thus the sine of *20d* is equal to the difference of the sines of *40d* and *80d* Arks of like distance from *60d* on each side , and so may be added either to *40d* forward , or the other way from the end of the Scale.

In

In finding the Hour and Azimuth by these Scales, not in the Versed sines, the Directions altogether prescribe a Parralel entrance; but if the Extent from the Altitude to the Declination be entred at the Cosine of the Altitude or of the Declination in the Line of sines according as the Case is, and the Thread laid to the other foot, the Hour and Azimuth may be found in the lesser-sines by a Lateral entrance.

Or if the said Extent be doubled and entred as before hinted, the answer will be found in the equal Limb.

Example to find the Suns Azimuth.

Declination 23*d* 31′ North.
Altitude——41 : 34

Having taken the distance between these two Tearms in the Azimuth Scale and doubled it, enter one foot in the Line of sines at 48*d* 26′, the Complement of the Altitude, and laying the Thread to the other according to nearest distance it will lye over 15*d* of the equal Limb for the Suns Azimuth from the East or West Southwards.

The Use of the Versed Sines in the Limbe.

It may be noted in the former general Proportion, I have used the word Azimuth from Noon or Midnight Meridian, though not so proper, because they are more universal and common to both Hemispheres, other expressions besides their Verbosity would be full of Caution for the following Proportion in our Northern Hemispere, without the Tropick that finds it from the South between the Tropick of *Cancer* and the Equinoctial, when the Sun comes to the Meridian between the Zenith and the Elevated Pole would find it from the North, wherefore it is fit to be retained.

A general Proportion for finding the Hour.

As the Cosine of the Declination, To the Secant of the Latitude:
Or, *As the Cosine of the Latitude; To the Secant of the Declination:*
So is the difference of the Sines of the Suns Altitude proposed, and of his Meridian Altitude;
To the Versed Sine of the hour from Noon:
And *So is the sum of the sines of the Suns proposed Altitude, and of his Midnight Depression,*

To the Versed sine of the hour from Midnight:
And *So is the sine of the Suns Meridian Altitude,*
To the Versed sine of the Semidiurnal Ark:
And *So is the sine of the Suns Midnight Depression,*
To the Versed sine of the Seminocturnal Ark.

The Operation will be like the former, I shall therefore onely illustrate it by one Example, the Meridian Altitude is got in Winter by differencing, in Summer by adding the Declination to the Complement of the Latitude, if the sum exceed 90*d* the Complement thereof to 180*d* is the Meridian Altitude.

An Example for finding the Hour from Noon.

Declination—— 23d 31′ North the 11th *June.*
Comp. Latitude—— 38 28 *London.*
61 59 Meridian Altitude.

Proposed Altitude—— 36 42, take the distance between the sines of these two Arks, and enter it once down the Line of sines from the Center, and take the distance to the Thread laid over the Secant, then enter one foot of that extent at the sine being the first Tearm, and to the other lay the Thread, and in the Versed sines in the Limb it will lye over the Versed Sine of the hour from Noon.

In this Example, if the Thread be laid over the Secant of

51d 32′ } the extent must be entred at the sine of 66d 29′
23 31 } 38 28

either way the answer will fall upon 60*d* of the Versed sine shewing the Hour to be either 8 in the forenoon, or 4 in the afternoon.

If the hour fall near noon, then the extent of the Compasses may be Quadrupled and entred as before, and look for the answer in the Versed Sines Quadrupled: Or before the distance be took to the Thread the extent of difference may be entred four times down from the Center.

The Converse of this Proposition will be to find the Suns Altitude on all Hours universally.

As the Secant of the Latitude, To Cosine Declination;
Or, *As the Secant of the Declination, To Cosine Latitude:*
So the Versed sine of the hour from Noon,

To

To the difference of the sines of the Suns Meridian Altitude, and of his Altitude sought, to be subtracted from the sine of the Meridian Altitude, and there will remain the sine of the Altitude sought.

So in Latitude of *London*, if the Suns Declination were 13*d* 00′, and the hour from noon 75*d*, that is either 7 in the morning, or 5 in the afternoon.

Lay the Thread over the Versed sine of the hour from noon, namely, 75*d*, and from the sine of 77*d* the Complement of the Declination, take the nearest distance to it, then lay the Thread to the Secant of the Latitude, and enter the former extent between the Scale and the Thread, and you will find a sine equal to the difference sought, which sine take between the Compasses and setting down one foot at the sine of 51*d* 28′ the Meridian Altitude, the other foot turned towards the Center will fall upon the sine of 19*d* 27′ the Altiude sought.

A General Proportion for the Azimuth.

Get the Remainder or Difference between these two Arks, the Suns Altitude and the Complement of the Latitude by Substracting the less from the greater, and then the Proportion will hold,

As the Cosine of the Latitude, Is to the Secant of the Altitude,
Or, *As the Cosine of the Altitude, To the Secant of the Latitude:*
So is the sum of the sines of the Suns Declination, and of the aforesaid Remainder, To the Versed Sine of the Azimuth from the Noon Meridian in Summer only when the Suns Altitude is less then the Complement of the Latitude. In all other Cases, *So is the difference of the said sines,*
To the Versed sine of the Azimuth, as before from Noon Meridian.

Example.

The 11[th] of *June* aforesaid, the ☉ having 23*d* 31′ of North declination, his Altitude was observed to be 18*d* 20′, which substracted from 38*d* 28′ the remainder is 20*d* 8′, take out the sine thereof, and set down one foot at the sine of 23*d* 31′, and set the other forwards towards 90*d*, then take the nearest to the Thread laid over the Secant of the Latitude 51*d* 32′, enter one foot of this Extent at the Complement of the Altitude by reckoning the Altitude it self from 90*d* towards the Center, and the Thread laid to the other foot cuts the Line of Versed sines at 105*d* the Azimuth from the South.

The

The same day when the Altitude was more then the Colatitude, suppose 60*d* 11′ the Remainder will be found to be 21*d* 43′, take the distance between the sine thereof and of 23*d* 31′, and because the Extent is but small enter it four times down the Line of sines from the Center, and take the nearest distance to the Thread laid over the Secant of the Latitude, which entred at the Cosine of the Altitude, the Thread laid to the other foot shews 25*d* in the Quadrupled Versed Sines for the Azimuth from the South.

The Proportion hence derived for the Amplitude.

As the Cosine of the Latitude, To Secant of the Declination,&c. as before.

So in Summer is the sum in Winter, the difference of the sines of the Suns Declination, and of the Complement of the Latitude,

To the Versed Sine of the Amplitude from Noon Meridian.

The Proportion for the Azimuth will be better exprest by making the difference to be a difference of Versed sines.

How the Versed sines in the Limbe may be spared in Case a Quadrant want them.

If a Quadrant can only admit of a Line of sines from the Center, the common Quadrant of Mr *Gunters* very well may, on the right edge above the Margent for the Numbers of the Azimuths, it may be easily fitted for any or many Latitudes by setting Marks or Pricks to the Tangent and Secant of the Latitudes in the Limbe, which may be taken out by help of the Limb, Line of Sines, or by Protraction, and either of these general Proportions wrought upon it, or those which follow, if it be observed that whensoever the Thread lyes over the Versed sine of any Ark in the Limbe, it also at the same Time lyeth over a Sine equal to half that Versed sine to the common Radius: Now because the sine of 30^d doubled is equal to the Radius, let it be observed whether the sine cut by the Thread be greater or less then 30 deg.

When it is less let the Line of Sines represent the former half of a Line of Versed Sines, and take the sine of the Ark the Thread lay over, and enter it twice forward from the end of the Scale towards the

the Center, and you will obtain the Versed sine of the angle sought.

When it is more take the distance between the sine of 30*d* and the said sine, and letting the Line of sines represent the latter half of a Line of Versed sines, enter the said distance twice from the Center, and you will obtain the Versed sine of the Arch sought, namely, the sine of an Arch, whereto 90*d* must be added.

Three sides to find an Angle, a general Proportion.

As the Sine of one of the Sides including an angle,
Is to the Secant of the Complement of the other including side:
So is the difference of the Versed Sines of the third side, and of the Ark of difference betwen the two including sides,
To the Versed Sine of the Angle sought.
And *So is the difference of the Versed sines of the third side, and of the sum of the two including sides,*
To the Versed sine of the sought angles Complement to 180*d.*

To repeat the Converse when two sides and the angle comprehended are given to find the third side were needless.

If one of the containing sides be greater then a Quadrant, instead of it in referrence to the two first Tearms of the Proportion, take the Complement thereof to 180^{d} for the reputed side, but in differencing or summing the two containing sides alter it not: And further note, that the same Versed sine is common to an Ark less then a Semicircle, and to its Complement to 360*d*.

The Operation of this Proportion will be wholly like the former, so that there needs no direction but only how to take out a difference of two Versed sines to the common Radius, seeing this Quadrant of so small a Radius is not capable of such a Line from the Center.

And here note that the difference of two Versed sines less then a Quadrant, is equal to the difference of the natural sines of the Complements of those Arks.

And the difference of two Versed sines greater then a Quadrant is equal to the difference of the natural sines of the excess of those Arks above 90*d*.

And by consequence the difference of the Versed sines of two Arks the one less, the other greater then a Quadrant is equal to the sum

 of

of the natural ſines of the leſſer Arks Complement to 90^{d}, and the greater Arks exceſs above it.

And ſo a difference of Verſed ſines may be taken out of the Line of natural ſines conſidered as ſuch.

Or the Line of ſines may be conſidered ſometimes to repreſent the former half of a Line of Verſed Sines as it is numbred with the ſmall figures by its Complements from the end of it to 90^{d} at the Center, and ſometimes the latter half of it, and then the graduations of it as Sines muſt be conſidered as numbred from 90*d* to 180*d* at the end of it, and ſo a difference to be taken out of it by taking the diſtance between the two Tearms, which if the two Arks fall the one to be greater the other leſs then 90*d* will be a ſum of two ſines, as before hinted, and in this Caſe the ſine of the greater Arks exceſs above 90*d* to be ſet down outwards if it may be, at the Verſed ſine of the leſſer Ark, or which is all one, at the ſine of that Arks Complement, and the diſtance from the Exterior foot of the Compaſſes to the Center will be equal to the difference of the Verſed ſines of the Arks propoſed.

To meaſure a difference of Verſed ſines to the common Radius.

In this Caſe alſo the Line of ſines muſt ſometimes repreſent the former, ſometimes the latter half of a Line of Verſed ſines, and then one foot of the difference applyed to one Ark, the other will fall in many Caſes upon the Ark ſought, each Proportion variouſly expreſt, ſo that poſſibly either one or the other will ſerve in all Caſes.

But if one of the feet of the Compaſſes falls beyond the Center of the Quadrant: To find how much it falls beyond it, bring the ſaid foot to the Center, and let the other fall backward on the Line, then will the diſtance between the ſaid other foot, where it now falleth, and the place where it ſtood before be equal to the exceſs of the former foot beyond the Center, which accordingly thence meaſured helps you to the Arch ſought, and its Complement both at once with due regard to the repreſentation of the Line; this ſhould be well obſerved, for it will be of uſe on other Inſtruments.

A difference of Verſed ſines thus taken out to the Common Radius muſt be entred but once down from the Center.

To

To take out a Difference of Versed sines to half the common Radius.

Count both the Arks proposed on the Versed sines in the Limbe, and find what Arks of the equal Limbe answer thereto, then out of the Line of sines take the distance between the said Arks, and you have the extent required, which being but half so large as it should be is to be entred twice down from the Center.

To measure a difference of Versed sines to half the common Radius.

The Versed sines are largest at that end numbred with 180^{d}, count the given Ark from thence, and laying the Thread over the equal Limb find what Ark answers thereto, then setting down the Compasses at the like Ark in the Line of sines from the end of it towards the Center mind upon what Ark it falls, the Thread laid to the like Ark in the Limb sheweth on the Versed sines the Ark sought.

To save the labour of drawing a Triangle, I shall deliver the Proportion for the Azimuth derived from the general Proportion,

As the Cosine of the Latitude, To the Secant of the Altitude,
Or, *As the Cosine of the Altitude, To the Secant of the Latitude:*
So is the difference of the Versed sines of the Sun or Stars distance from the Elevated Pole, and of the Ark of difference between the Latitude and Altititude.
To the Versed sine of the Azimuth sought; as it falls in the Sphœre that is from the Midnight Meridian.
And *So is the difference of the Versed sines of the Polar distance, and of the Ark of difference between a Semicircle and the sum of the Latitude and Altitude,*
To the Versed Sine of the Azimuth from Noon Meridian.

A Canon derived from the Inverse of the general Proportion to finde the Distance of places in the Ark of a great Circle.

As the Secant of one of the Latitudes,
To the Cosine of the other.
So the Versed sine of the difference of Longitude,
To the difference of the Versed sines of the Ark of distance sought, and

 of

of the Ark of difference between both Latitudes, when in the same Hemisphere, or the Ark of the sum of both Latitudes when in both Hemispheres, which difference added to the Versed sine of the said Ark gives the Versed sine of the Ark of distance sought.

And *So is the Versed sine of the Complement of the difference of Longitude to 180d.*

To the difference of the Versed sines of the Ark of distance sought, and of an Ark being the sum of the Complements of both Latitudes when in one Hemisphere; Or the sum of the lesser Latitude encreased by 90d, and of the Complement of the greater Latitude when in different Hemispheres, which difference substracted from the Versed sine of the said Ark, there will remain the Versed sine of the Ark of distance sought.

This Proportion is to be wrought after the same manner as we found the Suns Altitudes on all hours universally; and the difference to be measured in the Line of sines as representing the former half of a Line of Versed sines, according to the Directions given for measuring of a difference of Versed sines to the common Radius or Radius of the Quadrant.

By altring the two first Tearms of the Proportion above, we may work this Proposition by positive entrance.

As the Radius, To the Cosine of one of the Latitudes:
So the Cosine of the other Latitude, To a fourth.

Again.

As the Radius, To the Versed sine, as above expressed in both parts,
So is that fourth, To the difference as above expressed.

An Example for finding the distance between London *and* Bantum *in the Arch of a great Circle the same that was proposed in Page 56.*

Bantam	Longitude	140d	Latitude	5d 40′ *South.*
London		25, 50		51 32 *North.*
difference of Longitude		114 10	Sum	57 12

Lay the Thread to 5d 40′ in the Limb counted from left edge and from 38d 28′ in the sines the Complement of our Latitude take the nearest distance to it, then lay the Thread to 114d 10′ in the Versed sines and entring the former extent down the Line of sines from

from the Center take the nearest distance to it, then laying the Thread over 57^{1} 12′ in the Versed sine, it cuts the Limbe at 13^{d} 15′ from the right edge, at the like Ark set down one foot of the former extent in the Line of sines, and the other will reach to the Sine of 41^{1} 42′, then laying the Thread over the like Ark in the Limb, it will intersect the Versed sines at 109^{1} 18′ the Ark of distance sought to be converted into Leagues or Miles according to the number of Leagues or Miles that answer to a degree in each several Country.

Thus when we have two sides with the angle comprehended to find the third side, either to half or the whole common Radius without a Line of natural Versed sines from the Center, or by the Proportions in page 93, or a third way, which I pretermit to the great Quadrant; and thus the Reader may perceive this small Quadrant to be many ways both Universall and particular, which are of sudden performance though tedious in expression.

Three sides to find an angle.

Each of the Proportions in Rectangles and Squares before delivered for the Tables, may as before suggested be reduced to single Tearms, an instance shall be given in that which finds the Square of the sine of half the Angle sought.

Add the three sides of the Triangle together, and from the half sum subtract each of the sides including the angle sought, then it will hold,

As the sine of one of the Comprehending sides, (rather the greater that the entrance may be Lateral.)
Is to the sine of the difference of the same side from the half sum:
So is the sine of the difference of the other comprehending side,
To a fourth sine,

Again.

As the Sine of the other comprehending side,
Is to that fourth sine:
So is the Radius, To half the Versed sine of the angle sought, And *So is the Diameter, To the whole Versed sine.*

To work this on the Quadrant.

Upon the first Tearm in the Line of sines being the greatest containing

taining ſide, enter the extent of the ſecond, and to the other foot lay the Thread, then from the third Tearm in the ſines take the neareſt diſtance to it, Which extent enter at the ſine of the firſt Tearm in the ſecond Proportion, and to the other foot lay the Thread, and it will cut the Verſed ſine at the angle ſought.

Having ſhewed how all Proportions may be performed upon the Quadrant: I now proceed to the reſt of the Lines.

Of the Line of Chords on the left edge of the Backſide.

I ſhall not at preſent ſpeak any thing as to the uſe thereof, that is intended to be done in a Treatiſe of the general Scale; the principal uſe of a Line of Chords is to prick off readily the quantity of any Arch of a Circle, to do which take the Chord of 60^{d}, and draw a Circle with that Extent then any Arch being to be prict off it is to be taken out of the Line of Chords, and to be transferred into the Circle ſwept, this ſuppoſeth the Radius not to vary.

But to do it to any Radius that is leſſer take the Semidiameter of the Circle, and enter it at the Chord of 60^{d} laying the Thread to the other foot, and the neareſt diſtances to the Thread will be Chords to the Semidiameter aſſigned, and the Converſe will meaſure the Chord of any Arch by a Parralel entrance.

A Chord may be taken off though no Line of Chords be graduated.

The Bead whereſoever it be ſet carryed from one edge of the Quadrant to the other, the Thread being extended doth deſcribe the Quadrant of a Circle, if therefore extending the Thread down one edge of the Quadrant you ſet the Bead to the diſtance of the Radius (or Semidiameter of the Circle ſwept) from the Center, and at it ſet down one foot of the Compaſſes, and lay the Thread kept at a certainty in ſtretching over the Limb to any Arch, & then open the Compaſſes to the Diſtance of the Bead, you ſhall take out the Chord of the ſaid Arch.

A Chord may very conveniently be taken off from any Circle ſwept Concentrick to the Limbe, and divers ſuch there are upon both ſides of this Quadrant; Sweep a Circle of the like Radius on Paper as that on the Quadrant, and then ſetting one foot to the Interſection of

of the Concentrick Circle with the right edge, the Thread being laid over any Arch whatsoever in the Limbe, take the distance to the Intersection of the Thread with the Concentrick Circle, and transform the said Extent into the Circle drawn upon Paper.

Of the Versed Sines Augmented.

These are to be used with fitted Scales thereto, to stand upon a loose Ruler for the ready and more Exact finding the Hour and Azimuth near Noon, or at other times, and shall be treated of in the use of the Diagonal Scale.

Of the Line of Latitudes and Scale of Hours on the spare edges of the foreside:

The use of these Scales are for the ready pricking down of any Diall that hath a Center in an Equicrural Triangle from the Substile, as shall be shewed in the Use of the great Quadrant, though the Schems be fitted to small Scales.

The Scale of Hours standing on the Edge on the foreside may very well be supplyed from thence to another Radius, as shall in due Time be shewed, though it do not proceed from the Center, and therefore may be spared out of the Limbe on the Backside.

The Description of the Diagonal Scale.

THE particular Scales handled in Page 181 would find the hour and Azimuth in the equal Limbe without doubling the Extent, if laying the Thread over the Cosine of the Declination in the lesser sines when the hour is sought; or over the Cosine of the Altitude when the Azimuth is sought, it be minded what Ark of the Limbe the Thread intersects, and then make the entrance of one foot of the Extent at the like Ark in the sines laying the Thread to the other foot according to nearest distance.

But

But becauſe theſe Scales are more convenient being twice as long, there is accordingly a Diagonal Scale fitted to ſerve for our Engliſh Region, and may be accommodated to any 5 or 6 degrees of Latitude, and placed conveniently on any Inſtrument for Surveigh to give the hour and Azimuth in the Limbe of the Inſtrument or on the frame of the plain Table.

And here I am to intimate that either the hour or Azimuth Scale before deſcribed on the ſmall Quadrant, will ſerve to finde both the Hour and Azimuth, as conveniently as either; the foundation whereof is, that the ſame Proportion demonſtrated from the Analemma that finds the hour from ſix being applyed to other ſides of the Triangle will alſo find the Azimuth from the Eaſt or Weſt, an inſtance whereof I give in the Uſe of a great Quadrant for finding the Azimuth when the Declination of the Sun or Stars exceeds the Latitude of the place.

By the like parity of reaſon the Proportion that found the Azimuth is a general Proportion to find an angle, when the threee ſides are given the Canon will be,

As the Coſine of one of the including ſides,
Is to the Radius:
So is the Coſine of the ſide Oppoſite to the angle ſought,
To a fourth a Sine or Secant.

Again.

As the ſecond including ſide,
Is to the Cotangent of the firſt including ſide:
So when any one of the ſides is greater then a Quadrant is the ſum, but when all leſs the difference between the 4th aboveſaid, and the Coſine of the ſecond Includer,
To the Coſine of the angle ſought.

Wee do ſuppoſe but one of the three ſides given to be greater then a Quadrant, if there be any ſuch it ſubtends an Obtuſe angle, and both the other ſides being leſs then Quadrants ſubtend Acute Angles.

When the 4th Sine is leſs then the Coſine of the ſecond Includer, the Angle ſought is Obtuſe, other ways Acute.

Hence the peculiar Proportion educed for the hour will be,

As the Sine of the Latitude, Is to the Radius:
So is the Sine of the Altitude, To a 4th a Sine or Secant.

Again,

Again.

As the Cosin: of the Declination, Is to the Tangent of the Latitude,
So in Summer is the difference, but in Winter the sum of the sines of the Declination of the Sun or Stars, and of the 4th Sine,
To the Sine of the hour from six.

When the 4th Ark is less then the Declination the hour is Obtuse, when greater Acute, and in Winter always Acute:

But of this Proportion I make no use it being liable in some Cases to Excursion, and will not hold backward to find the Suns Altitudes the hour being assigned.

This Diagonal Scale is made after the same manner as the Hour Scale described in the small Quadrant, being but several Lines of sines the greater whereof are made equal to the Secant of the Latitude, whereto they are fitted, the Radius of which Secant is 5 Inches long.

The lesser Sines continued the other way, their respective Radii are made equal to the sine of the Latitude of that greater sine whereto they are continued.

The parralel Lines fitted to the respective Latitudes are not to be equidistant one from another; but having determined the distance between the two Extream Latitudes, to which they are fitted for the the larger sine it will hold,

As the difference of the Secants of the two extream Latitudes,
Is to the distance between the Lines fitted thereto:
So is the difference of the Secants of the lesser extream Latitude, and any other intermediate Latitude,
To the distance thereof from the lesser extream.

And so for the lesser sine continued the other way, having placed the two Extreams under the two former Extreams, to place the intermediate Lines, the Canon would be,

As the difference of the sines of the two extream Latitudes,
Is to the distance between the Lines fitted thereto:
So is the difference of the sines of the lesser extream Latitude; and of any other intermediate Latitude.
To the distance thereof from the lesser Extream.

Having fitted the distances of the greater sine, streight Lines drawn through the two extream sines shall divide the intermediate

Parralels also into Lines of sines, proper to the Latitudes to which they are fitted: Now for the lesser sines they are continued the other way at the ends of the former Parralells, the Line proper to each Latitude should be divided into a Line of sines, whose Radius should be equal to the sine of the Latitude of the other sine whereto it is fitted; and so Lines traced through each degree to the Extreams; but by reason of the small distance of these Lines, the difference is so exceeding small, that it may not be scrupled to draw Lines Diagonal wise from each degree of the two outward extream Sines, for being drawn true, they will not be perceived to be any other then streight Lines.

Whereas these Lines by reason of the latter Proportion should not fall absolutely to be drawn, at the ends of the former Lines whereto they are fitted, and then they would not be so fit for the purpose, yet the difference being as we said, so insensible that it cannot be scaled, they are notwithstanding there placed and crossed with Diagonals drawn through each degree of the Extreams..

The Uses of the Diagonal Scale.

1. *To find the time of Sun rising or setting.*

In the Parralel proper to the Latitude take out the Suns Declination out of the lesser continued sines, and enter one foot of this extent at the Complement of the Declination in the Line of sines, and in the equal Limb the Thread being laid to the other foot will shew the time sought.

In the Latitude of *York*, namely 54^{d}, if the Sun have 20^{d} of Declination $\left.\begin{array}{l}\text{Northward}\\ \text{Southward}\end{array}\right\}$ he rises at $\left\{\begin{array}{l}4\\ 8\end{array}\right\}$ and sets at $\left\{\begin{array}{l}8\\ 4\end{array}\right.$

2. *To find the Hour of the Day or Night for South Declination.*

In the Parralel proper to the Latitude account the Declination in the lesser continued sine, and the Altitude in the greater sine, and take their distance, which extent apply as before to the Cosine of the Declination in the Line of sines on the Quadrant, and laying the

Thread

Thread to the other foot according to nearest distance it shews the time sought in the equal Limbe:

Thus in the Latitude of *York* when the Sun hath 20^d of South declination, his Altitude being 5^d, the hour from noon will be found 45 minutes past 8 in the morning, or 15 minutes past 3 in the afternoon *ferè*.

For North Declination.

The Declination must be taken out of the lesser sine in the proper Parralel, and turned upward on the greater sine and there it shews the Altitude at six for the Sun or any Stars in the Northern Hemispere, the distance between which Point and the given Altitude must be entred as before at the Cosine of the declination, laying the thread to the other foot and it shews the hour in the Limb from six towards noon or midnight, according as the Sun or Stars Altitude was greater or lesser then its Altitude at six.

So in the Latitude of *York*, when the Sun hath 20^d of North declination if his Altitude be 40^d, the hour will be 46 minutes past 8 in the morning, or 14 minutes past 3 in the afternoon.

4. *The Converse of the former Proposition will be to find the Altitude of the Sun at any hour of the day, or of any Star at any hour of the night.*

I need not insist on this, having shewn the manner of it on the small quadrant, only for these Scales use the Limb instead of the lesser sines, for Stars the time of the night must first be turned into the Stars hour, and then the Work the same as for the Sun.

5. *To find the Amplitude of the Sun or Stars.*

Take out the Declination out of the greater sine in the Parralel proper to the Latitude, and measure it on the Line of sines on the lesser Quadrant, and it shews the Amplitude sought.

So in the Latitude of *York* 54^d when the Sun hath 20^d of Declination, his Amplitude will be $35^d\ 35'$.

6. *To find the Azimuth for the Sun or any Stars in the Hemisphere.*

For South Declination.

Account the Altitude in the leſſer ſine continued in the proper Parralel, and the Declination in the greater ſine, and take their diſtance enter one foot of this extent at the Coſine of the Altitude on the Quadrant, and lay the Thread to the other according to neareſt diſtance, and in the Limbe it ſhews the Azimuth from Eaſt or Weſt Southwards.

So in the Latitude of *York*, when the Sun hath 20^d of South Declination, his Altitude being 5^d, the Azimuth will be found to be 44^d 47′ to the Southwards of the Eaſt or Weſt.

For North Declination.

Account the Altitude in the leſſer ſine continued, and apply it upward on the greater ſine and it finds a Point thereon, from whence take the diſtance to the declination in the ſaid greater ſine in the Parralel proper to the Latitude of the place, and enter one foot of this Extent at the Coſine of the Altitude on the Line of ſines, and the Thread being laid to the other foot according to neareſt diſtance, ſhews the Azimuth in the Limbe from Eaſt or Weſt.

So in the Latitude of *York*, when the Sun hath 20^d of North Declination, and 40^d of Altitude, his Azimuth will be 23^d 16′ to the Southwards of the Eaſt or Weſt.

When the Hour or Azimuth falls near Noon, for more certainty you may lay the Thread to the Complement of the Declination for the Hour, or the Complement of the Altitude for the Azimuth, in the Limbe, and enter the reſpective extents Parralelly between the Thread and the Sines, and find the anſwer in the ſines.

We might have fitted one Scale on the quadrant to give both the houre and Azimuth in the Equall Limb by a Lateral entrance, and have enlarged upon many more Propoſitions, which ſhall be handled in the great Quadrants.

M^r *Sutton* was willing to add a Backſide to this Scale, and therefore hath put on particular Scales of his own for giving the requiſites of an upright Decliner in this Latitude, which he hath often made upon Rulers for Carpenters and other Artificers and Dialliſts, and where-

whereof he was willing to afford them a Print; whereto I have added other Scales for giving the Hour and Azimuth near Noon.

On the Backside are drawn these Lines,

A large Dyalling Scale of 6 hours or double Tangents, with a Line of Latitudes fitted thereto.
A large Chord.
A Line for the Substiles distance from the Meridian.
A Line for the Stiles height.
A Line for the angle of 12 and 6.
A Line for the inclination of Meridians.
All these Scales relate to Dyalling.

An Azimuth Scale being two Lines of natural sines of the same Radius set together at O, and thence numbred with Declinations, this Scale must be made of the same sine that the hour Scale following is made of continued from O one way to 38^{d} $28'$, and the other way to 23^{d} $31'$ or further at pleasure; but numbred from the beginning which is at the end of that 38^{d} $28'$ the Complement of the Latitude with 10^{d} $20'$, *&c.* up to 60^{d}.

The Hour Scale is no other then a Line of sines with the declinations set against the Meridian Altitudes in the Latitude of *London*, the Radius of which sine is equal in length to the Dyalling Scale of hours.

Of the Uses of these Scales.

The Line of Hours and Latitudes is general for pricking down all Dialls with Centers as will afterwards be shewed in the Use of the great Quadrant, and by help of the Scale of Hours may the Diameter of a such a Circle be graduated as is placed in on the back of the great Quadrant, and the Line of Latitudes will serve as a Chord to divide the upper Quadrant, and the Hour Scale or Line of Sines will serve as a Chord to divide a Semicircle, whose Diameter is equal to the Scale of Hours into 90 equal parts and their Subdivisions, and hereby may Proportions in sines and Tangents, or Tangents alone be wrought by Protraction, and so the necessary Arks in Dyalling found generally as is done by M^r^ *Foster* in the three last Schems of his *Posthuma*, this will easily be understood if the use of the Circle on this Quadrant be well apprehended.

The

The particular Scales give the requisite Arks of upright Decliners in this Latitude by inspection, for count the plaines Declination in the Line of Chords, and a Square laid over it intersects all those Arks or to be found by applying the Declination taken out of the Chords with Compasses to every other Line.

Example.

So if an upright Plain decline 35^d from the Meridian.

The Substiles distance from the Meridian will be	24^d 30′
The Stiles height	30 38
The Inclinations of Meridians	41 49′
The angle of 12 and 6	54 10

These particular Scales also resolve some of the Cases of right angled Sphoerical Triangles, relating to the Motion of the Sun or Stars thus,

Of the Line of the Stiles height.

Account the Declination in the Line for the Stiles height, and against it in the Chord stands the Amplitude of the Sun or Stars from the Meridian.

Example for Amplitude.

So when the Sun hath 18^d of Declination, his Amplitude will be 67^d 13′ from the Meridian, and 29^d 47′ from the Vertical.

The reason hereof is because the two first fixed Tearms of the Proportion that Calculate the Stiles height are the Radius and the Cosine of the Latitude, and the two first Tearms that Calculate the Amplitude are the Cosine of the Latitude and the Radius, and therefore must as well serve in this Case as in that.

On this Stile Line may be found the Suns Altitudes on all hours, when he is in the Equinoctial by applying the hour from six taken from the Chords to the other end of the Stile Line.

Of the Substiler Line.

Hereby we may find the time of Sun rising and setting, take the Declination out of the Substilar Line and measure it on the Line of Chords.

Example.

So when the Sun hath 18 of North Declination, the Ascensional difference is 24^d 9′ in time 1 hour 36 ⅗ minutes, and so much the Sun rises and sets from six.

Here-

Hereby may be also found the Equinoctial Altitudes to every Azimuth.

Of the Line for the Angle of 12 and 6.

Hereby we may find the time when the Sun will be due East or West.

Account the Complement of the Declination in this Scale, and against it in the Chords stands the hour from six.

Example.

So when the Sun hath 18ᵈ of North Declination, he will be East or West at 7 in the morning, or 5 in the afternon.

By these Scales the requisites of an East or West Reclining or Inclining Diall in this Latitude may be found.

1. *The Substiles distance from the Meridian.*

Account the Complement of the Re/In clination in the Chords, and against it in the Line for 12 and 6 stands the Complement of the angle sought.

2. *For the Stiles height.*

Apply the Reclination in the lesser sines on the Diagonal Scale in the Parralel proper to the Latitude to the greater sine and it shewes the Ark sought.

3. *For the Inclination of Meridians.*

This may be also found on the Diagonal Scale when the Substiles distance is not more then the Latitude, By Accounting the Substiles distance on the greater sine, and applying it to the lesser.

4. *For the Angle of 12 and six.*

Account the Complement of the Reclination in the Chords, and against it in the Substilar Line is the Complement of the angle sought.

So if an East or West Plain Recline or Incline 35ᵈ.

The Substiles distance from the Meridian will be ——	45ᵈ	52′
The Stiles height ——————————	26	41
The Inclination of Meridians ——————	66	27
And the angle of 12 and 6 ——————	56	55

Of the Hour and Azimuth Scales.

This Scale is fitted to find the Hour from Noon in the Versed sines

aug-

augmented, and the Proportion to be wrought by it the ſame as delivered in the uſe of the ſmall Quadrant.

As the Coſine of the Declination, Is to the Secant of the Latitude :
So is the difference of the ſines of the Suns propoſed and Meridian Altitude,
To the Verſed ſine of the hour from Noon.

And of this one Proportion we make two by introducing the Radius.

As the Radius, is to the Secant of the Latitude :
So is the former diſtance, To a fourth.

By fitting the Radius of the ſines equal in length to the Secant of the Latiude ; this firſt Proportion is removed for the ſaid difference of ſines taken out of this fitted Scale is the 4th Proportional, the Proportion that remains to be wrought upon the Quadrant is,

As the Coſine of the Declination, Is to the difference of the ſines taken out of this fitted Scale : So is the Radius, To the Verſed ſine of the hour from Noon.

By this means if in the ſame Proportion as we increaſe the length of the fitted Scale, we alſo increaſe the verſed ſines lying in the Limb, we may find the hour and Azimuth near noon with certainty if the Altitude be well given.

Theſe Scales in their Uſe preſuppoſe the Hour and Azimuth of the Sun to be nearer the noon Meridian then 60^{d}.

Operation to find the Hour.

Take the diſtance between the Altitude and the Declination proper to the ſeaſon of the year out of the Hour Scale, and enter one foot of this Extent at the Coſine of the Declination in the Line of ſines, and laying the Thread to the other foot according to neareſt diſtance, it ſhews the hour from noon in the Verſed ſines Quadrupled.

Example.

When the Sun hath 23^{d} $31'$ of North Declination, and 60^{d} of Altitude, the hour from noon will be 13^{d} $58'$ to be Converted into time.

When the hour is found to be leſs then 40^{d} from Noon, the former extent may be doubled and entred as before, and it ſhews the hour in the Verſed ſines Octupled.

And when the hour is leſs then 30^{d} from Noon, the former extent may be tripled and entred as before, and after this manner it is poſſible

fible to make the whole Limb give the hour next Noon, the Verfed Sine Duodecupled, lies on the other fide of the Quadrant; and in this cafe, an Ark muft firft be found in the Limb, and the Thread laid over the faid Ark, counted from the other edge, will interfect the faid Verfed Sine at the Ark fought.

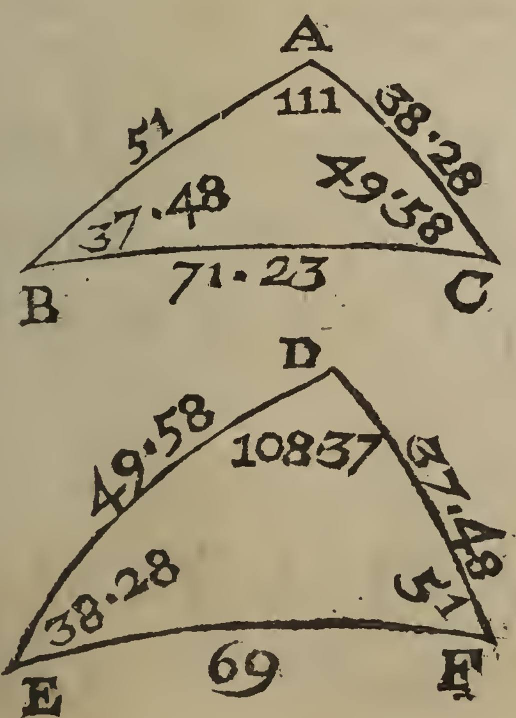
To find the Suns Azimuth.

TAke the diftance in the Azimuth Scale, between the Altitude and the Declination proper to the feafon of the year, and entring it at the Cofine of the Altitude, laying the Thread to the other foot, according to neareft diftance, it will fhew the Azimuth in the Verfed Sines quadrupled; or, when the Azimuth is near Noon, according to the former reftrictions for the hour, the extent may be doubled, or tripled, and the anfwer found in the Verfed Sines Octupled, or Duodecupled, as was done for the hour.

Example.

So when the Sun hath 23^d 31′ of North Declination, his Altitude being 60^d. The Azimuth will be found to be 26^d 21′ from the South.

By the like reafon, when we found the Hour and Azimuth in the equal Limb by the Diagonal Scale, if thofe extents had been doubled, the Hour and Azimuth near fix, or the Vertical, might have been found in a line of Sines of 30^d, put thorow the whole Limb, but that we thought needlefs.



FINIS.

THE DESCRIPTION AND VSES

Of a Great Universal Quadrant:

With a Quarter of *Stoflers* particular *Projection* upon it, Inverted.

Contrived and Written by *John Collins* Accomptant, and Student in the MATHEMATIQUES.

LONDON,
Printed in the Year, 1658.

The DESCRIPTION Of the Great Quadrant.

IT hath been hinted before, that though the former contrivance may serve for a small Quadrant, yet there might be a better for a great one.

The Description of the Fore-side.

On the right edge from the Center is placed a line of Sines.
On the left edge from the Center, a line of Versed Sines to 180^d.
The Limb, the same as in the small Quadrant.
Between the Limb and the Center are placed in Circles, a Line of Versed Sines to 180^d, another through the whole Limb to 90^d.
The Line of lesser Sines and Secants.
The line of Tangents.
The Quadrat and Shaddows.
Above them, the Projection, with the Declinations, Days of the moneth, and Almanack.
On the left edge is placed the fitted Hour, and Azimuth Scale.

Within the Projection abutting against the Sines, is placed a little Scale, called *The Scale of Entrance*, being graduated to 62^{d}, and is no other but a small line of Sines numbred by the Complements.

At the end of the Secant is put on the Versed Sines doubled, that is, to twice the Radius of the Quadrant, and at the end of the Tangents tripled, to some few degrees, to give the Hour and Azimuth near Noon more exactly.

The Description of the Back-side.

On the right edge from the Center, is placed a Line of equal parts, being 10 inches precise, decimally subdivided.

On the out-side next the edge, is placed a large Chord to 60^{d}, equal in length to the Radius of the Line of Sines.

On the left edge is placed a Line of Tangents issuing from the Center, continued to $63^{d}\ 26'$, and again continued apart from 60^{d}, to 75^{d}

The equal Limb.

Within it a Quadrant of Ascensions, divided into 24 equal hours and its parts, with Stars affixed, and Letters graved, to refer to their Names.

Between it and the Center is placed a Circle, whereof there is but three Quadrants graduated.

The Diameter of this Circle is no other then the Dyalling Scale of 6 hours, or double Tangents divided into 90^{d}.

Two Quadrants, or the half of this Circle beneath the Diameter, is divided into 90 equal parts or degrees.

The upper divided Quadrant, is called the Quadrant of Latitudes.

From the extremity of the said Quadrant, and Perpendicular to the Diameter, is graduated a Line of Proportional Sines; M *Foster* calls it *the Line Sol.*

Diagonal-wise, from one extremity of the Quadrant of Latitudes, to the other, is graduated a line of Sines; that end numbred with 90^{d}, that is next the Diameter, being of the same Radius with the Tangents.

Opposite

Opposite and parallel thereto from 45^d of the Semicircle, to the other extremity of the Diameter, is placed a Line of Sines equal to the former,

Diagonal-wise. from the beginning of the Line Sol, to the end of the Diameter, is graduated a Line of 60 Chords.

From the beginning of the Diameter, but below it, towards 45^d of the Semicircle, is graduated the Projection Tangent, *alias*, a Semi-tangent, to 90^d, being of the same Radius with the Tangents.

The other Quadrant of this Circle being only a void Line, there passeth through it from the Center, a Tangent of 45^d, for Dyalling, divided into 3 hours, with its quarters and minutes.

Below the Diameter is void space left, to graduate any Table at pleasure, and a Line of Chords may be there placed.

Most of these Lines, and the Projection, have been already treated upon in the use of the small Quadrant, those that are added, shall here be spoke to.

Of

Of the Line of Verſed Sines, on the left Edge, iſſuing from the Center.

THis Line, and the uſes of it, were invented by the learned Mathematician, M. *Samuel Foſter*, of *Greſham* Colledge. deceaſed, from whom I received the uſes of it, applyed to a Sector; I ſhall, and have added the Proportions to be wrought upon it, and in that, and other reſpects, diverſifie from what I received; wherein I ſhall not be tedious, becauſe there are other ways to follow, ſince found out by my ſelf.

The chief uſes of it are. to reſolve the two caſes of the fourth Axiom of Spherical Trigonometry; as, when three ſides are given to find an Angle, or two ſides with the Angle comprehended, to find the third ſide, which are the caſes that find the Hour and Azimuth generally, and the Suns Altitudes on all hours. For the Hour, the learned Author thought meet to add a Zodiaque of the Suns place annexed to it, both in the uſe of his Sector, as alſo in the uſe of his Scale, publiſhed ſince his death, entituled *Poſthuma Foſteri*, that the Suns place being given, which for Inſtrumental uſe might be obtained, by knowing on what day of each moneth the Sun enters into any Signe, and allowing a degree for every days motion, come by it *prope verum*, and being ſought in the annexed Zodiaque (which is no other then two lines of 90^{d}. Sines, each made equal to the Sine of 23^{d} 31$'$ the Suns greateſt Declination) juſt againſt it ſtands the Suns Declination, if accounted in the Verſed Sine, from 90^{d} each way; but this for want of room, and becauſe the Declination is more eaſily given by help of the day of the moneth, I thought fit to omit, the rather, becauſe it may alſo be taken from the Table of Declinations.

But from hence I firſt obſerved, that if the two firſt terms of a Proportion were fixed, if two natural Lines proper to thoſe terms, were

were fitted of an equal length, and poſited together, if any third term be given, to find a fourth in the ſame proportion, it would be given by inſpection, as ſtanding againſt the third; but if the Lines ſtand aſunder, or a difference be the third term, application muſt be made from one Line to the other with Compaſſes, as in the ſame Scale there is alſo fitted a Line of 60 parts, equal in length to the Radius of a ſmall Sine, ſerving to give the Miles in every ſeveral Latitude, anſwerable to one degree of Longitude.

Three ſides given to find an Angle, the Proportion,

As the difference of the Verſed Sines of the Sum, and difference of any two Sides including an Angle,
Is to the Diameter,
So is the difference of the Verſed Sines of the third ſide,
And of the Ark of difference between the two including Sides,
To the Verſed Sine of the Angle ſought;
And ſo is the difference of the Verſed Sines of the third ſide
And of the ſum of the two including ſides,
To the Verſed Sine of the ſought Angles, Complement to a Semicircle.

Corollary.

And ſeeing there is ſuch proportion between the latter terms of the fore-going Proportion, as between the former, omitting the two firſt terms, it alſo holds,

As the difference of the Verſed Sines of the third ſide, and of the Ark of difference between the two including ſides
Is to the Verſed Sine of the Angle ſought,
So is the difference of the Verſed Sines of the third ſide,
And of the ſum of the two including ſides,
To the Verſed Sine of the ſought Angles, Complement to 180^d.

And this is the Proportion M. *Foſter* makes uſe of in his Scale, page 25 and 27. to find the Hour and Azimuth by Protraction, as alſo in page 68. in Dyalling, when three ſides are given to find an Angle, by conſtituting two right angled equi angled plain Triangles, the legs whereof conſiſt of the 4 terms of this Proportion.

But in that Protraction work, the firſt and third terms of the Proportion are given together, with the ſum of the ſecond and fourth terms, to find out the ſaid terms reſpectively.

The Proportion for the Hour.

As the difference of the Verſed Sines of the Sum, and difference of the Complement of the Latitude, and of the Sun or Stars diſtance from the Elevated Pole;
Is to the Diameter or Verſed Sine of 180^{d},
So is the difference of the Verſed Sines of the Complement of the Altitude, and of the Ark of difference between the Complement of the Latitude, and of the Polar diſtance,
To the Verſed Sine of the Hour from Noon.

And if the latter clauſe of the third term be the Sum of the Co-latitude and Polar diſtance, the Proportion will find the Verſed Sine of the hour from midnight,

And if the ſum of any two Arks exceed a Semicircle, take its Complement to 360^{d}, for the ſame Verſed Sine is common to both.

When the Declination is towards the Elevated Pole, the Polar diſtance is the Complement of it to 90^{d}; and when towards the Depreſſed Pole, the Polar diſtance is equal to the Sum of 90^{d}, and of the Declination added together.

Example.

Let the Suns Declination be 15^{d} 46′ North, Complement,	74^{d}	14′
The Complement of the Latitude,	38	28
Sum	112 :	42
Difference	35 :	46
And let the Altitude be 20^{d}, Complement	70 :	00

Operation

Operation.

Take the diſtance between the Verſed Sines of 35^{d} 46′, and of 112^{d} 42′, and entring one foot of that extent at the end of the Verſed Scale at 180^{d}, lay the thred to the other foot, according to neareſt diſtance, then take the diſtance between the Verſed Sines of 35^{d} 46′, and 70^{d}, and entring that extent parallelly, between the Thred and the Scale, and the other foot will reſt upon the Verſed Sine of 77^{d} 32′, the quantity of the Hour from the Meridian being either 50′ paſt 6 in the morning, or 10′ paſt 5 in the afternoon.

The Reader may obſerve in this work, that the thred lies over a Star, by entring the firſt extent; as alſo, that there is the ſame Star graduated at 35^{d} 46′ of the Verſed Sine, and this no other then the Bulls eye, having 15^{d} 46′ of North Declination, for which Star in this Latitude, there needs be no ſumming or differencing of Arks, in regard the Stars declination varies not: So to find that Stars hour at any time, having any other Altitude, only lay the thred over that Star in the Quadrant, and take the diſtance between the Star in the Scale, and the Complement of its Altitude, and enter that extent parallelly between the Thred and the Scale, and it finds the Stars hour from the Meridian: Thus when that Star hath 39^{d} of Altitude, its hour from the Meridian will be found to be 45^{d} 54′, in time, 3 hours $3\frac{1}{2}$′, which to get the true time of the night, muſt be turned into the Suns hour by help of the Nocturnal on the Back-ſide: But admitting the Suns Declination and Altitude to have been the ſame with the Stars, the true time of the day thus found, would have been $56\frac{1}{2}$′ paſt 9 in the morning, or $3\frac{1}{2}$′ paſt 3 in the afternoon; and thus the Reader may have what Stars he pleaſes put on of any Declination, and for any Latitude; and they may be put on at ſuch a diſtance from the Center, that the diſtance from it to the Star, may be a Chord to be meaſured in the Limb, to give the Stars Aſcenſional difference, or the like concluſion: And thus the thred being once laid, and the former point found for one example to the Suns Declination, neither of them varies that day; which is a ready general way for finding the time of the day for the Sun.

To find the Semidiurnal, and Seminocturnal Arks.

Suppose the Sun to have no Altitude, and the Complement of it to be 90^d, and then work by the former precept, and you will find the Semidiurnal Ark from the beginning of the Line, and the Seminocturnal Ark from the end of the Line, which doubled, and turned into time, shews the length of the Day and Night, and the difference between 90^d, and either of those Arks is the Ascensional difference, or time of rising and setting from 6.

To find the Azimuth generally.

The Proportions for this purpose have been delivered before, from which it may be observed, that there are no two terms fixed, and therefore to every Altitude, the containing sides of the Triangle, namely, the Complements both of the Altitude and Latitude must be summed and differenced, when the Proposition is to be performed on this Line solely, and the Operation will be after the same manner, as for the hour, namely, with a Parallel entrance; and this is all I shall say of the Authors general way; and of any other that he used, I never heard of; those ways that follow, being of my own supply.

By help of this Line to work a Proportion in Sines alone, wherein the Radius leads.

As the Radius
Is to the Sine of any Ark,
So is the Sine of any other Ark
To the Sine of a fourth Ark.

This fourth Sine, as I have said before, is demonstrated by M. *Gellibrand*, to be equal to half the difference of the Versed Sines of the Sum, and difference of the two middle terms of the Proportion.

Operation

Operation.

Let the Proportion be,

As the Radius
Is to the Sine of ———————— 40^d
So is the Sine of ———————— 27
To a fourth Sine ————————

Sum ——— 67
Difference —— 13

Take the distance between the Versed Sines of the said sum and difference, and measure it on the Line of Sines from the Center, and it will reach to 17^d, the fourth Sine sought.

By help of this Line may the Divisions of the line Sol, or Proportional Sines, be graduated to any Radius less then half the Radius of the Quadrant, the Canon is,

As the Versed Sine of any Ark added to a Quadrant,
Is to the Radius, or length of the Line Sol,
So is the Versed Sine of that Arks Complement to 90^d
To that length which pricked backward from the end of the Radius of the said Line, shall graduate the Arch proposed.

Example.

Suppose you would graduate 20 of the Line Sol, enter the Radius of the said Line upon the Versed Sine of 110, laying the thred to the other foot; and from the Versed Sine of 70^d, take the nearest distance to the thred, which prick from the end of the Line Sol, towards the beginning, and it shall graduate the said 20^d.

This Line Sol is made use of by M. *Foster* in his Scale for Dyalling.

The Line of Versed Sines was placed on the left edge of the foreside of the Quadrant, for the ready taking out the difference of the Versed Sines of any two Arks, and to measure a difference of two Versed Sines upon it, which are the chief uses I shall make of it; whereas to Operate singly upon it, it would be more convenient for the hand to have it lie on the right edge of the Quadrant.

An-

An example for finding the Azimuth generally, by help of Verſed Sines in the Limb, and of other Lines on the Quadrant.

I ſhall rehearſe the Proportion,

As the Coſine of the Latitude is to the Secant of the Altitude,
Or, *As the Coſine of the Altitude is to the Secant of the Latitude,*
So is the difference of the Verſed Sines of the Suns diſtance from the Elevated Pole, and of the Ark of difference between the Latitude and Altitude,
To the Verſed Sine of the Azimuth from the midnight meridian.

And making the latter clauſe of the third term the Complement of the Sum of the Latitude and Altitude to a Semicircle, the Proportion will find the verſed Sine of the Azimuth from the noon Meridian.

Example.

Altitude,	$51^d\ 32'$	
Latitude	$34:32$	Complement $55^d\ 2[illegible]$
Difference	$17:00$	
☉ diſtance from elevated Pole,	$66:29$	

Operation in the firſt Terms of the Proportion.

On the Line of Verſed Sines, take the diſtance between 17^d, and $66^d\ 29'$, and entring it twice down the line of Sines, from the Center, take the neareſt diſtance to the thread laid over the Secant of $51^d\ 32'$, the given Altitude, and entring one foot of this Extent at the Sine of $55^d\ 28'$ the Complement of the Latitude, lay the thred to the other foot, according to neareſt diſtance, and in the line of Verſed Sines in the Limb, it will lie over 95^d, for the Suns Azimuth from the midnight meridian.

And the Suns declination ſuppoſed the ſame, he ſhall have the like

like Azimuth from the North, in our Latitude of *London*, when his Altitude is 34^{d} 32′, for the ſides of the Triangle are the ſame.

Another Example.

To find it in the verſed Sine of 90^{d}

Latitude	47^{d} 27′
Altitude	51 : 32
Sum	98 : 59
Complement	81 : 1
Polar diſtance	66 : 29

Take the diſtance in the Line of Sines, as repreſenting the former half of a Line of Verſed Sines, between theſe two Arks counted towards the Center, *viz.* 66^{d} 29′, and 81^{d} 01′, and enter this extent twice down the Line of Sines from the Center, and take the neareſt diſtance to the thred lying over the Secant of the Latitude 47^{d} 27′, then enter one foot of this extent at 51^{d} 32′ counted from the end of the Sines towards the Center, laying the thred to the other foot, according to neareſt diſtance, and in the Verſed Sine of 90, it ſhews the Azimuth to be 65^{d} from the South in this our Northern Hemiſpere.

Of

Of the fitted Particular Scale, and the Line of Entrance thereto belonging.

THis Scale ſerves to find both the Hour and Azimuth in the Latitude of *London*, to which it is fitted, in the equal Limb, by a Lateral or poſitive Entrance, it conſiſts of two Lines of Sines.

The greater is 62' of a Sine, as large as can ſtand upon the Quadrant, the Radius of the leſſer Sine is made equal to 51' 32' of this greater, being fitted to the Latitude: The Scale of Entrance ſtanding within the Projection, and abutting on the Line of Sines, is no other but a portion of a Line of Sines, whoſe Radius is made equal to 38' 28' of the greater Sine of the fitted Scale; and this Scale of Entrance is numbred by its Complements up to 62', as much as is the Suns greateſt meridian Altitude in this Latitude.

The ground of this Scale is derived from the Diagonal Scale, the length whereof bears ſuch Proportion to the Line of Sines whereto it is fitted, as the Secant of the particular Latitude doth to the Radius, which is the ſame that the Radius bears to the Coſine of the Latitude, and conſequently, making the Line of Sines to repreſent the fitted Scale, the Radius of that Sine whereto it is fitted, muſt be equal to the Coſine of the Latitude: and ſo we needed no particular Scale, but this would remove the particular Scale, or Scale of Entrance, nearer the Center, and would not have been ſo ready as this fitted Scale; however, hence I might educe a general method for finding the hour and Azimuth in the Limb, without Tangents or Secants. The firſt Work would be to proportion out a Sine to a leſſer Radius, which would find the point of Entrance, the next would be to finde the Altitude, or Depreſſion, at 6. the third would be to enter the ſum, or diffe ence of the Sines of the Altitude, or depreſſion at 6 at the point of Entrance, and to lay the thred to the other foot; but I ſhall demonſtrate it from other grounds.

1. To

1. *To find the time of Sun Rising, or Setting.*

Take the Declination from the leſſer Sine, and enter it at the Declination in the Scale of Entrance, laying the thred to the other foot, according to neareſt diſtance, and it ſhews the time of Riſing or ſetting in the equal Limb.

So when the Sun hath 13 of South Declination, he riſeth at 8′ paſt 7 in the morning *fere*, and ſets at 52′ paſt 4 in the afternoon.

2. *To find the true time of the day.*

In Summer, or Northwardly Declination, take the diſtance between the Altitude in the greater Sine, and the Declination in the leſſer Sine.

In Winter, take the Declination in the leſſer Sine, and with your Compaſſes add it to the Altitude in the greater Sine.

Theſe extents enter at the Declination in the Scale of Entrance, and lay the thred to the other foot, according to neareſt diſtance, and in the equal Limb, it will lye over the true time of the day.

In Summer, when the Declination in the fitted Scale is above the Altitude, the hour is found from 6 towards midnight, when below it, towards Noon.

Example.

When the Sun hath 13 of North Declination, his Altitude being 39 10′ will be a quarter paſt 9 in the morning, or 3 quarters paſt 2 in the afternoon; and when he hath the ſame South Declination, his Altitude being 16 14′ the time of the day will be found the ſame.

The Converſe will find the Suns Altitudes on all hours by this fitted Scale, which I ſhall handle the general way.

3. *To find the Amplitude.*

Take the Declination from the greater Sine, and enter it at the beginning of the Scale of Entrance, laying the thred to the other foot, according to nearest distance, and it shews it in the Limb.

When the Sun hath 13^d of Declination, his Amplitude will be 21^d 12′.

4. *To find the Azimuth of the Sun.*

In Summer, take the distance between the Altitude in the lesser Sine, and the Declination in the greater.

In Winter, or South Declinations, take the Declination from the greater Sine, and add it to the Altitude in the lesser Sine with your Compasses.

These Extents, enter at the Altitude in the Scale of Entrance, and lay the thred to the other foot, according to nearest distance, and in the equal Limb, it shews the Azimuth from the East or West.

In Summer, when the Altitude falls below the Declination, the Azimuth is found from the East or West, Northwards; when above it, Southwards.

So when the Sun hath 13^d of North Declination, his Altitude being 43^d 50′ the Azimuth will be found to be 45^d from East or West, Southwards; and when he hath the same South Declination, his Altitude being 14^d 50′ he shall have the same Azimuth.

These Scales are fitted to give the Altitude at six, and the Vertical Altitude by Inspection.

Against the Declination in the $\left.\begin{matrix}\text{greater}\\\text{lesser}\end{matrix}\right\}$ Sine

stands the $\left\{\begin{matrix}\text{Vertical Altitude or Depression,}\\\text{Altitude or Depression at six.}\end{matrix}\right.$

When the Hour or Azimuth falls near Noon, mind against what Arch of the Line of Sines the point of Entrance falls, the thred may be laid to the like Arch in the Limb, and the respective extents entred

entred parallelly between the Scale and the thred, and the answer found in the Line of Sines.

But we have a better remedy by help of the Versed Sine of 90^d put thorow the whole Limb.

The joynt use of the Fitted Scale, with the Versed Sine of 90^d in the Limb.

IN the following Propositions, I shall make no use of the lesser Sine of the Fitted Scale.

Get the Summer and Winter Meridian Altitude, by summing and differencing the Declination, and the Complement of the Latitude, which may be done with Compasses in the equal Limb, by applying the Chord of the Declination both ways from the Colatitude.

To find the Hour of the Day in Winter.

Take the distance between the Meridian Altitude, and the given Altitude, out of the greater Sine of the fitted Scale, and as before, enter it at the Declination in the Scale of Entrance, laying the thred to the other foot, according to nearest distance; and in the Versed Sine of 90^d it shews the hour from Noon.

So if the Sun have 13^d of South Declination, the Meridian Altitude is 25^d 28$'$, if the given Altitude be 17^d 44$'$ the time of the day will be half an hour past 9 in the morning, or as much after 2 in the afternoon.

To find the Hour of the Day in Summer.

Take the distance between the Summer Meridian Altitude, and the proposed Altitude, and if this extent be less then the distance of the Declination in the Scale of Entrance from the Center, enter it

it at the Declination in the said Scale, and laying the thred to the other foot, it will in the Versed Sine of 90^d shew the Hour from Noon.

If the Sun have 23^d 31′ of North Declination, his Meridian Altitude will be 61^d 59′, if his given Altitude be 47^d 51′, the time of the day will be a quarter past 9 in the morning, or three quarters of an hour past 2 in the afternoon.

If the Extent be larger then the distance of the point of Entrance, to wit, the distance of the Declination in the Scale of Entrance from the Center, the hour must be found from midnight.

In this case, with your Compasses add the Sine of the Winter Meridian Altitude, taken from the greater Sine of the fitted Scale, to the Sine of the Altitude in the said Scale, and enter the said whole extent at the point of Entrance, as before; and in the Versed Sine of 90^d, the thred will shew the hour from midnight.

When the Sun hath 23^d 31′ of North Declination, if his Altitude be 5^d 24′, the time of the day will be half an hour past 4 in the morning, or half an hour past 7 in the evening, the Winter Meridian Altitude to this Declination being 14^d 57′. When the hour in these examples falls near Noon, the extent of the Compasses may be doubled, or tripled, and an Ark first found in the Limb, then if the thred be laid over the like Ark from the other edge, it will accordingly in the Versed Sines doubled or tripled, shew the time sought; and the like may be done for the Azimuth.

To find the Azimuth of the Sun in Winter.

Get the Ark of difference between the Suns Altitude, and the Complement of the Latitude, and in the greater Sine of the fitted Scale, take the distance between the said Ark, and the Suns Declination, and enter one foot of this Extent at the Altitude in the Scale of Entrance, laying the thred to the other foot, and in the Versed Sine of 90^d, it shews the Azimuth from Noon Meridian.

Example.

Example.

Colatitude,	38^d 28′	The Azimuth to this example, will be 50^d from the South.
Altitude,	12 : 13	
Ark of Difference	26 : 15	
Declination	13 : 00	

In Summer, get the Ark of difference between the Altitude, and the Complement of the Latitude, then when the Suns Altitude is the lesser of the two, take the sum, but when the greater, the difference of the Sines of the Suns Declination, and of the said Ark, and enter it at the Altitude on the Scale of Entrance, and you will find the Azimuth from the noon Meridian, as before; but when either of those extents are larger then the distance between the point of Entrance and the Center, the Azimuth must be found from the midnight Meridian.

In this case, take the difference, that is, the distance of the Sines of the Suns Declination, and of the Ark, being the sum of the Altitude and Colatitude, out of the greater Sine of the fitted Scale, and enter it at the Altitude in the Scale of Entrance, laying the thred to the other foot, and in the Versed Sine it shews the Azimuth from the North.

Example for finding the Azimuth from the North.

Colatitude	38^d 28′	The Azimuth to this example, will be found to be 70^d from the North.
Altitude	14 : 15	
Sum	52 : 43	
Declination,	23 : 31	

Of

Of the joynt use of the Diagonal Scale, with the Line of Sines on this Quadrant.

If the respective extents that found the Hour and Azimuth in the Limb on the small Quadrant, be doubled, and applyed here to the Line of Sines issuing from the Center, which in this case becomes the Scale of Entrance, the Hour and Azimuth will be also found in the equal Limb of this Quadrant, for all those respective Latitudes to which the Diagonal Scale is accommodated.

Of the Hour and Azimuth Scales on the Back-side thereof.

THose Scales were fitted to the Versed Sines quadrupled on that small Quadrant, and consequently, are fitted to the Versed Sine of 90^{d}, and the Line of Sines on this Quadrant, which is just double the Radius of that Quadrant.

Those Scales are peculiarly fitted for the Latitude of *London*, and thereby we may alwaies find the Hour and Azimuth in the Versed Sine of 90', without the trouble of summing or differencing of Arks.

1. *By the Hour Scale, to find the Hour of the Day.*

Take the distance between the Declination, proper to the season of the year, and the Altitude, and entering one foot of that extent at the Complement of the Declination in the Sines, lay the thred to the other foot, according to nearest distance, and it shews the hour from Noon.

Example.

Example.

When the Sun hath 13 of North Declination, his Altitude being 47 24′, the Hour will be 30′ past 10 in the morning, or as much past 1 in the afternoon.

In Summer, when this extent is greater then the Cosine of the Declination, and that it will be, when the Sun hath less Altitude then he hath at 6.

The Declination is graduated against the Meridian Altitudes. In this case, add the Sine of the Altitude given, to the Sine of the Meridian Altitude in Winter, to that Declination, with your Compasses, and enter that whole extent at the Declination counted in the Line of Sines from 90 laying the thred to the other foot, according to nearest distance, and in the Versed Sine of 90, it will shew the hour from midnight.

Declination,—23 31′ North;	The hour will be found either 4 in the morning, or 8 at night.
Altitude—— 1 : 34	

2. *By the Azimuth Scale, to find the Azimuth of the Sun.*

Take the distance between the Declination proper to the season of the year, and the Altitude, and entering one foot of this extent at the Complement of the Altitude in the Lines of Sines issuing from the Center, to the other lay the thred according to nearest distance, and it shews the Azimuth from the noon Meridian in the Versed Sine of 90.

Declination—23 31′ North,	The Azimuth hereto will be found 65 from the South.
Altitude——47 : 27	

In Summer, when this extent is greater then the Cosine of the Altitude, and that it will be, when the Sun hath less Altitude then he hath in the Vertical, the Azimuth must be found from the midnight Meridian. In this case, because the Azimuth Scale is not continued

continued far enough, the ſum of the Altitude and Colatitude muſt be gotten, and the diſtance taken between the ſaid Ark and the Declination, counted in the hour-Scale as a Sine, and that extent entred at the Altitude counted from 90^{d} in the Line of Sines, and the thred laid to the other foot, will ſhew the Azimuth from the North in the Verſed Sine of 90^{d} in the Limb.

Colatitude,	38 : 28	The Azimuth to this example will be 65^{d} from the North.
Altitude	10 : 19	
Sum	48 : 47	
Declination North.	23 : 31	

General Proportions.

It now remains to be ſhewed, how the Hour, and Azimuth, *&c.* may be found generally, either in the equal Limb, or in the Verſed Sine of 90^{d}, and that without the help of Tangents or Secants, and poſſibly with more convenience then with them.

In page 55. I have aſſerted, that the fourth term in any direct Proportion, bears ſuch Proportion to the firſt term, as the Rectangle of the two middle terms doth to the ſquare of the firſt term.

And in page 105. That the Sine of any Arch bears ſuch proportion to the Secant of the Complement of another Ark, as the Rectangle of the Sines of both thoſe Arks, doth to the Square of the Radius.

Whence it follows,

That, *As the Radius,*
Is to the Sine of one of the ſides including an Angle,
So is the Sine of the other containing ſide,
To a fourth Sine.

I ſay then, that this fourth Sine bears ſuch Proportion to the Radius, as the Sine of one of thoſe including ſides, doth to the ſecant of the Complement of the other.

And therefore, when three ſides are given to find an Angle, it will hold,

As the Radius,
Is to the Sine of one of those including sides,
So is the Sine of the other including side,
To a fourth sine.

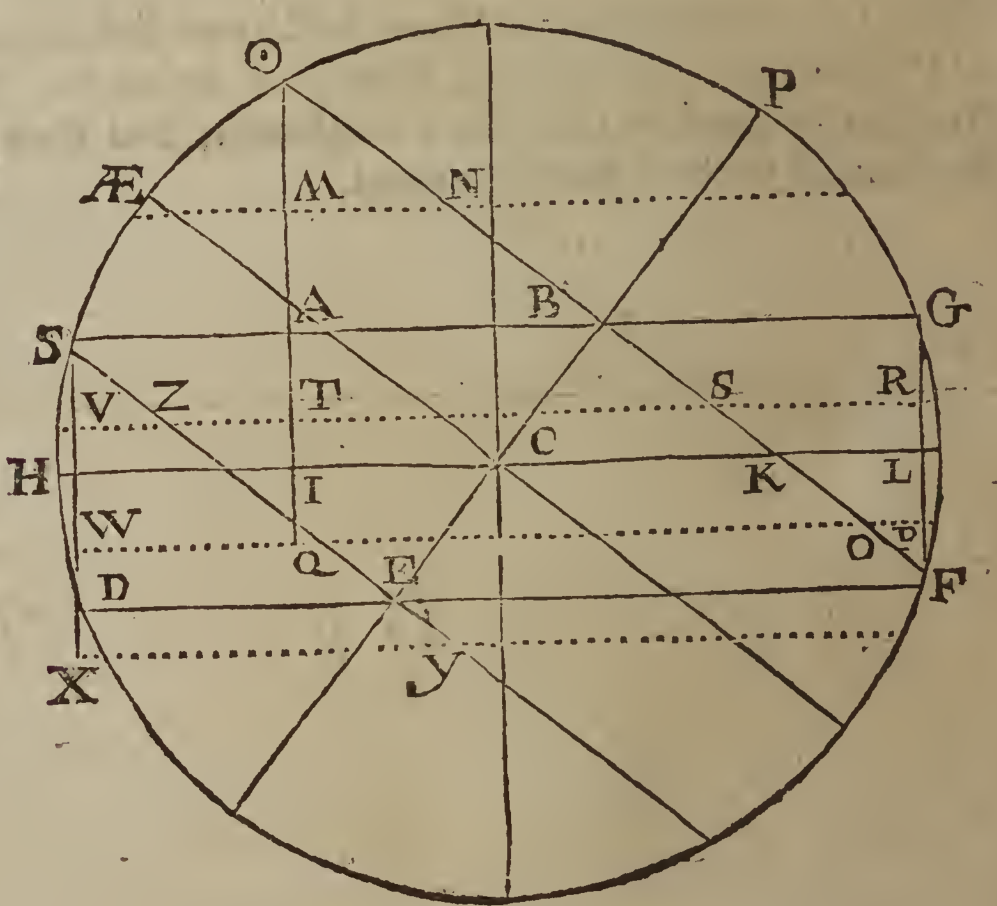
Again,

As that fourth Sine,
Is to the difference of the Versed Sines of the third side, and of the Ark of difference between the two including sides,
So is the Radius,
To the Versed Sine of the Angle sought.
And as that fourth Sine,
Is to the difference of the Versed Sines of the third side, and of the sum of the two including sides,
So is the Radius,
To the Versed Sine of the sought Angles, Complement to 180^{d}, *or a Semicircle.*

Thus we are freed from a Secant in the two first terms of 3 several Proportions that find the Hour and Azimuth: All which I shall further confirm from the *Analemma*, and then proceed to Application, in the Scheme annexed.

Proportions in the Analemma.

UPon the Center C, draw a Circle, and let N C be the Axis of the Horizon, and E P the Axis of the World, Æ C the Equator, ☉ F and Z Y two Parallels of Declination on each ſide the Equator, alike equidiſtant, S G the parallel of Altitude at 6, and D E F the parallel of Depreſſion at 6; draw a parallel of Altitude leſs then the Altitude at 6 V R, and another greater M N continued; alſo a parallel of Depreſſion leſs then the Depreſſion at 6 W P, and another greater X Y, and there will be conſtituted diverſe right lined, right angled Triangles, relating to the motion of the Sun or Stars, in which it will hold.



As Radius is to the Cosine of the Declination,
So is the Cosine of the Latitude
To a fourth.

Namely the difference of the Sines of the Meridian Altitude, and Altitude at 6 in Summer, equal to the Sum of the Sines of the Meridian Altitude and Depression at 6 in Winter, which is equal to the sum of the Sines of the midnight Depression and Altitude at 6 in Summer.

A : B ☉ :: B : A ☉. Again the same, As D : S E :: E : D S
Again the same ——— G : B F :: B : F G.

As that fourth is to the Radius,
So is the Sine of the Meridian Altitude,
To the Versed Sine of the Semidiurnal Ark
A☉ : ☉B :: ☉I ; ☉K.

The two first terms are common to all the rest of the following Proportions.

And so is the Sine of the midnight Depression,
To the Versed Sine of the Seminocturnal Ark.
G F : B F :: F L to K F.

And so is the Sine of the Altitude,
To the difference of the Versed Sines of the Semidiurnal Ark, and Hour sought from Noon,
☉A : B☉ :: I M : K N.

And so is the Sine of the Depression,
To the difference of the Versed Sines of the Seminocturnal Ark, and of the hour from Midnight,
F G : F B :: P L : K O

And so is the difference of the Sines of the Suns Meridian, and given Altitude,
To the Versed Sine of the hour from Noon,
☉A : ☉B :: ☉M : ☉N.

 If

If the Sun have Depreſſion,

So is the ſum of the Sines of the Suns Meridian Altitude, and propoſed Depreſſion,
To the Verſed Sine of the hour from Noon,

A☉ : ☉B :: ☉Q : ☉O.

And ſo is the difference of the Sines of the Midnight and propoſe Depreſſion,
To the Verſed Sine of the hour from Midnight,

FG : FB :: PF : OF.

But ſuppoſing the Sun to have Altitude, retaining ſtill the two firſt terms, it holds.

And ſo is the ſum of the Sines of the Midnight Depreſſion and given Altitude,
To the Verſed Sine of the hour from Midnight.

FG : FB :: FR : FS.

And ſo is the Sine of the Altitude, or Depreſſion at ſix,
To the Sine of the Aſcenſional difference,

A☉ : ☉B :: AI : BK.

In Summer, if the Sun have Altitude,

So is the difference of the Sines of the Altitude at ſix, and of the given Altitude,
To the Sine of the hour from ſix, towards Noon, if the given Altitude be greater then the Altitude at ſix, otherwiſe towards Midnight.

A☉ : ☉B :: AM : BN. Alſo A☉ : ☉B :: AT : BS.

If he have Depreſſion,

So is the ſum of the Sines of the Altitude at ſix, and the given Depreſſion,
To the Sine of the Hour from ſix, towards Midnight.

A☉ : ☉B :: AQ : BO.

In

In Winter, if the Sun have Altitude:

So is the the sum of the Sines of his Depression at 6, and of his given Altitude,

To the Sine of the hour from 6 toward Noon,

SD : SE :: DV : ZE.

If he have Depression.

So is the difference of the Sines of his Depression at 6, and of his given Depression.

To the Sine of the hour from 6 towards Noon, when the Depression is less then the Depression at 6, otherways towards Midnight.

SD : SE :: WD : QE.

SD : SE :: DX : EY.

When two terms of a Proportion happen in the common Radius, and two in a Parallel, there needs no Reduction.

In Latitudes nearer the Poles then the Polar Circles, the Semidiurnal Arks, when the Declination is towards the Elevated Pole, will be more then the Diameters of their Parallels; in that case, the difference, is the difference of the Versed Sine of the Hour, and of the fourth Proportional, found by the Proportion that finds the Semidiurnal Ark.

General Proportions for the Hour.

The Proportion selected for the Hour is,

As the Radius, Is to the Cosine of the Latitude,

So is the Cosine of the Declination,

To a fourth:

Namely, the difference of the Sines of the Meridian Altitude, and of the Altitude at 6.

Again,

Again,

1. *As that fourth, Is to the Radius,*
So in Summer, *is the difference*; *but* in VVinter, *the sum of the Sines of the Suns Altitude or Depression at 6,*
To the Sine of the Hour from 6 towards Noon or Midnight, according as the Altitude or Depression is greater or less then the Altitude or Depression at 6.
2. *And so is the difference of the Sines of the Meridian, and proposed Altitude,*
To the Versed Sine of the Hour from Noon:
And so is the sum of the Sines of the Midnight Depression, and given Altitude,
To the Versed Sine of the Hour from Midnight.
3. *And so is the Sine of the Altitude,*
To the difference of the Versed Sines of the Semidiurnal Ark, and of the Hour sought.

By the first Proportion, the hour may be found generally, either in the equal Limb, or Line of Sines.

By the second Proportion, it may be found generally, either in the Versed Sines of 90^{d}, or 180^{d}.

By the third Proportion, it may be found in the Line of Versed Sines issuing from the Center in many cases. I shall add a brief Application of all three ways.

The first Work will be to find the point of Entrance.

Example, For the Latitude of Nottingham, 53^{d}:

Lay the thred to the Declination, admit 20 in the Limb, counted from the left edge, and from the Latitude in the Line of Sines, counted towards the Center from 90^{d}; take the nearest distance to the thred, the said extent measured from the Center, will fall upon 34^{d} 25′, and there will be the point of Entrance; let it be recorded, or have a mark set to it.

If the Suns Declination be North, the Meridian Altitude in that Latitude, will be 57^{d}, the said extent will reach from the Sine thereof, to the Sine of the Suns Altitude, or Depression at 6, to that Declination, namely, to 15^{d} 51′: which may also be found with-

out

out the Meridian Altitude, by taking the distance from 20' in the Sines, to the thred laid over the Arch 53', counted from the right Edge, and by measuring that extent from the Center, the point thus found, I call the Sine point.

Thirdly, If the respective distances between the Sine point, and the Sine of the given Altitude, be taken and entred upon the point of Entrance, laying the thred to the other foot, according to nearest distance, the hour may be found all day for that Declination, when it is North in the equal Limb.

Example, For the Latitude of Nottingham *, to the former Declination, being North.*

When the Sun hath 11' 31' / 20 17 } of Altitude, the Hour in each Case will be found half an hour from 6, to the lesser Altitude beyond it, towards Midnight; to the greater, towards Noon.

And when the Altitude is 38d 19', the time of the day will be either half an hour past 8 in the morning, or half an hour past 3 in the Afternoon.

An Example for the Latitude of Nottingham, *when the Declination is as much South.*

Let the Altitude be 10d 6', In this case add the Sine thereof to the Sine of 15' 51', the whole extent will be equal to the Sine of 26' 39'; Enter this Extent upon the point of Entrance at 34' 25' laying the thred to the other foot, according to nearest distance, and the time of the day found in the Limb, will be either half an hour past 9 in the morning, or half an hour past 2 in the afternoon.

An Example for working the second Proportion.

The Summer Meridian Altitude is 57', if the given Altitude be 46d 11'; take the distance between the Sines of these two Arks, and entring this extent upon the point of Entrance, lay the thred to the other foot, according to nearest distance, it will in the Versed

Sine

Sine of 90^{d}, ſhew the Hour from Noon to be 37^{d} 30′, that is, either half an hour paſt 10 in the morning, or half an hour paſt 1 in the Afternoon.

And when the Hour falls near Noon, we may double or triple the extent of the Compaſſes, and find an Ark in the Limb, which if counted from the other edge, and the thred laid over it, will give anſwer in the Verſed Sines doubled or tripled accordingly.

A third Example.

If the Altitude were 3^{d} 15′, in this caſe the diſtance between it and the Meridian Altitude being greater then the diſtance of the point of Entrance from the Center, the hour muſt be found from Midnight; add the Sine thereof to the Sine of 17^{d}, the Winter Meridian Altitude, the whole extent will be equal to the Sine of 20 25′; Enter the ſaid extent upon the point of Entrance, as before, and in the Verſed Sine of 90^{d}, the hour will be found to be either half an hour paſt 4 in the Morning, or half an hour paſt 7 in the Evening.

Examples for working the third Proportion.

Take the Sine of 30^{d}, and enter it upon the point of Entrance, laying the thred to the other foot, according to neareſt diſtance, and there keep it; then take the neareſt diſtance to it from the Sine of 57, the Meridian Altitude; and the ſaid Extent prick upon the Line of Verſed Sines on the left edge, and it will reach to 118^{d} 54′, ſet a mark to it. Laſtly, the neareſt diſtance from the Sine of each reſpective Altitude to the thred, being pricked from the ſaid mark, will reach to the Verſed Sine of the hour from Noon, for North Declinations.

So when the Sun hath 24^{d} 48′ of Altitude, the Hour from

Noon will be found to be——————75^{d} (7 : 17 above; 105 below)

A

A Winter Example for that Declination.

The nearest distance from the Sine of 17^{d}, the Winter Meridian Altitude, to the thred, will reach to the Versed Sine of $61^{d}\,6'$, the Complement of the former to a Semicircle, at which set a mark; then if the Altitude were ——— $\left\{\begin{matrix} 12 & 30' \\ 14: & 26 \end{matrix}\right\}$ the nearest distances to the thred prickt from the latter mark, would shew the hours to these Altitudes to be $\left\{\begin{matrix} 2 & \text{hours} \\ 1\frac{1}{2} & \text{hour} \end{matrix}\right\}$ from Noon

This last Proportion in some cases will be inconvenient, being liable to excursion in Latitudes more Northwardly.

Two sides with the Angle comprehended, to find the third side.

As the Radius,
Is to the Sine of one of the Includers,
So is the Sine of the other Includer,
To a fourth.

Again,

As the Radius,
Is to the Versed Sine of the Angle included,
So is that fourth,
To the difference of the Versed Sines of the third side, and of the Ark of difference between the two including sides,
And so is the Versed Sine of the Included Angles Complement to 180.
To the difference of the Versed Sines of the third side, and of the sum of the two including sides.

Another Proportion for finding it in Sines, elsewhere delivered.

By the former Proportion, having the advantage both of lesser and greater Versed Sines, we may find the side sought, either in the line of Sines, or in the line of Versed Sines on the the left edge, issuing from the Center.

The Converſe of the Proportion that found the Hour, will find the Suns Altitudes on all Hours.

As the Radius,
Is to the Coſine of the Latitude,
So is the Coſine of the Declination,
To a fourth Sine.

Namely, The difference of the Sines of the Suns Meridian Altitude, and of his Altitude at 6 in Summer, but the ſum of the Sines of his Depreſſion at 6, and Winter Meridian Altitude, hereby we may obtain the point of Entrance and Altitude, or Depreſſion at 6, as before, and let them be recorded, then it holds,

As the Radius,
Is to the Sine of the Hour from 6,
So is that fourth Sine,
To the difference of the Sines of the Suns Altitude at 6, and of his Altitude ſought; But in Winter, *To the ſum of the Sines of his Depreſſion at 6, and of the Altitude ſought.*

Hereby we may find two Altitudes at a time.

Lay the thred to the Hour in the Limb, and from the point of Entrance, take the neareſt diſtance to it, the ſaid Extent being ſet down at the Altitude at 6, ſhall reach upward to the greater Altitude, and downward, to the leſſer Altitude.

Example.

Admit the hour to be 5 and 7 in the morning, the Altitudes thereto for 20 North Declination for the Latitude of *Nottingham*, will be found to be $7^{d}\ 17'$, and $24^{d}\ 48'$

If the Hour be more remote from 6 then the time of Riſing, we may find a Winter Altitude to as much South Declination, and a Summer Altitude, to the ſaid North Declination.

Thus if the Hour be 45^{d} from 6, that is either 9 in the morning, or 3 in the afternoon, the neareſt diſtance from the point of Entrance to the thred, will reach from the Sine of $15^{d}\ 51'$, the Altitude

tude at 6 upwards, to the Sine of 42^{d} 18′, the Summer Altitude to that Declination: But downwards, it reaches beyond the Center: In this case measure, that extent from the Center, and take the distance between the inward foot of the Compasses, and the Altitude at 6, which measured on the Sines, will be found to be 7^{d} 17′ for the Winter Altitude to that Hour.

So if the hour were 60^{d} from 6, that is either 10, or 2, the Summer Altitude would be found to be 49^{d} 42′, and the Winter Altitude 12^{d} 30′.

And this may be found in the Versed Sines on the left edge, accounted as a Sine each way from the middle, if use be made of the lesser Sines, instead of the Limb, in finding the point of Entrance, as also, in laying it to the Sine of each hour from 6, in which case the Compasses will alwaies find two Altitudes at once; for when they fall beyond the midst of the said Line, it shews the Winter Altitudes counted from thence towards the end of the said Versed Sines.

Having found the fourth Sine, which gives the point of Entrance as before, the Altitudes on all hours may be found by the Versed Sines of 90^{d} in the Limb, the Proportion will be,

As the Radius,
Is to the Versed Sine of the Hour from Noon,
So is the fourth abovesaid,
To the difference of the Sines of the Meridian Altitude, and of the Altitude sought.

But for hours beyond 6, the Proportion will be,

As the Radius,
Is to the Versed Sine of the Hour from Midnight,
So is the fourth abovesaid,
To the sum of the Sines of the Suns Depression at Midnight (equal to his Winter Meridian Altitude,) and of his Altitude sought,

Hereby also we may find two Altitudes at once.

Operation.

Lay the thred to the Versed Sine of the Hour from Noon, and from the point of Entrance at 34^{d} 25′, take the nearest distance to

it, the ſaid Extent ſhall reach from the Summer Meridian Altitude, accounted in the Sines to the Altitude ſought, alſo from the Winter Meridian Altitude, to the Altitude ſought.

Example.

Latitude of *Nottingham* is 53^{d}, Complement - —— 37^{d}
Suns Declination,—— ·—— —— ——20

Sum being the Summer Meridian Altitude—·57^{d}
Difference being Winter Meridian Altitude 17

If it were required to find the Altitudes for the hours of

11		1	The Extents ſo	55 00′	And the Winter	15 51
10	and	2	taken out will find	49, 42	Altitudes to the	12,30
9		3	the Summer Alti-	42, 18	ſame hours and	7,17
8		4	tudes to be——	33, 47	Declination——	00, 32

But for hours more remote from the Meridian then 6, as admit for 5 in the morning, or 7 at night, which is 75^{d} from the North Meridian; lay the thred to the ſaid Ark in the Verſed Sine of 90^{d}, and the diſtance from the point of Entrance to it, ſhall reach from the Sine of 57^{d}, the Meridian Altitude, to the Sine of 24^{d} 48′, the Summer Altitude for the Hour 75^{d} from Noon, and if that Extent be pricked from the Winter Meridian Altitude, it will reach beyond the Center, in which caſe, enter that Extent upon the Line of Sines, and take the diſtance between the point of limitation and 17^{d}, which will (being meaſured) be found to be the Sine of 7^{d} 17′, the Altitude belonging to the hour 105^{d} from Noon.

In like manner, the Altitudes for the hours $\left.\begin{matrix}97^{d}\ 30\\112,30\end{matrix}\right\}$ from Noon

that is $\left\{\begin{matrix}82^{d}\ 30'\\67,30\end{matrix}\right\}$ from Midnight, will be $\left\{\begin{matrix}11^{d}\ 31'\\3:15\end{matrix}\right\}$ and for the like hours from Noon $\left\{\begin{matrix}20^{d}\ 17'\\29:19\end{matrix}\right.$

In

In like manner, it might have been found in the Versed Sines issuing from the Center, if in finding the point of Entrance, and in laying the thred to the Versed Sine of the Hour, we make use of the lesser Sines, and of the Versed Sine of 180^d in the Limb.

For the Azimuth.

Two of the former Proportions may be conveniently applied to other sides, for finding the Azimuth universally.

As the Radius,
Is to the Cosine of the Latitude,
So is the Cosine of the Altitude,
To a fourth Sine.

Get the sum of the Altitude and Colatitude; or which is all one, the sum of the Latitude and Colatitude; and if it exceeds a Quadrant, take its Complement to a Semicircle: This fourth Sine is equal to the difference of the Sines of this Compound Ark, and of another Ark to be thereby found, called the latter Ark.

Then it holds,

As the fourth Sine,
Is to the Radius,
So in Summer is the difference, but in Winter, the sum of the Sines of this latter Ark, and of the given Declination,
To the Sine of the Azimuth from the Vertical.

When the latter Ark is more then the Declination, the Azimuth will be found from the Vertical towards the Noon Meridian, otherwise towards the Midnight Meridian, and in winter, always towards the Noon Meridian.

For such Stars as come to the Meridian between the Zenith and the elevated Pole, the fourth Ark will never exceed the Stars declination, and their Azimuth will be alwaies found from the Vertical towards the Meridian they come to, above the Horizon.

Example.

Example for the Latitude of Nottingham.

Complement of the Latitude is ———————— 37^{d}
Altitude is 40^{d} ——————————————40

Sum ——————————77.

Let the Declination be 20^{d} North.

To find the point of Entrance, take the neareſt diſtance to the thred laid over 50 in the Limb, counted from right edge from the Sine of 37^{d}, the ſaid Extent meaſured from the Center, falls upon the Sine of 27^{d} 26′, and there will be the point of Entrance; the ſaid Extent prickt from 77^{d} in the Sines, will reach to the Sine of 30 51′, where the Sine point falls.

Laſtly, The diſtance between the Sine point, and the Sine of 20^{d} being entred at the point of Entrance, and the thred laid to the other foot, the Azimuth will be found in the equal Limb to be 21^{d} 48′ from the Eaſt or Weſt Southwards, becauſe the Sine point fell beyond the Declination.

Another Example for that Latitude, the Declination being 20^{d} *South Altitude.* 12^{d} 30′

The point of Entrance will fall at the Sine of 36^{d}

The Sine point may be found without ſumming or differencing of Arks, by taking the neareſt diſtance from the Sine of the Latitude, to the thred laid over the Altitude, counted in the Limb from the right edge; which Extent being added to the Sine of 20^{d} the Declination, the whole Extent will be equal to the Sine of 31^{d}, this being entred on the point of Entrance, and the thred laid to the other foot, the Azimuth will be found to be 61^{d} 14′ from the Eaſt or Weſt Southwards.

A third Example for the Latitude of London, 51 32′.

Let it be required to find the Azimuth of the middlemoſt Star in the great Bears tail, Declination is 56′ 45′, let the Altitude be 44′ 58′.

The neareſt diſtance from the Sine of 38′ 28′ to the thred laid over the Altitude counted from the right edge, will find the point of Entrance to be at the Sine of— —— ——————————26d 6′.

The neareſt diſtance from the Sine of 51d 32′ to the thred laid over the Altitude,counted from the right edge,need not be known, but the diſtance between that Extent, and the Sine of 56′ 45′, the Stars Declination being entred on the point of Entrance, will find the Azimuth of that Star, by laying the thred to the other foot, to be 40 from the Eaſt or Weſt Northwards.

Thus we find it the general way, and ſo it will alſo be found by the fitted particular Scale; for the Hour, the point of Entrance,and Sine point, vary not till the Declination change; but for the Azimuth, they vary to every Altitude.

To find the Azimuth in the Verſed Sines.

As the fourth, found by the former Proportion; namely,where the point of Entrance hapned,

Is to the Radius;

So is the difference of the Verſed Sines of the Polar diſtance, and of the Ark of Difference between the Altitude and the Latitude,

To the Verſed Sine of the Azimuth from Midnight Meridian.

This finds the Angle it ſelf in the Sphere.

And ſo is the difference of the Verſed Sines of the Polar diſtance, and of the Ark of reſidue of the ſum of the Latitude and Altitude taken from a Semicircle.

To the Verſed Sine of the Azimuth from Noon Meridian. This finds the Complement of the Angle in the Sphere to a Semicircle.

The

The Proportion to find it from Midnight Meridian, the third term being express'd in Sines, will be thus.

Get the ſum of the Altitude and Colatitude, and when it exceeds a Quadrant, take its Complement to a Semicircle, the Ark thus found, is called the Compound Ark. Then it holds,

As the fourth found before.
Is to the Radius,
So in Summer Declinations, *is the difference, but* in Winter Declinations, *the ſum of the Sines of the Suns or Stars declination, and of the compound Ark,*
To the Verſed Sine of the Azimuth from the Midnight Meridian of the place.

Uſe this Proportion alwaies for the Sun or Stars, when they come to the Meridian between the Zenith and elevated Pole.

And to find it from the Noon Meridian,

Get the difference between the Altitude and Colatitude, and then it holds,

As the fourth Sine found before,
Is to the Radius,
So is the ſum of the Sines of the ſaid Ark of Difference, and of the Suns Declination,
To the Verſed Sine of the Azimuth from the Noon Meridian, in Summer only, when the Suns Altitude is leſs then the Colatitude.

In all other caſes,

So is the difference of the ſaid Sines,
To the Verſed Sine of the Azimuth, as before, from Noon Meridian.

If by the former Proportion it be required to find the Azimuth in the Verſed Sine of 90^{d}, a difference of Verſed Sines taken out of the Line of Verſed Sines on the left edge muſt be doubled, and being taken out of the Line of Sines, as ſometimes repreſenting the former, ſometimes the latter half of a Verſed Sine, needs not be doubled.

Example.

Example:

Latitude of *Nottingham*- ——53°
Altitude of the Sun ———4

Ark of difference ———49
☉ Declination 20° North, the
Polar diſtance is————70

The Point of entrance will fall at the Sine of 36° 54′

And the difference of the Verſed Sines of 49° and 70°, equal to the diſtance between the Sines of 41° and 20° being entred at the Point of entrance, and the Thread laid to the other foot will lye over 61° 30′ of the Verſed Sine of 90°, and ſo much is the Suns Azimuth from the North.

Another Example for finding it from the South when the Altitude is more then the Colatitude.

Altitude———47°
Colatitude———37 of *Nottingham.*

difference———10

The Point of entrance will fall at the ſine of 24° 14′ found by taking the neareſt extent from ſine of 37° to the Thread lying over 43^{d} of the Limb the Coaltitude.

Then the diſtance between the ſines of 10°, the Ark of difference as above, and the ſine of 20° the Suns North Declination being entred at the Point of entrance, and the Thread laid to the other foot, will ſhew 53° 55′ in the Verſed ſine of 90° for the Suns Azimuth from the South.

A third Example when the Altitude is leſs then the Colatitude in Summer.

Complement Latitude 37° of *Nottingham.*
Altitude————34

difference————3

 The

The Point of entrance will fall at the fine of 29^{d} 55′, and the fum of the fines of 3^{d}, and of 20^{d} the Suns declination fuppofed North, is equal to the fine of 23^{d} 13′: Which Extent entred at 29^{d} 55′, the Point of entrance, and the Thread laid to the other foot according to neareft diftance, it will interfect the Verfed fine of 90^{d} at the Ark of 77^{d} 57′, and fo much is the Suns Azimuth from the South.

And if there were no Verfed fines in the Limbe, find an Ark of the equal Limbe, and enter the fine of the faid Ark down the Line of fines from the other end, and you may obtain the Verfed fine of the Ark fought.

More Examples need not be infifted upon, having found the Point of entrance, the diftance between the Verfed fines of the Bafe or fide fubtending the angle fought, and of the Ark of difference between the two including fides, being taken out of the ftreight Line of Verfed Lines on the left edge, and entred at the Point of entrance, laying the Thread to the other foot fhews in the Verfed Sine of 180^{d} in the Limb the angle fought; and if the faid diftance or Extent be doubled, and there entred it fhews the angle fought in the Verfed Sine of 90^{d}, when the Angle is lefs then a Quadrant, when more, the diftance between the Verfed Sines of the Bafe and the fum of the Legs, will find the Complement of the angle fought to a Semicircle without doubling in the Verfed Sine of 180^{d} in the Limb; with doubling in the Verfed Sine of 90^{d}.

Laftly, Three fides, *viz.* all lefs then Quadrants, or one of them greater, generally to find an angle in the equal Limb, the Proportion will be,

As the Radius,
Is to the Cofine of one of the including fides:
So is the Cofine of the other Includer,
To a fourth Sine.

Again,

As the Sine of one of the Includers,
To the Cofecant of the other:

So

So when any one of the sides is greater then a Quadrant is the sum but when all less, the difference of the fourth Sine, and of the Cosine of the third side,
To the Cosine of the angle sought.

If any of the three sides be greater then a Quadrant, it subtends an Obtuse angle, the other angles being Acute; But when they are all less then Quadrants, if the 4th Sine be less then the Cosine of the third side, the angle sought is Acute, if equal thereto, it is a right angle, if greater an Obtuse angle.

From the Proportion that finds the Hour from six, we may educe a single Proportion applyable to the Logarithms without natural Tables for Calculating the Hour of the day to all Altitudes, By turning the third Tearm, being a difference of Sines or Versed Sines into a Rectangle, and freeing it from affection.

The two first Proportions to be wrought are fixed for one Declination; The first will be to find the Suns Altitude or Depression at six.

The second will be to find half the difference of the Sines of the Suns Meridian Altitude, and Altitude sought, &c. as before defined, the Proportion to find it is,

As the Secant of 60d,
To the Cosine of the Declination:
So is the Cosine of the Latitude,
To the Sine of a fourth Arch.

Lastly, *To find the Hour.*

Get the sum and difference of half the Suns Zenith distance at the hour of six; and of half his Zenith distance to any other proposed Altitude or Depression.

Then, *As the Sine of the fourth Arch,*
Is to the Sine of the sum:
So is the Sine of the difference,
To the Sine of the hour from six towards Noon or Midnight, according as the Altitude or Depression was greater or lesser then the Altitude or Depression at six.

Obſerving that the Sine of an Arch greater then a Quadrant, is the Sine of that Arks Complement to a Semicircle.

Of the Stars placed upon the Quadrant below the Projection.

ALL the Stars placed upon the Projection are ſuch as fall between the Tropicks and the Hour may be found by them with the Projection, as in the Uſe of the ſmall Quadrant: Which may alſo be found by the fitted particular Scale, not only for Stars within the Tropicks, but for all others without, when their Altitude is leſs then 62$^{\text{d}}$, and likewiſe their Azimuth may be thereby found when their Declination is not more then 62$^{\text{d}}$.

For other Stars without the Tropicks, they may be put on below the Projection any where in ſuch an angle that the Thread laid over the Star ſhall ſhew an Ark in the Limb, at which in the Sines the Point of entrance will always fall; And again, the ſame Star is to be graved at its Altitude or Depreſſion at ſix in the Sines, and then to find the Stars hour in that Latitude whereto they are fitted, will always for Northern Stars be to take the diſtance in the Line of Sines between the Star and its given Altitude, and to enter that Extent at the Point of entrance, laying the Thread to the other foot according to neareſt diſtance, and it gives the Stars hour in the equal Limb from ſix, which may alſo be found in the Sines by a Parrallel entrance, laying the Thread over the Star.

Example.

Let the Altitude of the laſt in the end of the great Bears Tail be 63$^{\text{d}}$, take the diſtance between it and the Star which is graved at 37$^{\text{d}}$ 30′ of the Sines, the ſaid Extent entred at the Sine of 23$^{\text{d}}$, the Ark of the Limb the Thread interſects when it lies over the ſaid Star, and by laying the Thread to the other foot you will find that Stars hour to be 46$^{\text{d}}$ 11′ from ſix towards Noon Meridian, if the Altitude increaſe, and in finding the true time of the night, the Stars hour muſt be always reckoned from the Meridian it was laſt upon; in this Example it will be 5 minutes paſt *9 ferè.*

Of

Of the Quadrant of Ascensions on the backside,

This Quadrant is divided into 24 Hours with their quarters and ſubdiviſions, and ſerves to give the right Aſcenſion of a Star, as in the ſmall Quadrant to be caſt up by the Pen.

It alſo ſerves to find the true Hour of the night with Compaſſes.

Firſt having found the Stars hour, take the diſtance on the Quadrant of Aſcenſions in the ſame 12 hours between the Star and the Suns Aſcenſion (given by the foreſide of the Quadrant) the ſaid Extent ſhall reach from the Stars hour to the true hour of the night, and the foot of the Compaſſes always fall upon the Quadrant; Which Extent muſt be applyed the ſame way it was taken, the Suns foot to the Stars hour.

Example.

If upon the 30th of *December* the laſt in the end of the Bears Tail were found to be 9 hours 05′ paſt the Meridian it was laſt upon, the true time ſought would be 16 minutes paſt 3 in the morning.

Another Example for the Bulls Eye.

Admit the Altitude of that Star be 39*d*, that Stars hour as we found it by the Line of Verſed Sines was 3 ho 3′ from the Meridian, if the Altitude increaſe, then that Stars hour from the Meridian it was laſt upon was 57 minutes paſt 8 ———————— 8h : 57′

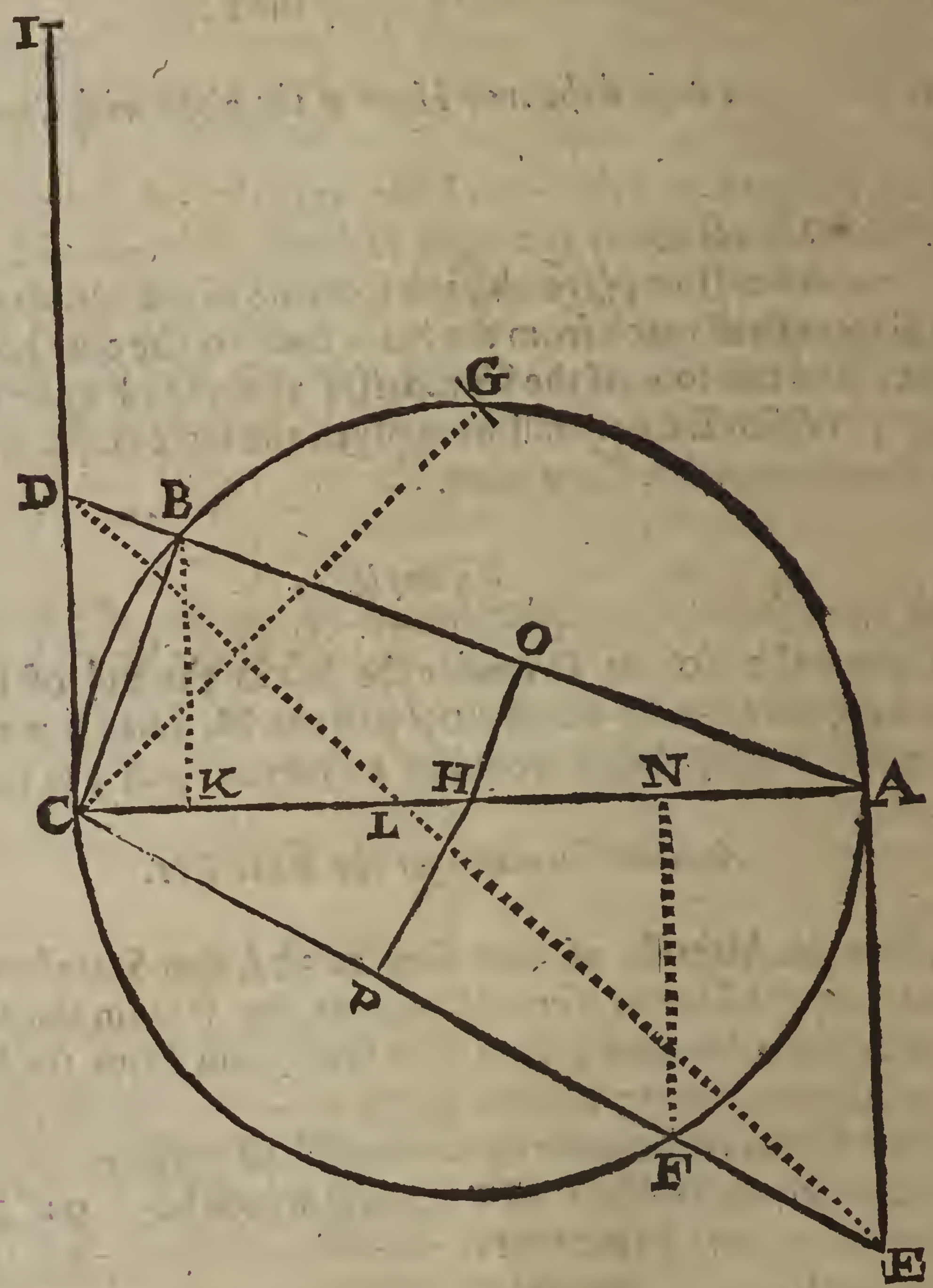
If this Obſervation were upon the 23d of *October*,
the Complement of the Suns Aſcenſion would be— 9 : 30
The Aſcenſion of that Star is ———————————4 : 16

The true time of the night would be forty——10 : 43
three minutes paſt ten.

The diſtance between the Star and the Suns Aſcenſion being applyed the ſame way, by ſetting the Sun foot at the Stars hour will ſhew the true time ſought.

When

When the Star is paſt the Meridian, having the ſame Altitude, the Stars hour will be 3′ paſt 3, and the true time ſought, will be 49′ paſt 4 in the next morning.



The

11 000 : 19 : 11

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The Geometrical Construction of Mr Fosters Circle.

1 0000

from

THe Circle on the Back side of the Quadrant, whereof one quarter is only a void Line, is derived from M. *Foster's* Treatise of a Quadrant, by him published in *An o* 1638. the foundation and use whereof being concealed, I shall therefore endeavour to explain it.

from

Upon the Center H describe a circle, and draw the Diameter A C, passing through the Center, and perpendicularly thereto, upon the point C, erect a Line of Sines C I, whose Radius shall be equal to the Diameter A C, let 90d of the Sine end at I; I say then, if from the point A, through each degree of that Line of Sines, there be streight lines drawn, intersecting the Quadrant of the circle C G, as a line from the point D doth intersect it at B the Quadrant C G, which the Author calls the upper-Quadrant, or Quadrant of Latitudes shall be constituted, and if C I be continued as a Secant, by the same reason the whole Semicircle C G A may be occupied; hence it will be necessary to educe a ground of calculation for the accurate dividing of the said Quadrant, and that will be easie; for A C being Radius, the Sine C D doth also represent the Tangent of the Angle at A, therefore seek the natural Sine of the Ark C D in the Table of Natural Tangents, and the Ark corresponding thereto, will give the quantity of the Angle D A C, then because the point A falls in the circumference of the Circle, where an Angle is but half so much as it is at the Center, by 31 Prop. 3. *Euc.* double the Angle found, and from a Quadrant divided into 90 equal parts, and their subdivisions, by help of a Table so made, may the Quadrant of Latitudes be accurately divided: but the Author made his Table in page 5. without doubling, to be graduated from a Quadrant divided into 45 equal parts,

Again, If upon the Center C, with a pair of Compasses, each degree of the line of Sines be transferred into the Semicircle C G A it shall divide it into 90 equal parts; the reason whereof is plain, because

because the Sine of an Arch is half the chord of twice that Arch, and therefore the Sines being made to twice the Radius of this circle, ſhall being transferred into it,become chords of the like Arch, to divide a Semicircle into 90 equal parts.

Again, upon the point A, erect a line of Tangents of the ſame Radius with the former Sine,which we may ſuppoſe to be infinitely continued, here we uſe a portion of it A E.

If from the point C, the other extremity of the Diameter lines be drawn, cutting the lower Semicircle (as a line drawn from E interſects it at F) through each degree of the ſaid Tangent, the ſaid lower Semicircle ſhall be divided into 90 equal parts; the reaſon is evident a line of Tangents from the Center ſhall divide a Quadrant into 90 equal parts, and becauſe an Angle in the circumference is but half ſo much as it is in the Center, being transferred thither, a whole Semicircle ſhall be filled with no more parts.

The chief uſe of this Circle, is to operate Proportions in Tangents alone, or in Sines and Tangents joyntly, built upon this foundation, that equiangled plain Triangles have their ſides Proportional.

In ſtreight lines, it will be evident from the point D to E, draw a ſtreight line interſecting the Diameter at L, and then it lies as C L to C D; ſo is A L to A E: it is alſo true in a Circle, provided it be evinced, that the points B L F fall in a ſtreight line.

Hereof I have a Geometrical Demonſtration, which would require more Schemes, which by reaſon of its length and difficulty, I thought fit at preſent not to inſert, poſſibly an eaſier may be found hereafter: As alſo, an Algebraick Demonſtration, by the Right Honourable, the Lord *Brunkard*, whereby after many Algebraick inferences it is euinced, that as L K is to K B :: ſo is L N to L F: whence it will follow, that the points B, L, F, are in a right line.

If a Ruler be laid from 45^{d} of the Semicircle, to every degree of the Quadrant of Latitudes, it will conſtitute upon the Diameter, the graduations of the Line *Sol*, whereby Proportions in Sines might be operated without the other ſupply.

From

From the same Scheme also follows the construction of the streight line of Latitudes, from the point G, at 90^{d} of the Quadrant of Latitudes, draw a streight Line to C, and tranfer each degree of the Quadrant of Latitudes with Compasses, one foot resting upon C into the said streight line, and it shall be constituted.

To Calculate it.

The Line of Latitudes C G bears such Proportion to C A as the Chord of 90^{d} doth to the Diameter, which is the same that the Sine of 45^{d} bears to the Radius; or which is all one that the Radius bears to the Secant of 45*d*, which Secant is equal to the Chord of 90*d*; from the Diagram the nature of the Line of Latitudes may be discovered.

Any two Lines being drawn to make a right angle, if any Ark of the Line of Latitudes be pricked off in one of those Lines retaining a constant Hipotenusal A C, called the Line of Hours, equal to the Diameter of that Circle from whence the Line of Latitudes is constituted, if the said Hipotenusal from the Point formerly pricked off, be made the Hipotenusal to the Legs of the right angle formerly pricked off, the said Legs or sides including the right angle shall bear such Proportion one to another, as the Radius doth to the sine of the Ark so prickt off; and this is evident from the Schem for such Proportion as A C bears to C D, doth A B bear to B C, for the angle at A is Common to both Triangles, and the angle at B in the circumference is a right angle, and consequently the angle A C B will be equal to the angle A D C, and the Legs A C to C D bears such Proportion by construction, as the Radius doth to the Sine of an Ark, and the same Proportion doth A B bear to B C, in all ases retaining one and the same Hypotenuial A C, the Proportion therefore lies evident.

As the Radius, the sine of the angle at B,
To its opposite side A C, the Secant of 45*d*:
So is the sine of the angle at A,
To its opposite side B C *sought.* Now the quantity of the angle at A was found by seeking the natural Sine of the Ark proposed in

the Table of natural Tangents; and having found what Ark anſwers thereto, the Sine of the ſaid Ark is to become the third Tearm in the Proportion.

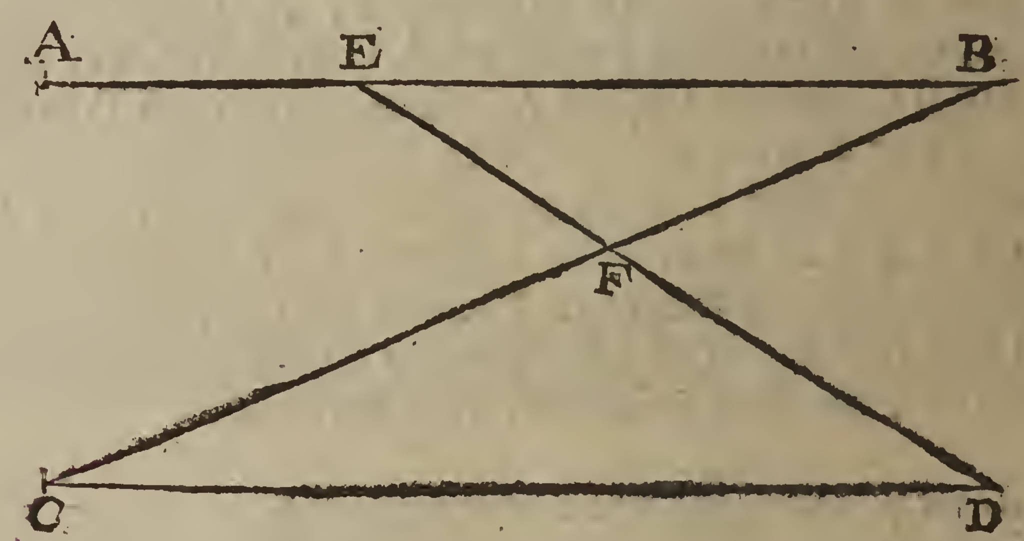
But the Cannon preſcribed in the Deſcription of the ſmall Quadrant is more expedite then this, which Mr *Sutton* had from Mr *Dary* long ſince, for whom, and by whoſe directions he made a Quadrant with the Line *Sol*, and two Parrallel Lines of Sines upon it, as is here added to the backſide of this Quadrant.

Of the Line of Hours, alias, *the Diameter or Proportional Tangent.*

This Scale is no other then two Lines of natural Tangents to 45*d*, each ſet together at the Center, and from thence beginning and continued to each end of the Diameter, and from one end thereof numbred with 90*d* to the other end.

This Line may fitly be called a Proportional Tangent, for wherſoever any Ark is aſſumed in it to be a Tangent, the remaining part of the Diameter is the Radius to the ſaid Tangent.

So in the former Schem, if C L be the Tangent of any Ark, the Radius thereto ſhall be A L.



In the Schem annexed, let A B be the Radius of a Line of Tangents equal to C D, and alſo parralel thereto, and from the Point

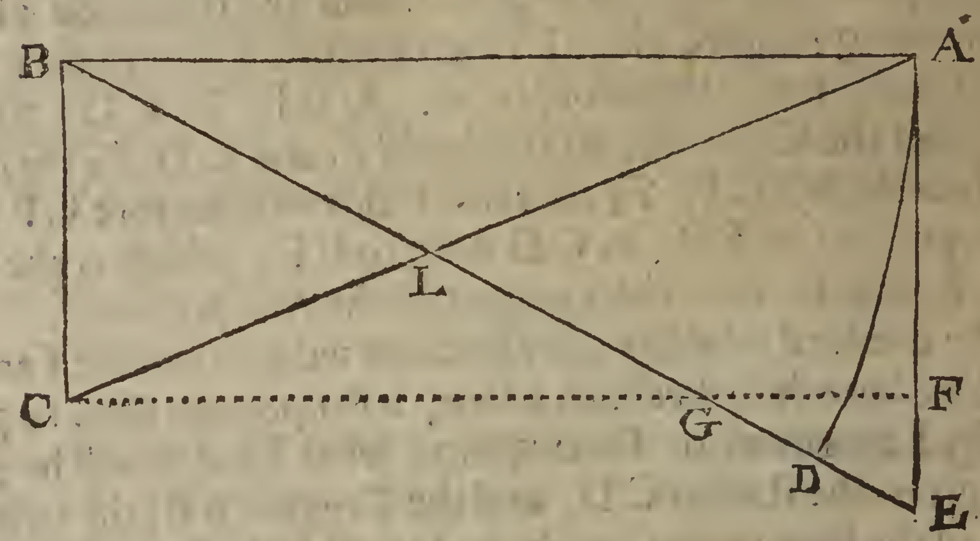
Point B to C draw the Line B C, and let it be required to divide the ſame into a Line of Proportional Tangents: I ſay, Lines drawn from the Point D to every degree of the Tangent, A B ſhall divide one half of it as required from the ſimilitude of two right angled equiangled plain Triangles, which will have their ſides Proportional, it will therefore hold, *As* C F, *To* C D: *So* F B, *To* B E, and the Converſe, *As the ſecond Tearm* C D, *To the fourth* B E: *So is the firſt* C F, *To the third* F B, and therefore C F bears ſuch Proportion to F B, as C D doth to B E, which is the ſame that the Radius bears to the Tangent of the Ark propoſed.

If it be doubted whether the Diameter wil be a double Tangent or the Line here deſcribed ſuch a Line, a Proportion ſhall be given to find by Experience or Calculation, what Line it will be; for there is given the Radius C D, and the Tangent B E, the two firſt Tearms of the Proportion, with the Line C B the ſum of the third and fourth Tearms, to find out the ſaid Tearms reſpectively; and it will hold by compounding the Proportion,

As the ſum of the firſt and ſecond Tearm,
Is to the ſecond Tearm:
So is the ſum of the third and fourth Tearm,
To the fourth Tearm, that is,
As C D + B E, *Is to* B E:
So is C F + F B = C B, *To* F B, ſee 18 *Prop.* of 5 of *Euclid*, or page 18 of the Engliſh *Clavis Mathematicæ*, of the famous and learned Mr *Oughtred.*

After the ſame manner is the Line *Sol*, or Proportional Sines made, that being alſo ſuch a Line, that any Ark being aſſumed in it to be a Sine, the diſtance from that Ark to the other end of the Diameter, ſhall be the Radius thereto.

A Demonstration to prove that the Line of Hours and Latitudes Will jointly prick off the hour Distances in the same angles as if they Were Calculated and prickt of by Chords.



Draw the two Lines A B and C B crosing one another at right angles at B, and prick off B C the quantity of any Ark out of the line of Latitudes, and then fit in the Scale of Hours; so that one end of it meeting with the Point C, the other may meet with the other Leg of the right angle at A, from whence draw A E parralel to B C; So A B being become Radius, B C is the Sine of the Arch first prickt down from the line of Latitudes; from the Point B through any Point in the line of Proportional Tangents, at L draw the Line B L E, and upon B with the Radius B A draw the Arch A D, which measureth the Angle A B E to the same Radius: I say, there will then be a Proportion wrought, and the said Arch measureth the quantity of the fourth Proportional, the Proportion will be,

As the Radius,

To the Sine of the Ark prickt down from the Line of Latitudes:

So is any Tangent accounted in the Scale, beginning at A,

To the Tangent of the fourth Proportional; in the Schem it lies evident in the two opposite Triangles L C B and L A E, by construction equiangled and consequently their sides Proportional.

Assuming A L to be the Tangent of any Ark, L C becomes the Radius, according to the prescribed construction of that Line, it then lies evident,

As

As L C the Radius,
To C B the Sine of any Ark,
So is L A, the Tangent of any Ark,
To A E, the Tangent of the fourth Proportional.

Namely, of the Angle A B E, and therefore it pricks down the Hour-lines of a Dyal most readily and accurately: the Proportion in pricking from the Substile being alwaies,

As the Radius,
To the Sine of the Stiles height,
So the Tangent of the Angle at the Pole,
To the Tangent of the Hour-line from the Substile.

Uses of the Graduated Circle.

To work Proportions in Tangents alone.

In any Proportion wherein the Radius is not ingredient, it is supposed to be introduced by a double Operation, and the Poportion will be,

As the first term, To the second,
So the Radius to a fourth.

Again,

As the Radius is to that fourth,
So is the third Term given,
To the fourth Proportional sought.

In illustrating the matter, I shall make use of that Theoreme for varying of Proportions, that the Tangents of Arches, and the Tangents of their Complements are in reciprocal Proportions.

As Tangent 23^d, to Tangent 35^d, So Tangent 55^d to the Tangent of 67^d.

In working of this Proportion, the last term may be found on the equal Semicircle, or on the Diameter.

1. *In the Semicircle.*

Extend the thred through 23^d on the Diameter, and through 35^d in the Semicircle, and where it intersects the Circle on the opposite

ſite ſide, there hold one end of it, then extend the other part of it over 55 in the Diameter, and in the Semicircle, it will interſect 67^d for the term ſought.

2. On the Diameter.

Extend the thred over 23^d in the Semicircle, and 35^d on the Diameter, and where it interſects the void circular line on the oppoſite ſide, there hold it, then laying the other end of it over 55*d* in the Semicircle, and it will cut 67*d* on the Diameter.

If the Radius had been one of the terms in the Proportion, the operation would have been the ſame, if the Tangent of 45*d* had been taken in ſtead of it.

To work Proportions in Sines and Tangents joyntly.

1. If a Sine be ſought, the middle terms being of a different ſpecies.

Extend the thred through the firſt term on the Diameter, being a Tangent, and through the Sine, being one of the middle terms, counted in the unequal Quadrant, and where it interſects the Oppoſite ſide of the Circle hold it, then extend the thred over the Tangent, being the other middle term counted on the Diameter, and it will interſect the graduated Quadrant at the Sine ſought.

Example.

If the Proportion were as the Tangent of 14^d to the Sine of 29^1 So is the Tangent of 20^1 to a Sine, the fourth Proportional would be found to be the Sine of 45^1.

2. If a Tangent be ſought, the middle terms being of ſeveral kinds, Extend the thred through the Sine in the upper Quadrant, being the firſt term, and through the Tangent on the Diameter, being one of the other middle terms, holding it at the Interſection of the Circle on the oppoſite ſide, then lay the thred to the other middle term in the upper Quadrant, and on the Diameter, it ſhews the Tangent ſought.

Example.

Example.

If the Suns Amplitude and Vertical Altitude were given, the Proportion from the *Analemma* to find the Latitude would be,

As the Sine of the Amplitude to Radius,
So is the Sine of the Vertical Altitude,
To the Cotangent of the Latitude

Let the Amplitude be————————39d 54′
And the Suns Altitude being East or West——30d 39′

Extend the thred through 39d 54′, the Amplitude counted in the upper Quadrant, and through 45d on the Diameter, holding it at the intersection with the Circle on the Opposite side, then lay the thred over 30d 39′, the Vertical Altitude, and it will intersect the Diameter at 38d 28′, the Complement of the Latitude sought.

But Proportions derived from the 16 cases of right angled Spherical Triangles, having the Radius ingredient, will be wrought without any motion of the thred.

An Example for finding the Suns Azimuth at the Hour of 6.

As the Radius to the Cosine of the Latitude,
So the Tangent of the Declination,
To the Tangent of the Azimuth, from the Vertical towards Midnight Meridian.

Extend the thred over the Complement of the Latitude in the upper Quadrant, and over the Declination in the Semicircle, and on the Diameter. it shews the Azimuth sought.

So when the Sun hath 15d of Declination, his Azimuth shall be 9d28′ from the Vertical at the hour of 6 in our Latitude of *London.*

Another

Another Example to find the time when the Sun will be due East or West.

Extend the thred over the Latitude in the Semicircle, and over the Declination on the Diameter, and in the Quadrant of Latitudes it ſhews the Ark ſought.

The Proportion wrought, is,

As the Radius to the Cotangent of the Latitude,
So is the Tangent of the Declination,
To the Sine of the Hour from 6.

Example.

So when the Sun hath 15^{d} of North Declination, in our Latitude of *London*, the Hour will be found 12^{d} 18′ from 6 in time 49 $\frac{1}{5}$′ paſt 6 in the morning, or before it in the afternoon.

Another Example to find the Time of Sun riſing.

As the Cotangent of the Latitude, to Radius,
So is the Tangent of the Declination,
To the Sine of the Hour from 6 before or after it.

Lay the thred to the Complement of the Latitude in the Semicircle, and over the Declination on the Diameter, and in the Quadrant of Latitudes, it ſhews the time ſought in degrees, to be converted into common time, by allowing 15^{d} to an hour, and 4′ to a degree.

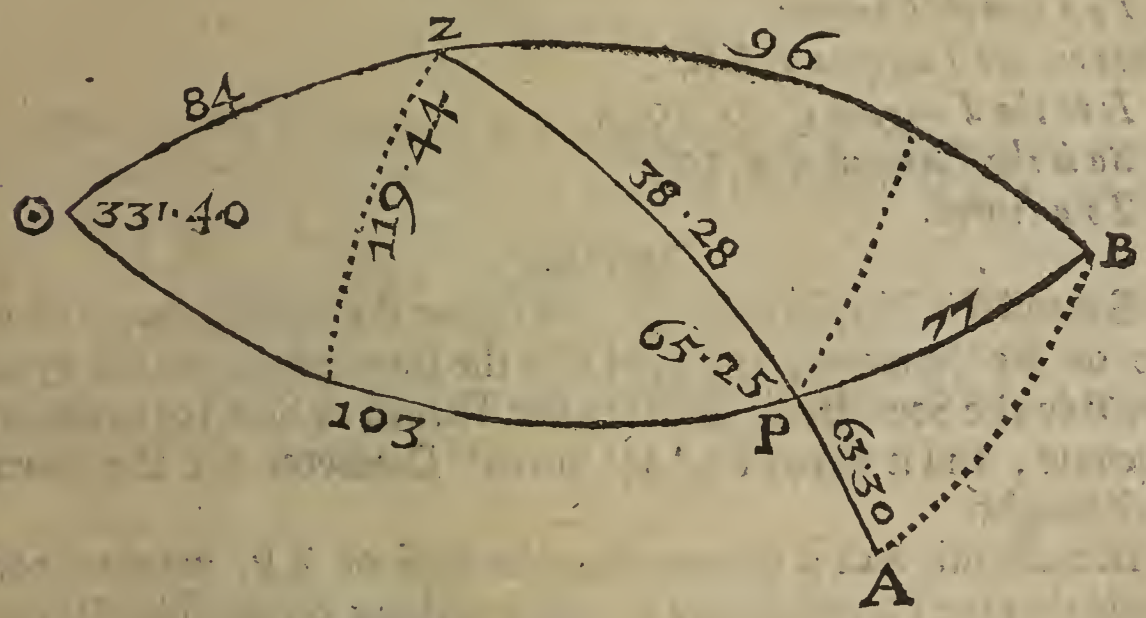
So in the Latitude of *London*, 51^{d} 32′ when the Sun hath 15^{d} of Declination, the aſcenſional difference or time of riſing from 6, will be 19^{d} 42′, to be converted into common time, as before.

By what hath been ſaid. it appears, that the Hour and Azimuth may be found generally by help of this Circle and Diameter.

For the performance whereof, we muſt have recourſe to the Proportions delivered iu page 123. whereby we may alwaies find the two Angle adjacent to the ſide on which the Perpendicular fal-

let,

ſeth, which may be any ſide at pleaſure; for after the firſt Proportion wholly in Tangents is wrought, to find either of thoſe Angles, will be agreeable to the ſecond caſe of right angled Spherical Triangles, wherein there will be given the Hypotenuſal, and one of the Legs, to find the adjacent Angle, only it muſt be ſuggeſted, that when the two ſides that ſubtend the Angle ſought, are together greater then a Semicircle, recourſe muſt be had to the Oppoſite Triangle, if both thoſe Angles are required to be found by this Trigonometry, otherwiſe one of them, and the third Angle may be found by thoſe directions, by letting fall the perpendicular on another ſide, provided the ſum of the ſides ſubtending thoſe Angles be not alſo greater then a Semicircle; or, having firſt found one Angle, the reſt may be found by Proportions in Sines only.



IN the Triangle ☉ Z P, if it were required to find the angles at Z and ☉, becauſe the ſum of the ſides ☉ P and Z P are leſs then a Semicircle they might be both found by making the half of the Baſe ☉ Z the firſt Tearm in the Proportion, and then becauſe the angles ☉ Z are of a different affection, the Perpendicular would fal without on the ſide ☉ Z continued towards B, as would be evinced

by

by the Proportion, for the fourth Ark discovered, would be found greater then the half of ⊙ Z; hence we derived the Cannon in *page* 124, for finding the Azimuth; Whereby might also be found the angle of Position at ⊙; so if it were required to find the angles at ⊙ and P, the sides ⊙ Z and Z P being less then a Semicircle the Perpendicular would fall within from Z on the side ⊙ P, as would also be discovered by the Proportion, for the fourth Ark would be found less then the half of ⊙ P.

But if it were required to find both the angles at Z and P, in this Case we must resolve the Opposite Triangle Z B P, because the sum of the sides ⊙ Z and ⊙ P are together greater then a Semicircle, and this being the most difficult Case, we shall make our present Example. The Proportion will be,

As the Tangent of half Z P,
Is to the Tangent of the half sum of Z B *and* P B:
So is the Tangent of half their difference,
To a fourth Tangent.

That is, *As Tangent* 19d 14′,
Is to the Tangent of 86d 30′,:
So is the Tangent of 9d 30′,
To a fourth.

Operation.

Extend the Thread through 19*d* 14′ on the Semicircle, and 9*d* 30′ on the Diameter, and hold it at the Intersection on the opposite side the Semicircle, then lay the Thread to 86*d* 30′ in the Semicircle, and it shews 82*d* 44′ on the Diameter for the fourth Ark sought.

Because this Ark is greater then the half of Z P, we may conclude that the Perpendicular B A falls without on the side Z P continued to A.

fourth Ark	82d	44′	
half of Z P is	19	14	
sum	101	58	is Z A
difference	63	30	is P A

Then in the right angled Triangle B P A, right angled at, A we have P A and B P the Hypotenusal, to find the angle B P A, equal to the angle ⊙ Z P. The Proportion is

As

As the Radius,
Is to the Tangent of 13*d*, *the Complement of* BP:
So is the Tangent of P A 63*d* 30′,
To the Cosine of the angle at P.

Extend the Thread through 13*d* on the Diameter, and through 63*d* 30′ in the Semicircle counted from the other end, and in the upper Quadrant, it shews 27*d* 35′ for the Complement of the angle sought.

And letting this Example be to find the Hour and Azimuth in our Latitude of *London*, so much is the hour from six in Winter when the Sun hath 13*d* of South Declination, and 6*d* of Altitude, in time 1 ho 50 $\frac{1}{3}$ minutes past six in the morning, or as much before it in the afternoon.

To find the Azimuth.

Again, in the Triangle Z A B right angled at A, there is given the Leg or Side Z A 101*d* 58′, and the Hipotenusal Z B 96*d*, to find the angle B Z P; here noting that the Cosine or Cotangent of an Ark greater then a Quadrant is the Sine or Tangent of that Arks excess above 90*d*, and the Sine or Tangent of an Ark greater then a Quadrant, the Sine or Tangent of that Arks Complement to 180*d*, it will hold,

As the Radius,
To the Tangent of 6*d*:
So is the Tangent 78*d* 2′,
To the Sine of 29*d* 44′, found by extending the Thread through 78*d* 2′ on the Semicircle, counted from the other end, *alias*, in the small figures, and in the Quadaant it will intersect 29*d* 44′; now by the second Case of right angled Sphœrical Triangles, the angle A Z B will be Acute, wherefore the angle ☉ Z B is 119*d* 44′ the Suns Azimuth from the North, the Complement being 60*d* 16′ is the angle A Z B, and so much is the Azimuth from the South.

To work Proportions in Sines alone.

THat this Circle might be capacitated to try any Case of Sphærical Triangles, there are added Lines to it, namely, the Line *Sol* falling perpendicularly on the Diameter from the end of the Quadrant of Latitudes, whereto belongs the two Parrallel Lines of Sines in the opposite Quadrants, the upermost being extended cross the Quadrant of Latitudes.

The Proportion not having the Radius ingredient, and being of the greater to the less.

Account the first Tearm in the line *Sol*, and the second in the upper Sine extending the Thread through them, and where it intersects the opposite Parrallel hold it; then lay the Thread to the third Tearm in the line *Sol*, and it will intersect the fourth Proportional on the upper Parrallel.

As the Sine of 30^{d},
To the sine of any Arch:
So is the Cosine of that Arch,
To the sine of the double Arch and the Converse.

By trying this Canon, the use of these Lines will be suddenly attained.

Example.

As the sine of 30^{d},
To the sine of 20^{d}:
So is the sine of 70^{d},
To the sine of 40^{d}.

But if it be of the less to the greater, the answer must be found on the Line *Sol*.

Account the first Tearm on the upper Sine, and the second in the Line *Sol*, and hold the Thread at the Intersection of the opposite Parrallel, then lay the Thread to the third Tearm on the uper Parrallel, and on the line *Sol* it will intersect the fourth Proportional if it be less then the Radius.

But

But Proportions having the Radius ingredient, will be wrought without any Motion of the Thread.

As the Cosine of the Latitude,
To Radius:
So is the sine of the Declination,
To the sine of the Amplitude.

So in our Latitude of *London*, when the Declination is 20 12′ the Amplitude will be found to be 33 42′.

Extend the Thread through 38*d* 28′ on the line *Sol.*and through the Declination in the upper Sine, and it will intersect the opposite Parrallel Sine at 33*d* 42′, the Amplitude sought.

The use o the Semi-Tangent and Chords are passed by at present.

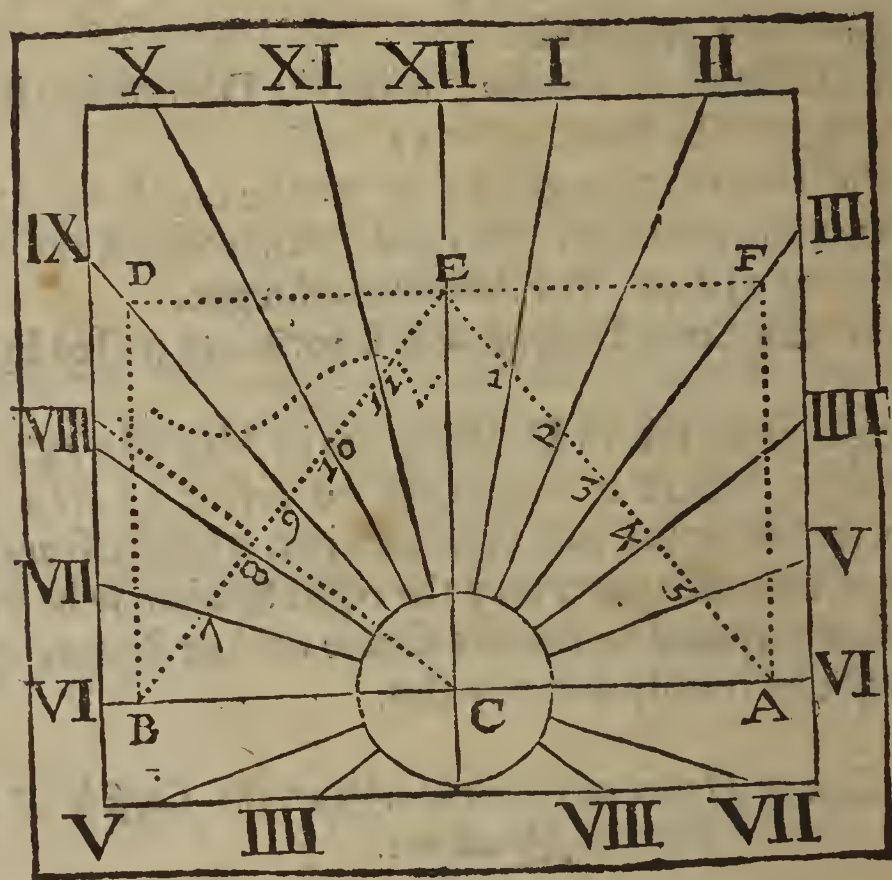
The line *Sol* is of use in Dyalling, as in Mr *Fosters Posthuma*, page 70 and 71, where it is required to divide a Circle into 12 equal parts for the hours, and each part into 4 subdivisions for the quarters, and into such parts may the equal Semicircle be divided; that if it were required to divide a Circle of like Radius into such parts, it might be readily done by this.

Of the Line of Hours on the right edge of the foreside of the Quadrant.

This is the very same Scale that is in the Diameter on the Backside, only there it was divided into degrees, and here into time, and placed on the outermost edge; there needs no line of Latitudes be fitted thereto, for those Extents may be taken off as Chords from the Quadrant of Latitudes; by help of these Scales thus placed on the outward edges of the Quadrants may the hour-lines of Dyals be prickt down without Compasses.

To

To Draw a Horizontal Dyal.



FIrst draw the line C E, for the Hour-line of 12, and cross it with the Perpendicular A B, then out of a Scale or Quadrant of Latitudes set of C B and C A, each equal to the Stiles height, or Latitude of the place, then place the Scale of 6 hours on the edge of the Quadrant, whereto the Line of Latitudes was fitted, one extremity of it at A, and move the Quadrant about, till the other end or extremity of it will meet with the Meridian line C E; then in regard the said Scale of Hours stands on the very brink or outwardmost edge of the Quadrant, with a Pin, Pen, or the end of a black-lead pen, make marks or points upon the Paper or Dyal against each hour (and the like for the quarters, and other lesser parts)

parts) of the graduated Scale, and from thoſe marks draw lines into the Center, and they ſhall be the hour-lines required, without drawing any other lines on the Plain, the Scale of Hours on the Quadrant is here repreſented by the lines A E, and E B, the hour lines above the Center, are drawn by continuing them out through the Center.

And thoſe that have Paper prints of this line, may make them ſerve for this purpoſe, without pricking down the hour points by Compaſſes, by doubling the paper at the very edge or extremity of the Scale of Hours.

Otherwiſe to prick down the ſaid Dial without the Line of Latitudes and Scale of hours in a right angled Parallellogram.

Having drawn C E the Meridian line, and croſſed it with the perpendicular C A B, and determining C E to be the Radius of any length, take out the Sine of the Latitude to the ſame Radius, and prick it from C to A and B, and ſetting one foot at E, with the ſaid Extent ſweep the touch of an Arch at D and F; then take the length of the Radius C E, and ſetting down one foot at B, ſweep the touch of an Ark at D, interſecting the former, alſo ſetting down the Compaſſes at A, make the like Arch at F, and through the points of Interſection, draw the ſtreight lines A F, B D, and F E D, and they will make a right angled Parallellogram, the ſides whereof will be Tangent lines.

To draw the Hour lines:

Make E F, or E D Radius, and proportion out the Tangents of $\left.\begin{array}{l}15^{d}\\30\end{array}\right\}$ and prick them down from E to $\left\{\begin{array}{l}1\\2\end{array}\right.$ and $\left.\begin{array}{l}11\\10\end{array}\right\}$ and draw lines through the points thus found, and through the points F and D, and there will be 3 hours drawn on each ſide the Meridian line.

Again, make A F or B D Radius, and proportion out the Tangent of 15^{d}, and prick it down from A to 5, and from B to 7.

Alſo proportion out the Tangent of 30^{d}, and prick it down from A to 4, and from B to 8, and draw lines into the Center, and ſo the Hour-lines are finiſhed, and for thoſe that fall above the 6 of clock

line,

line, they are only the opposite hours continued, after the like manner are the halfs and quarters to be prickt down.

Laftly, By chords prick off the Stiles height equal to the Latitude of the place, and let it be placed to its due elevation over the Meridian line.

Of Upright Decliners.

DIvers Arks for fuch plains are to be calculated, and may be found on the Circle before defcribed.

1. *The Subftiles diftance from the Meridian.*

By the Subftilar line is meant, a line over which the Stile or cock of the Dyal directly hangeth in its neareft diftance from the Plain, by fome termed the line of deflexion, and is the Ark of the plain between the Meridian of the Plain, and the Meridian of the place. The diftance thereof from the Hour line of 12, is to be found by this Proportion.

As the Radius,
To the Sine of the Plains Declination,
So the Cotangent of the Latitude,
To the Tangent of the Subftile from the Meridian.

2. *For the Angle of* 12 *and* 6.

An Ark ufed when the Hour-lines are pricked down from the Meridian line in a Triangle or Parallellogram, (and not from the Subftile,) without collecting Angles at the Pole.

As the Radius,
Is to the Sine of the Plains Declination,
So is the Tangent of the Latitude,
To the Tangent of an Ark, the Complement whereof is the Angle of 12 *and* 6.

3. *Incli-*

3. *Inclination of Meridians,*

Is an Ark of the Equinoctial, between the Meridian of the plain, and the Meridian of the place, or it is an Angle or ſpace of time elapſed between the paſſage of the ſhaddow of the Stile from the Subſtilar line into the Meridian line, by ſome termed the Plains difference of Longitude; and not improperly, for it ſhews in what Longitude from the Meridian where the Plain is; the ſaid Plain would become a Horizontal Dyal, and the Stiles height ſhews the Latitude, this Ark is uſed in calculating hour diſtances by the Tables and in pricking down Dyals by the Line of Latitudes, and hours from the Subſtile.

As the Radius,
Is to the Sine of the Latitude,
So the Cotangent of the Plains Declination,
To the Cotangent of the Inclination of Meridians.

Or,

As the Sine of the Latitude to Radius,
So is the Tangent of the Plains Declination,
To the Tangent of Inclination of Meridians.

4. *The Stiles height above the Subſtile.*

As the Radius,
Is to the Coſine of the Latitude,
So is the Coſine of the Plains Declination,
To the Sine of the Stiles height.

Or the Subſtiles diſtance being known,

As the Radius,
To the Sine of the Subſtiles diſtance from the Meridian,
So is the Cotangent of the Declination,
To the Tangent of the Stiles height.

 Or,

Or, The Inclination of Meridians being known.

As the Radius,
To the Coſine of the Inclination of Meridians,
So is the Cotangent of the Latitude,
To the Tangent of the Stile's height:

5. Laſtly, For the diſtances of the Hour-lines from the Subſtilar Line.

As the Radius,
Is to the Sine of the Stiles height above the Plain;
So is the Tangent of the Angle at the Pole,
To the Tangent of the Hours diſtance from the Subſtilar Line.

By the Angle at the Pole, is meant the Ark of difference between the Ark called the Inclination of Meridians, and the diſtance of any hour from the Meridian, for all hours on the ſame ſide the Subſtile falls, and the ſum of theſe two Arks for all hours on the other ſide the Subſtile.

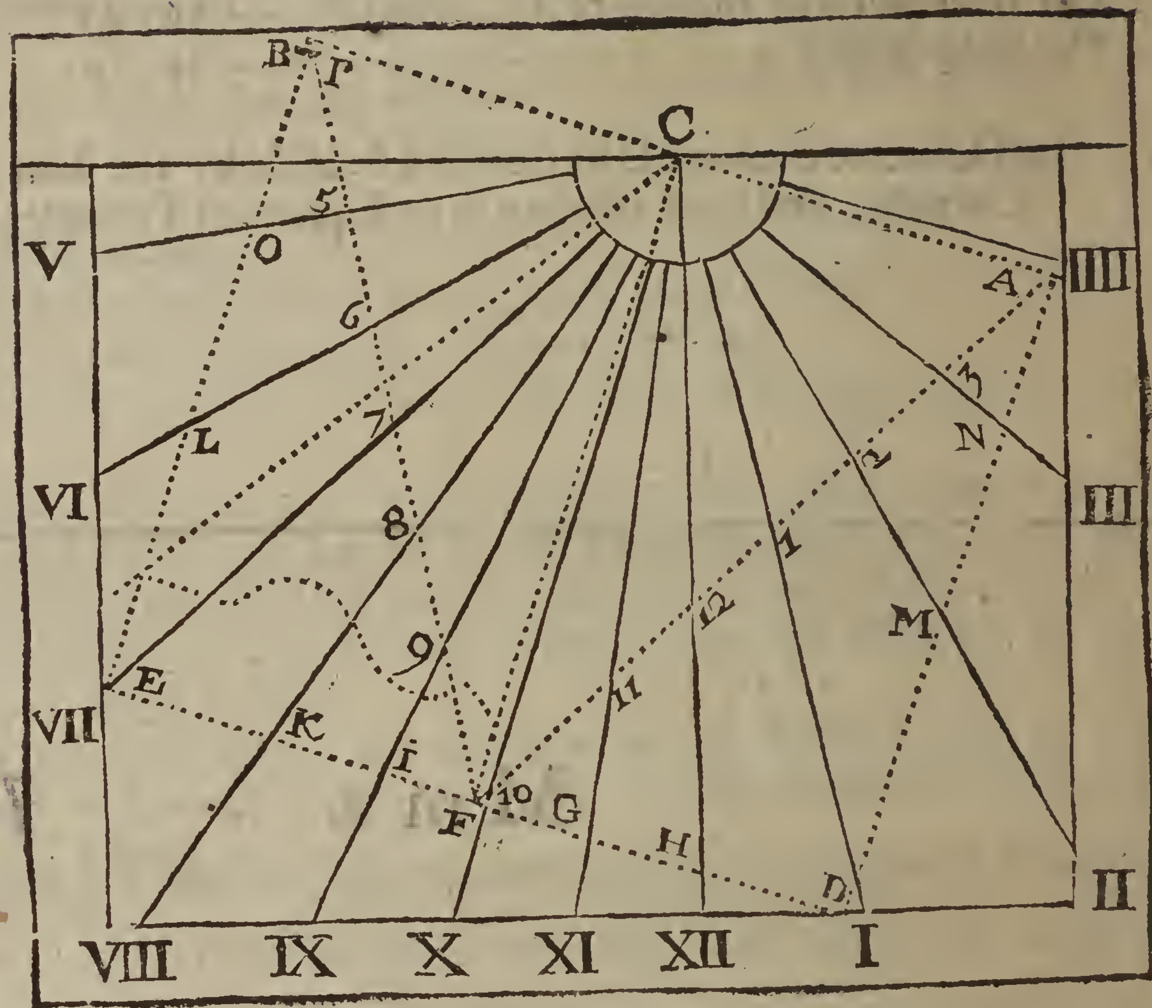
Theſe Proportions are ſufficient for all Plains to find the like Arks, without having any more, if the manner of referring Declining $\left\{\begin{matrix}\text{Re}\\\text{In}\end{matrix}\right.$clining Plains to a new Latitude, and a new Declination in which they ſhall ſtand as upright Plains, be but well explained, for Eaſt or Weſt $\left\{\begin{matrix}\text{Re}\\\text{In}\end{matrix}\right.$clining Plains, their new Latitude is the Complement of their old Latitude, and their new Declination, is the Complement of their $\left\{\begin{matrix}\text{Re}\\\text{In}\end{matrix}\right.$clination, which I count always from the Zenith, and upon ſuch a ſuppoſition, taking their new Latitude and Declination, thoſe that will try, ſhall find that theſe Proportions will calculate all the Arks neceſſary to ſuch Dials.

So if an Upright Plain decline 25 d in our Latitude of *London* from the Meridian.

The Subſtiles diſtance from the Meridian is	18 d 34′
The Angle of 12 and 6 is	62 : 00
The Inclination of Meridians is	30 : 47
The Stiles height is	34 : 19

To Delineate the ſame Dial from the Subſtile by the Line of Latitudes, and Scale of hours in an Equicrutal Triangle.

To Draw an Upright Decliner.



An Upright South Plain for the Latitude of London, *Declining 25*d *Eastwards.*

TO prick down this Dial by the line of Latitudes, and Scale of Hours in an *Isoceles* Triangle.

Draw C 12 the Meridian Line perpendicular to the Horizontal line of the Plain, and with a line of Chords, make the Angle

F C 12, equal to the Subſtiles diſtance from the Meridian, and draw the line F C for the Subſtile; Draw the line B A perpendicular thereto, and paſſing through the Center at C, and out of the line of Latitudes on the other Quadrants, or out of the Quadrant of Latitudes on this Quadrant, ſet off B C and C A each equal to the Stiles height, then fit in the Scale of 6 hours, proper to thoſe Latitudes, ſo that one Extremity meeting at A, the other may meet with the Subſtilar line at F.

Then get the difference between 30^{d} 47′, the inclination of Meridians, and 30^{d} the next hours diſtance leſſer then the ſaid Ark, the difference is 47′ in time, 3′ neareſt then fitting in the Scale of hours as was preſcribed.

Count upon the ſaid Scale,

Hour.	Min.		
0	3		10
1	3		11
2	3	from F to	12
3	3		1
4	3		2
5	3		3

And make points at the terminations with a pin or pen, & draw lines from thoſe points into the Center at C, & they ſhall be the true hour-lines required on this ſide the (Subſtile.

Again, Fitting in the Scale of Hours from B to F, count from that end at B the former Arks of time.

Ho	Min		
00,	03		4
1,	3		5
2,	3	from B to	6
3,	3		7
4,	3		8
5,	3		9

And make Points at the Terminations, through which draw Lines into the Center, and they ſhall be the hour Lines required on the other ſide the Subſtile.

The like muſt be done for the halfs and quarters, getting the difference between the half hour next leſſer (in this Example 22^{d} 30′) under the Ark called the inclination of Meridians, the difference is 1^{d} 17′ in time 33′ neareſt to be continually augmented an hour

hour at a time, and so prickt off as before was done for the whole hours.

By three facil Proportions, may be found the Stiles height, the Inclination of Meridians, and the Substiles distance from the Plains perpendicular, for all Plains Declining, Reclining, or Inclining, which are sufficient to prick off the Dyal after the manner here described, which must be referred to another place.

If the Scale of hours reach above the Plain, as at B, so that B C cannot be pricked down, then may an Angle be prickt off with Chords on the upper side the Substile, equal to the Angle F C A, on the under side, and thereby the Scale of hours laid in its true situation, having first found the point F on the under side.

To prick down the former Dyal in a Rectangular oblong, or long square Figure from the Substile.

Having set off the Substilar F C, assume any distance in it, as at F to be the Radius, and through the same at right Angles, draw the line E F D, then having made F C any distance Radius, take out the Sine of the Stiles height to the same Radius, and entring it at the end of the Scale of three hours, make it the Radius of a Tangent, and proportion out Tangents to

	3′	and set them off from F	to	10
1 hour	3			G
2	3			H

Again, Take out the Tangents of the Complement of the first Ark, increasing it each time by the augmentation of an hour,

namely		57′	and prick them from F to	I	and from the points
	1 ho.	57		K	
	2	57		E	

thus found, draw lines into the Center.

Then for the other sides of the Square, make C F the Radius of the Dyalling Tangent of 3 hours, and proportion out Tangents to the former Arks, namely,

	3′	and prick them from B to	P	Also to the latter Arks.		57′	and prick them from A to ——	N
1 ho.	3		O		1 h.	57		M
2	3		L		2	57		D

and

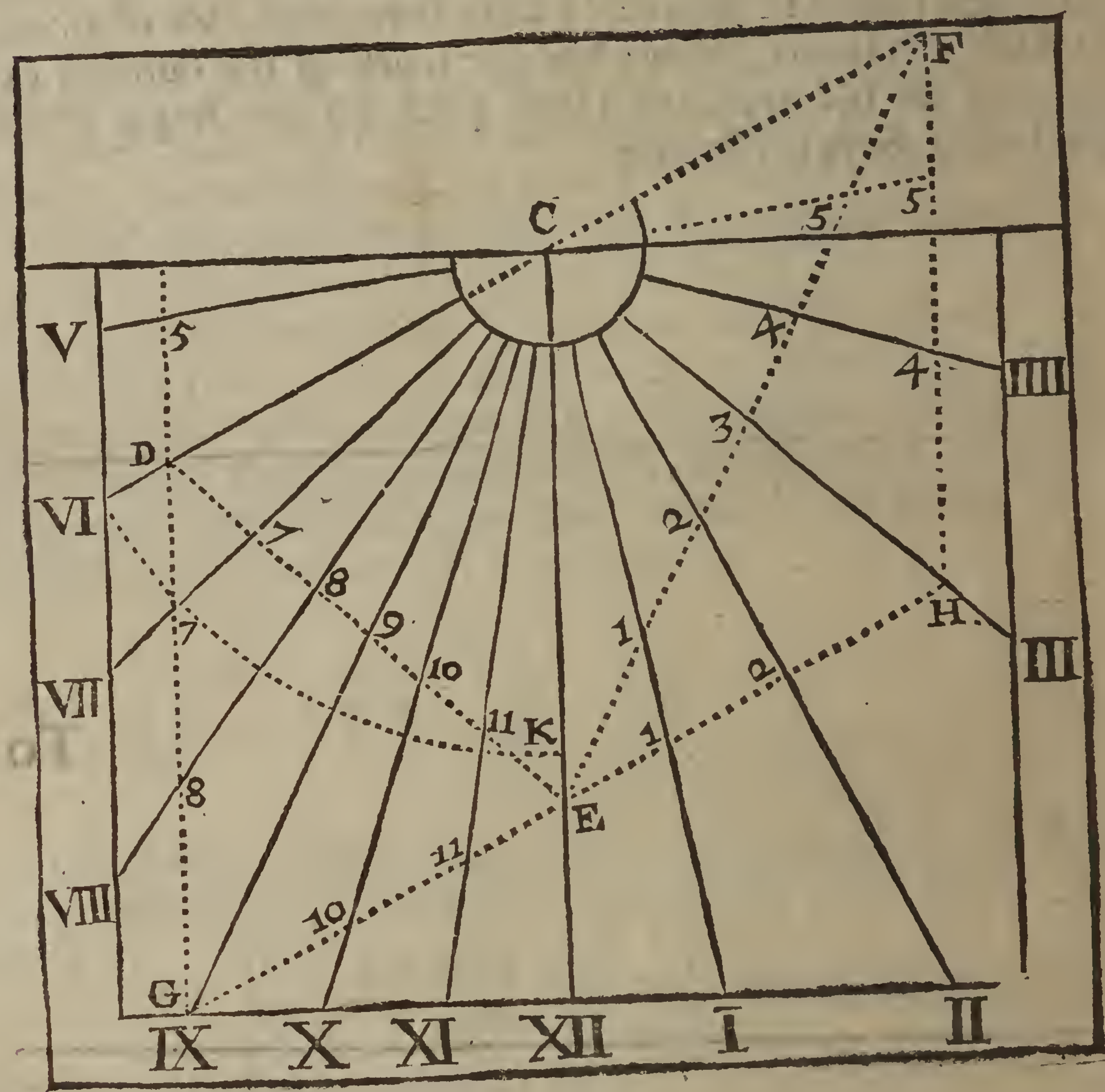
and draw lines from these terminations into the Center, and the Hour-lines are finished; after the same manner must the halfs and quarters be finished.

And how this trouble in Proportioning out the Tangents may be shunned without drawing any lines on the Plain, but the hour-lines, may be spoke to hereafter, whereby this way of Dyalling, and those that follow, will be rendred more commodious.

Lastly, the Stile may be prickt off with Chords, or take B C, and setting one foot in F, with that Extent sweep the touch of an occult Arch, and from C, draw a line just touching the outward extremity of the said Arch, and it shall prick off the Angle of the Stiles height above the Substile.

To

To prick off the former Dyal in an Oblique Parallellogram, or Scalenon alias *unequal sided Triangle from the Meridian.*



First, In an Oblique Parallellogram.

DRaw CE the Meridian line, and with 60[d] of a line of Chords, draw the prickt Arch, and therein from K, contrary to the Coast of Declination, prick off 62[d], the angle of 12 and 6, and draw

draw the line C D for the ſaid hour line continued on the other ſide the Center, and out of a line of Sines, make C E equall to 65^d the Complement of the Declination; then take out the ſine of $38^d\ 28'$ the Complement of the *Latitude*, and enter it in the line D C, ſo that one foot reſting at D, the other turned about, may but juſt touch the Meridian line, the point D being thus found, make C F equall to C D, and with the ſides C F and C E make the Parallellogram D G F H, namely, F H and G D equal to C E: and E G and E H equal to D C. And where theſe diſtances (ſweeping occult arches therewith) interſect will find the points H and G limiting the Angles of the Parallellogram.

Then making E H or C D Radius, proportion out the Tangents of $\left.\begin{matrix}15^d\\30\end{matrix}\right\}$ and prick them down from E to $\left\{\begin{matrix}1 \text{ and } 11\\2 \qquad 10\end{matrix}\right\}$ and draw lines into the Center through thoſe points, and the angular points of the Parallellogram at H and G, and there will be 6 hours drawn, beſides the Meridian line or hour line of 12. Then making D G Radius, proportion out the Tangent of 15^d, and prick it down from D upwards to 5, and downward to 7, alſo proportion out the tangent of 30^d and prick it from D to 8, and from F to 4, and draw lines into the Center, and ſo the hour lines are finiſhed; after the ſame manner are the halfs and quarters to be proportioned out and pricked down: and if this Work is to be done upon the Plain it ſelfe, the Parallel F H will excur above the plain, in that caſe, becauſe the Parallel diſtance of F H from the Meridian, is equal to the parallel diſtance of D G the ſpace G. 8. may be ſet from H to 4, and ſo all the hour lines prickt down.

To prick down this Dyal in a Scalenon, or unequal ſided triangle from the Meridian, from E to D draw the ſtreight line D E, and from the ſame point draw another to F, and each of them (the former hour lines being firſt drawn) ſhall thereby be divided into a line of double tangents, or ſcale of 6 hours, ſuch a one as is in the Diameter of the Circle on this quadrant, or on the right edge of the foreſide; and therefore by helpe of either of them lines, if it were required to prick down the Dyal, it might be done by Proportioning them out, take the extent D E, and prick it from one extremity of the Diameter in the Semicircle on the qua-

Nn drant,

drant, and from the point of Termination draw a line with black Lead to the other extremity, (which will eaſily rub out again either with bread or leather parings) and take the neareſt diſtance from $\left.\begin{matrix}15\\30\\45\end{matrix}\right\}$ of the Diameter to the ſaid line, and the ſaid extents

ſhall reach from, $\left\{\begin{matrix}E\\E\\E\end{matrix}\right\}$ to $\left.\begin{matrix}11\\10\\9\end{matrix}\right\}$ and from $\begin{matrix}D\\D\\D\end{matrix}$ to $\left\{\begin{matrix}7\\8\\9\end{matrix}\right.$

and the like muſt be done for the line E F, entring that in the Semicirle as before; or without drawing lines on the quadrant, if a hole be drilled at one end of the Diameter, and a thred fitted into it, lay the thred over the point in the Diameter, and take the neareſt diſtances thereto.

Laſtly, from a line of Chords, prick off the ſubſtilar line, and the ſtiles height as we before found it.

This way of Dyalling in a Parallellogram, was firſt invented by *John Ferrereus* a *Spaniard*, long ſince, and afterwards largely handled by *Clavius*, who demonſtrates it, and ſhews how to fit it into all plains whatſoever, albeit they decline, recline, or incline, without referring them to a new Latitude; the Triangular way is alſo built upon the ſame Demonſtration, and is already publiſhed by Mr *Foſter* in his *Poſthuma*, for it is no other then Dyalling in a Parallellogram, if the Meridian line C E be continued upwards, and C E ſet off upwards, and lines drawn from the point, ſo found to D and F, ſhall conſtitute a Parallellogram.

An

An Advertisement about observing of Altitudes.

IMagine a line drawn from the beginning of the line Sol, to the end of the Diameter, and therein suppose a pair of sights placed with a thred and bullet hanging from the begining of the said line, as from a Center; I say the line wherein the sights are placed, makes a right angle with the line of sines on the other side of Sol, and so may represent a quadrant, the equal Limbe whereof is either represented by the 90^d of the equal Semicirle, or by the 90^d of the Diameter and thereby an Altitude may be taken. Now to make an Isoceles equicrural, or equal legged triangle made of three streight Rulers, the longest whereof will be the Base or Hipotenusal line; thus to serve for a quadrant to take Altitudes withal, will be much cheaper, and more certain in Wood, then the great Arched wooden framed quadrants. Moreover, the said Diameter line supplies all the uses of the Limbe, from it may be taken off Sines, Tangents, or Secants, as was done from the Limbe; and therein the Hour and Azimuth, found generally by helpe of the line of Sines on the left edge, as is largely shewed in the uses of this quadrant, besides its uses in Dyalling, onely when such an Instrument is made apart, it will be more convenient to have the line of Sines to be set on the right edge, and the Diameter numbred also by its Complements; this Diameter or double Tangent, or Hipotenusal line being first divided on, all the other lines may from it, by the same Tables that serve to graduate them from the equal Limbe be likewise inscribed: and here let me put a period to the uses of this quadrant.

Gloria Deo.

The Description of an Universal small Pocket Quadrant.

THis quadrant hath only one face.

On the right edge from the Center is placed a line of fines divided into degrees and half degrees up to 60 d. afterwards into whole degrees to 80 d.

On the left edge issueth a line of 10 equal parts, from the Center being precise 4 inches long, each part being divided into 10 subdivisions and each subdivision into halfs.

These two lines make a right angle at the Center, and between them include a Projection of the Sphere for the Latitude of *London.*

Above the Projection are put on in quadrants of Circles a line of Declinations 4 quadrants for the dayes of the moneth, above them the names of 5 Stars with their right Ascensions graved against them, and a general *Almanack.*

Beneath the Projection are put on in quadrants of Circles a particular sine and secant, so called, because it is particular to the Latitude of *London.*

Below that the quadrat, and shadowes.

Below that a line of Tangents to 45 d.

Last of all the equal Limbe.

On the left edge is placed the Dialling scale of hours 4 Inches long, outwardmost on the right edge a line of Latitudes fitted thereto.

Within the line of sines close abutting thereto is placed a small scale called the scale of entrance beginning against 52 d. 35 min. of the sines numbred to 60 d.

The line of sines that issueth from the Center should for a particular use have been continued longer to wit to a secant of 28 d. because this could not be admitted, the said secant is placed outward at the end of the scale of entrance towards

 the

the Limbe, and as much of the sine as was needful placed at its due distance, at the other end of the scale of entrance.

Of the uses of the said quadrant.

THe *Almanack* hath been largely spoke to in pag. 12, and 13, also again in the uses of the Horizontal quadrant pag. 11, 12.

The quadrat and shadowes from pag. 35 to 44.

The line of Latitudes and scale of hours pag. 250. Again, from page to 262 to 274, also the line of sines, equal parts, and Tangents, in other parts of the Booke.

The use of the Projection.

THis projection is only fitted for finding the hour in the limb, and not the Azimuth, all the Circular lines on it are parallels of Altitude or Depression except the Ecliptick and Horizon, the Ecliptick, is known by the Characters of the signes, and the Horizon lyeth beneath it, being numbred with 10, 20, 30, 40.

The parallels of Altitude are the Winter parallels of *Stofler's Astrolable*, and are numbred from the Horizon upwards towards the Center, the parallels of Depression which supply the use of *Stoflers* Summer Altitudes are numbred downwards from the top of the Projection towards the limb.

To find the time of Sun rising, and his Amplitude.

LAy the thread over the day of the moneth, and set the Bead to the Ecliptick, then carry the thread and Bead to the Horizon, and the thread in the limb, shewes the time of rising, and the Bead on the Horizon the quantity of the Amplitude.

Example

Example.

So on the second of *August* the Suns Declination being 15 d. his Amplitude will be 24 d. 35 min. and the time of rising 41' past 4 in the morning.

To find the Hour of the Day.

HAving taken the Suns Altitude and rectified the Bead as before shewed, if the Sun have South Declination bring the Bead to that parallel of Altitude on which the Suns height was observed, amongst those parallels that are numbred upward towards the Center, and the thread in the limb sheweth the time of the day.

Eample.

So when the Sun hath 15 d. of South Declination, as about 28th of *January*, if his Altitude be 15 deg. the time of the day will be 39 m. past 2 in the afternoon, or 21 m. past 9 in the morning.

But in the Summer half year bring the Bead to lye on those parallels that are numbred downwards to the limb, and the thread sheweth therein the time of the day sought.

Example.

If on the second of *August* his Declination being 15 d. his Altitude were 40 d. the true time of the day would be 8 m. past 9 in the morning, or 52 m. past 2 in the afternoon.

If the Bead will not meet with the Altitude given amongst those parallels that run donwnwards towards the right edge, then it must be brought to those parallels that lye below the Horizon downward towards the left edge, and the thread in the Limb shewes the time of the day before six in the morning or after it in the evening in Summer.

Example.

When the Sun hath 15 d. of North Declination as on the ſecond of *Auguſt* if his Altitude be 5 d. the time of the Day will be 44 m. before 6 in the morning, or after it in the evening.

Of the general lines on this quadrant.

THe line of ſines on the right edge is general for finding either the hour or the Azimuth in the equal limb, or in the ſaid line of ſines, as I have largely ſhewed in page 231 for finding the hour, alſo for finding the Suns Altitudes on all hours, as in page 234, for finding the Azimuth from page 237 to pag. 239.

Though this quadrant hath neither ſecants nor verſed ſines as the reſt have, yet both may be eaſily ſupplyed, let it be required to work this Proportion.

As the Co-ſine of the declination, Is to the ſecant of the Latitude, So is the difference of the ſines of the Suns Meridian and given Altitude, To the verſed ſine of the hour from noon, before or after ſix the hour may be found from midnight by the proportions in page 230.

Let the Radius of the ſines be aſſumed to repreſent the ſecant of the Latitude, the Radius to that ſecant will be the coſine of the Latitude, then lay the thread to the complement of the Declination in the limb counted from right edge, and take the neareſt diſtance to it. I ſay that extent ſhall be the coſine of the Declination to the Radius of the Secant enter this at 90 d. of the line of ſines laying the thread to the other foot according to neareſt diſtance, then in the ſines take the diſtance between the Meridian Altitude and the given Altitude, and enter that extent ſo upon the ſines that one foot reſting thereon, the other turned about may juſt touch the thread the diſtance between the reſting foot and the Center is equal to the verſed ſine of the ark ſought and being meaſured from the end of the line of ſines towards the Center ſhewes the ark ſought.

Exam-

Example.

When the Suns Declination is 15 d. North if his Altitude were 35 d. 21 m. the time of the day would be found 45 d. from noon, that is 9 in the morning or 3 in the afternoon.

Of finding the Azimuth generally.

THough this may be found either by the ſines alone in the equal limbe as before mentioned, or by verſed ſines as was inſtanced for the hour, ſee alſo page 239, 240, 241, yet where the Sun hath vertical Altitude or Depreſſion, as in places without the Tropicks towards either of the Poles, it may be found moſt eaſily in the equal limb by the joynt help of ſines and tangents by the proportions in page 175.

Firſt, find the vertical Altitude as is ſhewed in page 174. Then for Latitudes under 45 d.

Enter in Summer Declinations the difference, but in Winter Declinations the ſum, of the ſines of the vertical Altitude, and of the propoſed Altitude once done the line of ſines from the Center, and laying the thread over the Tangent of the Latitude take the neareſt diſtance to it, then enter that Extent at the complement of the Altitude in the Sines, and lay the thread to the other foot, and in the limb it ſhewes the Azimuth from the Eaſt or Weſt.

Example. For the Latitude of *Rome* to witt 42 d. If the Sun have 15 d. of North Declination his vertical Altitude is 22 d. 45 m. If his given Altitude be 40 d. the Azimuth of the Sun will be 17 d. 33 m. to the Southward of the Weſt. If his declination were as much South and his propoſed Altitude 18 d. his Azimuth would be 41 d. 10 m. to the Southwards of the Eaſt or Weſt. For Latitudes above 45.

If we aſſume the Rad. of the quadrant to be the tangent of the Latit. the Rad. to that Tang. ſhall be the co-tangent of the Latit. wherefore lay the thread to the complement of the Latitude in the line of Tangents in the limb, and from the complement of the Altitude in the Sines take the neareſt diſtance to it, I ſay

that

that extent ſhall be coſine of the Altitude to the leſſer Radius which meaſure from the Center , and it finds the Point of entrance whereon enter the former Sum or difference of ſinesas before directed , and you will find the Azimuth in the equal limb.

Or if you would find the anſwer in the ſines , enter the firſt extent at 90 d. laying the thread to the other foot , then enter the Sum or difference of the Sines of the vertical and given Altitude, ſo between the ſcale and the thread,that one foot turned about may but juſt touch the thread , the other reſting on the ſines, and you will find the ſine of the Azimuth ſought.

Example. For the Latitude of *Edinburg* 55 d. 56 m. If the Sun have 15 d. of Declination, his vertical Altitude or depreſſion is 18 d. 14 m.the Declination being North,if his propoſed Altitude were 35 d. the Azimuth of the Sun would be 28 d. to the South-wards of the Eaſt or Weſt.

But if the Declination were as much South,and the Altitude 10 d. the Azimuth thereto would be 46 d. 58 m. to the South-wards of the Eaſt or Weſt.

The firſt Operation alſo works a Proportion to witt.

As the Radius Is to the cotangent of the Latitude.

So is the coſine of the Altitude , To a fourth ſine.

I ſay this 4th ſine beares ſuch Proportion to the Radius as the coſine of the Altitude doth to the tangent of the Latitude, for the 4th term of every direct Proportion beares ſuch Proportion to the firſt terms thereof, as the Rectangle of the two middle terms doth to the ſquare of the firſt term.

But as the rectangle of the co-tangent of the Latitude and of the coſine of the Altitude is to the ſquare of the Radius , So is the coſine of the Altitude is to the tangent of the Latitude , or which is all one , So is the co-tangent of the Latitude , To the ſecant of the Altitude , as may be found by a common diviſion of the rectangle , and ſquare of the Radius by either of the terms of the ſaid rectangle,by help of which notion I firſt found out the particular ſcales upon this quadrant.

All

All Proportions in sines and Tangents may be resolved by the sine of 90 d. and the Tangent of 45 d. on this quadrant if what hath been now wrote, and the varying of Proportions be understood, as in page 72 to 74 it is delivered.

Because the Projection is not fitted for finding the Azimuth there are added two particular scales to this quadrant, namely, the particular sine in the limb, and the scale of entrance abutting on the sines fitted for the Latitude of *London*.

Lay the thread to the day of the moneth, and it shewes the Suns Declination in the scales proper thereto.

Then count the Declination in the Limbe laying the thread thereto, and in the particular sine, it shewes the Suns Altitude or Depression being East or West.

To find the Suns Azimuth.

FOr North Declinations take the distance between the sines of the vertical Altitude and given Altitude; but for South Declinations adde with your compass the sine of the given Altitude to the sine of the vertical Altitude, enter the extent thus found, at the Altitude in the scale of entrance laying the thread to the other foot according to nearest distance, and in the equal limb it shewes the Azimuth sought from the East or West, or it may be found in the sines by laying the thread to that arch in the limb that the Altitude in the scale of entrance stands against in the sines; and entring the former extent paralelly between the thread and the sines.

Example. So when the Sun hath 13 d. of Declination his vertical Altitude or Depression is 16 d. 42 m. If the Declination were North and his Altitude 8 d. 41 m. his Azimuth would be 10 d. to the North-wards of the East or West. But if it were South and his Altitude 12 d. 13 m. the Azimuth would be 40 d to the South-wards of the East or West.

By the same particular scales the hour may be also found.

To find the time of Sun rising or setting.

TAke the sine of the Declination, and enter it at the Declination in the Scale of entrance and it shewes the time sought in the equal limbe from six.

Exam-

Example. When the hath 10 d. of Declination the Afcenfional difference is 49 m. which added to, or fubftracted from fix fhewes the time of rifing and fetting.

To find the hour of the day for South Declination.

IN taking the Altitude, mind what Ark in the particular Sine the thread cut, adde the Sine of that Ark to the Sine of the Declination, and enter that extent at the Declination in the Scale of entrance laying the thread to the other foot according to neareft diftance, and in the equal limb it fhewes the hour from fix. So if the Declination were 13 d. South and the Suns Altitude 14 d. 38 m. the thread in the particular Sine would cut 18 d. 49 m. and true time of the day would be 9 in the morning or 3 in the morning.

To find the time of the day for North Declination.

HAving obferved what Ark the thread in taking the Altitude hung over in the Particular Sine take the diftance between the Sine of the faid Arke, and the Sine of the Declination and entring that extent at the Declination in the Scale of entrance the thread in the limbe fhewes the hour from fix.

Example. If the Suns Declination were 23 d. 31 m. North, and his Altitude 39 d. the Arch in the particular Sine would be 53 d. 32 m. and the time of the day would be about 3 quarters paft 3 in the afternoon or a quarter paft 8 in the morning.

When the Altitude is more then the Latitude the thread will hang over a Secant in the particular Scale, this happens not till the Sun have more then 13 d. of North Declination, in this cafe take the diftance between the Secant before the beginning of the Scale of entrance, and the Sine of the Declination at the end of the fame and enter it as before.

Example. The Suns declination being 23 d. 31 m. North if his Altitude were 55 d. 29 m. the thread in the particular Scale would hang over the Secant of 18 d. 11 m. and the true time of the day would be a quarter paft 10 in the morning, or 3 quarters paft 1 in the afternoon. The Proportions here ufed are expreffed in Page 193.

The Stars hour is to be found by the projection by rectifying the Bead to the Sar and then proceed as in finding the Suns hour, afterwards the true time of the night is to be found as in page 32.

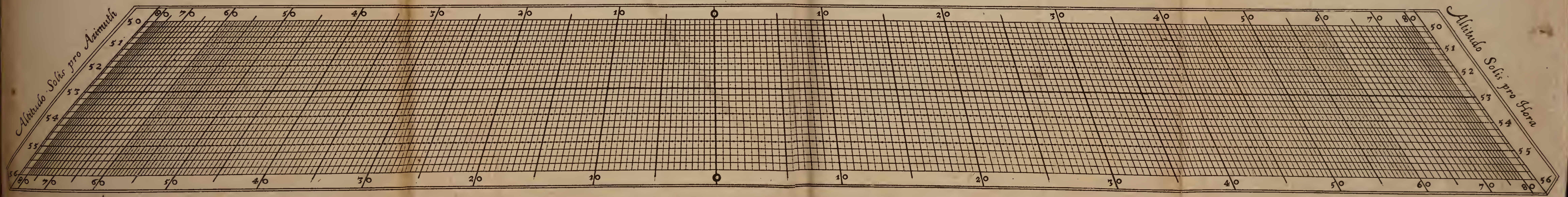
ERRATA.

In the Treatife of the Horizontal quadrant, pag. 43 line *6 for the 3 January read the 30th. In the Reflex Dialling* pag. 5, *adde to the laft line thefe words,* As *Kircher* fheweth in his *Ars Anaclaftica.*

FINIS.

page 11.

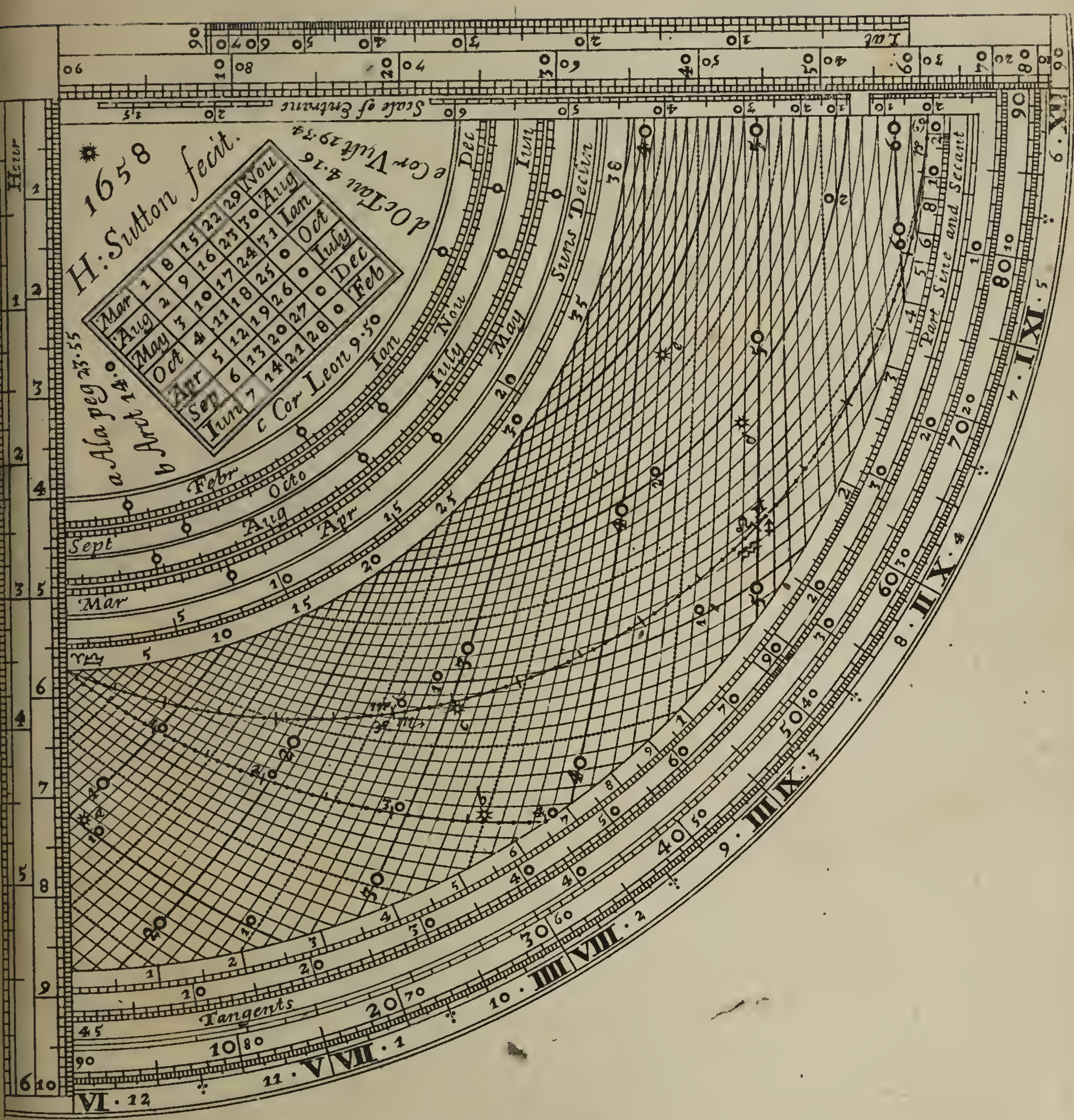
Hour Scale
Latitudes
Chords Declin
Chord
Substile
38·28 Stile
Henricus Sut fecit
90 Angle 12 & 6
Hour
Inclin Mer
Azimuth Scale
Altitudes
Hour Scale
Sines altitudes



Altitudo Solis pro Azimuth
Altitudo Solis pro Hora

Nouwember ye 12 1718 Recd £ 16 s 00 d 00

Which Made it £ 32 s 00 00 in full


1658
H: Sutton fecit.
a Ala peg 23.55
b Arct 14.0
c Cor Leon 9.50
d Oc Tau 4.16
Scale of Entrance
Suns Declin
Part Sine and Secant
Tangents
Hour
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If 36 of Cements Cost 36 ...

Shall 36767963 Cost at that Rate

36 — 56 — 6 — 36767963
[illegible]

72 — 29172102

009702[illegible]9

2597262

5971[illegible]2142

5971[illegible]2142

THE DESCRIPTION AND USES OF A GENERAL QUADRANT, WITH THE HORIZONTAL PROJECTION, UPON IT INVERTED.

Written and Published

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LONDON,

Printed Anno M. DC. LVIII.

The Deſcription
OF THE
HORIZONTAL QUADRANT.

THis Denomination is attributed to it becauſe it is derived from the Horizontal projection inverted.

Of the Fore-ſide.

On the right edge is a Line of natural Sines. On the left edge a Line of Verſed-Sines. Both theſe Lines iſſue from the Center where they concurre and make a right Angle, and between them and the Circular Lines in the Limb is the Projection included, which conſiſts of divers portions and Arkes of Circles.

Of the Parallels of Declination.

THeſe are portions of Circles that croſſe the quadrant obliquely from the left edge, towards the right.

To describe them.

OBserve that the left edge of the quadrant is called the Meridian Line, and that every Degree or Parallel of the Suns Declination if continued about would crosse the Meridian in two opposite points, the one below the Center towards the Limbe, and the other above, and beyond the Center of the quadrant, the distance between these two points is the Diameter of the said Parallel, and the Semidiameters would be the Center points.

It will be necessary in the first place, to limit the outwardmost Parallel of Declination, which may be done in the Meridian Line at any point assumed.

The distance of this assumed point from the Center in any Latitude, must represent the Tangent of a compound Arke, made by adding halfe the greatest Meridian Altitude to 45 Deg. which for *London* must be the Tangent of 76 Degr.

And to the Radius of this Tangent must the following work be fitted.

In like manner, the Semidiameters of all other Parallels that fall below the Center, are limited by pricking downe the Tangents of Arkes, framed by adding halfe the Meridian Altitude suitable to each Declination continually to 45 Degr.

Now to limit the Semidiameters above or beyond the Center onely prick off the respective Tangents of half the Suns mid-night Depression from the Center the other way, retaining the former Radius, by this meanes there will be found two respective points limiting the Diameters of each Parallel, which had, the Centers will be easily found falling in the middle of each Diameter.

But to doe this Arithmetically, first, find the Arke compounded of halfe the Suns meridian Altitude, and 45 Degr. as before, and to the Tangent thereof, adde the Tangent of halfe the Suns mid-night depression, observing that the Suns mid-night depression in Winter/Summer, is equal to his Meridian Altitude in Summer/Winter, his declination being alike in quantity, though in different Hemispheres, the halfe summe of these two Tangents are the respective Semidiameters sought, and being prickt in the meridian line

either

either way from the former points limiting the Diameters, will find the Centers.

Or without limiting those Points for the Diameters: first, get the Difference between the Tangents of those Arkes that limit them on either side, and the halfe summe above-said, the said difference prickt from the Center of the quadrant in the meridian line finds the respective Centers of those Parallels, the said halfe summes being the respective Semidiameters wherewith they are to be described.

Of the Line or Index of Altitudes.

THis is no other then a single prickt line standing next the Meridian line, or left edge of the quadrant, to which the Bead must be continually rectified, when either the houre or Azimuth is found by help of the projection.

To graduate it.

ADde halfe the Altitudes respectively whereto the Index is to be fitted to 45 Degr. and prick downe the Tangents of these compound arkes from the Center.

Example.

To graduate the Index for 40 Degr. of Altitude, the halfe thereof is 20, which added to 45 Degr. makes 65 Degr. which taken from a Tangent to the former Radius, and prickt from the Center, gives the point where the Index is to be graduated with 40 Degrees.

Hence it is evident that where the divisions of the Index begin marked (o) the distance of that point from the Center is equal to the common Radius of the Tangents. Because this quadrant (as all natural projections) hath a reverted taile, the graduations of the Index are continued above the Horizontal point (o) towards the Center to 30 Degr. 40′ as much as is the Sunnes greatest Vertical Altitude in this Latitude, and the graduations of the Index are set off from the Center by prick-

 ing

ing downe the Tangents of the arkes of difference between half the proposed Altitude, and 45 Deg. thus to graduate 20 deg. of the Index the halfe thereof is 10 Degrees, which taken from 45 Degrees, the residue is 35 Degrees, the Tangent thereof prickt from the Center gives the point where the Index is to be graduated with 20 Degrees.

Of the houre Circles.

These are knowne by the numbers set to them by crossing the Parallels of Declination, and by issuing from the upper part of the quadrant towards the Limbe.

To describe them.

Let it be noted that they all meet in a point in the Meridian Line below the Center of the quadrant: the distance whereof from the Center is equal to the Tangent of halfe the Complement of the Latitude taken out of the common Radius, which at *London* will be the Tangent of 19 Deg. 14′.

The former point which may be called the Pole-point, limits their Semidiameters, to find the Centers prick off the Tangent of the Latitude and through the termination raise a line Perpendicular to the Meridian line, the distance from the Pole-point being equal to the Secant of the Latitude, must be made Radius. And the Tangents of 15 Degrees, 30 Degrees &c. prickt off on the former raised line, gives the respective Centers of the houre Circles, the distances whereof from the Pole point are the Semidiameters wherewith those houre Circles are to be drawne.

Of the reverted Tail.

This needs no Rule to describe it, being made by the continuing of the parallels of Declination to the right edge of the quadrant and the houre Circles up to the Winter Tropick or parallel of Declination neerest the Center, however the quantity of it may be knowne by setting one foot of a paire of Compasses in the

Center

Center of the quadrant, and the other extend to 00 Degrees of Altitude in the Index; an Arch with that extent ſwept over the quadrant as much as it cuts off will be the Reverted Taile, and ſo much would be the Radius.

Of a Quadrant made, of this Projection not inverted.

BY what hath been ſaid it will be evident to the judicious that this inverſion is no other then the continance of the extents of one quarter of the Horizontal projection.

Which otherwiſe could not with convenience be brought upon a quadrant.

Hence it may be obſerved that.

Having aſſigned the Radius, a quadrant made of the Horizontal Projection without inverſion, to know how big a Radius it will require when inverted the proportion will hold.

AS the Radius, is to the diſtance of the interſection of the Æquinoctial point with the Horizon from the Center equall to the Radius of the ſaid Projection when not inverted, in any common meaſure.

So is the Tangent of an Arke compounded of 45 Degrees, and of half the Suns greateſt Meridian Altitude.

To the diſtance between the Center and the out-ward Tropick next the Limbe in the ſaid known meaſure when inverted, whence it followes that between the Tropicks this projection cannot be inverted, but the reverted taile will be but ſmall, and may be drawne with convenience without inverſion.

Of the Curved Line and Scales belonging to it.

BEyond the middle of the Projection ſtands a Curved or bending Line, numbred from the O or cypher both wayes, one way to 60 Degrees, but divided to 62 Degrees, the other way to 20 Degr. but divided to 23 Deg. 30′.

The

The Invention of this Line ownes Mr. *Dary* for the Author thereof, the Use of it being to find the houre or Azimuth in that particular latitude whereto it is fitted by the extension of a threed over it, and the lines belonging to it.

The lines belonging to it are two, the one a Line of Altitudes, and Declinations standing on the left edge of the quadrant, being no other but a line of Sines continued both wayes, from the beginning one way to 62 Degrees, the other way to 23. Degrees 30′.

The other line thereto belonging is 130 Deg. of a line of Versed Sines, which stands next without the Projection being parallel to the left edge of the quadrant.

To draw the Curve.

DRaw two lines of Versed Sines, it matters not whether of the same Radius or no, nor how posited; provided they be parallel, let each of them be numbred as a Sine both ways, from the middle at (o) and so each of them will containe two lines of Sines, to the right end of the uppermost set C, to the left end D, and to the right end of the undermost set A, and to the left end B.

First, Note that there is a certaine point in the Curve where the Graduations will begin both upwards and downwards, this is called the Æquinoctial point; to find it, lay a ruler from A to the Complement of the Latitude counted from (o) in the upper Scale towards D, and draw a line from A to it, then count it the other way towards C, *viz.* 38 Degrees 28′. for the Co-latitude of *London*, and lay a ruler over it, and the point B, and where it intersects the line before drawn, is the Æquinoctial point to be graduated.

Then to graduate the Division on each side of it, requires onely the making in effect of a Table of Meridian altitudes to every degree of Declination (which because the Curve will also serve for the Azimuth in which case the graduations of the Curve, which in finding the houre were accounted Declinations must be accounted

Alti-

Altitudes) muſt be continued to 62 Degrees for this Latitude, and further alſo if it be intended that the Curve ſhall find a Stars houre that hath more declination.

To make this Table.

GEt the Summe and difference of the Complement of the latitude and of the Degrees intended to be graduated, and if the ſumme exceed 90 Degrees, take its complement to 180 degrees inſtead of it: being thus prepared the Curve will be readily made.

To graduate the under part of the Curve.

Account the ſumme in the upper line from O towards D, and from the point A in the under line draw a line to it.

Account the difference in the upper line when the degree propoſed to be graduated is leſſe then the complement of the Latitude from O towards C: but when it is more towards D, and from the point B lay a Ruler over it, and where the Ruler interſects, the line formerly drawn is the point where the degree propoſed is to be graduated.

Example.

Let it be required to find the point where 60 deg. of the Curve is to be graduated.

Arke propoſed	60 deg.
Co-latitude	38 : 28
	98 : 28
Summe	81 : 32
Difference	21 32

Count 81 deg. 32′ in the upper line from O towards D, and from the point A draw a line to it.

Count the difference 21 degrees 32′ from O towards D, becauſe the co-latitude is leſſe then the arke propoſed, and lay a Ruler over it, and the point B, and where it interſects the for-

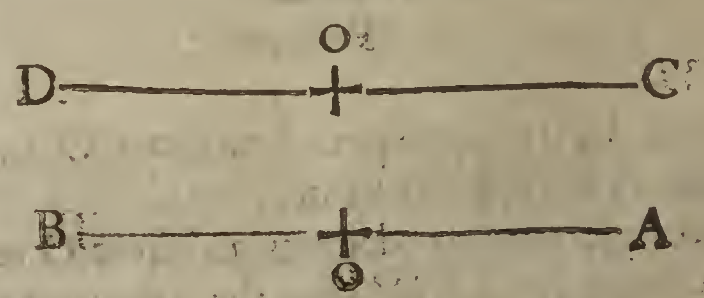
mer line is the point where 60 deg. of the Curve is to be graduated on the lower side.

Another Example.

Let it be proposed to graduate the same way,

The arke of 30 degr.	30 degr.
Co-latitude	38 : 28
Summe	68 : 28
Difference	8 : 28

Count 68 deg. 28′ from O towards D, and from the point A draw a line to it. Again in the said upper line, count 8 deg. 28′ upwards from O towards C, & from the point B lay a ruler over it, & where it intersects the line last drawn is the point where 30 d. of the curve is to be graduated. To graduate the upper part of the curve requires no other directions, the same arkes serve, if the account be but made the other way, and in accounting the summe the ruler laid over B in the lower line instead of A, and in counting the difference over A, instead of B, neither is there any Scheme given hereof, the Practitioner need onely let the upper line be the line of altitudes on the left edge of the quadrant continued out to 90 deg. at each end, and to that end next the Center set C, and to the other end D. So likewise let that end of the Versed Scale next the right edge of the quadrant be continued to 180 deg. whereto set A, and at the other end B, and then if these directions be observed, and the same distance and position of the lines retained, it will not be difficult to constitute a Curve in all respects agreeing with that on the fore-side of the quadrant.



Of

Of the houre and Azimuth Scale on the right edge of the Quadrant.

THis Scale ſtands outwardmoſt on the right edge of the quadrant, and conſiſts of two lines, the one a line of 90 ſines made equal to the coſine of the Latitude, namely, to the ſine of 38 deg. 28′, and continued the other way to 40 deg. like a Verſed ſine.

The annexed line being the other part of this Scale, is a line of natural Tangents beginniug where the former ſine began, the Tangent of 38 deg. 28′ being made equal to the ſine of 90 deg. this Tangent is continued each way with the ſine; towards the Limbe of the quadrant it ſhould have been continued to 62 deg. but that could not be without excurſion, wherefore it is broken off at 40 degrees, and the reſidue of it graduated below, and next under the Verſed ſine belonging to the Curve that runnes croſſe the quadrant being continued but to halfe the former Radius.

Of the Almanack.

NExt below the former line ſtands the Almanack in a regular ob-long with moneths names graved on each ſide of it.

Below the Almanack ſtands the quadrat, and ſhadowes in two Arkes of circles terminating againſt 45 deg. of the Limbe, below them a line of 90 ſines in a Circle equal to 51 deg. 32′ of the Limbe broken off below the ſtreight line, and the reſt continued above it.

Below theſe are put on in Circles a line of Tangents to 60 degrees.

Alſo a line of Secants to 60 deg. with a line of leſſer ſines ending againſt 30 deg. of the Limbe (counted from the right edge) where the graduations of the Secant begins.

Laſt of all the equal Limbe.

Prickt with the pricks of the quadrat.

Abutting upon the line of ſines, and within the Projection ſtands a portion of a ſmall ſine numbred with its Complements beginning againſt 38 deg. 28′ of the line of ſines, this Scale is

called the Scale of entrance. Upon the Projection are placed divers Stars, how they are inſcribed ſhall be afterwards ſhewne.

The deſcription of the Back-ſide.

Put on in quarters or Quadrants of Circles.

1 THe equal Limbe divided into degrees; as alſo into houres and halves, and the quarters prickt to ſerve for a Nocturnal.

2 A line of Equal parts.
3 A line of Superficies or Squares.
4 A line of Solids or Cubes.
5 A Tangent of 45 degrees double divided to ſerve for a Dyalling Tangent, and a Semitangent for projections.
6 The line Sol, *aliàs* a line of Proportional Sines.
7 A Tangent of 51 degrees 32′ through the whole Limbe.
8 A line of Declinations for the Sun to 23 deg. 31′.
9, 10, 11, 12 } Foure quadrants with the days of the Moneth.
13 The Suns true place, with the Charecters of the 12 Seignes.
14 The line of Segments, with a Chord before they begin.
15 The line of Metals and Equated bodies.
16 The line of Quadrature.
17 The line of Inſcribed bodies.
18 A line of 12 houres of Aſcenſion with Stars names, Declinations, and Aſcenſional differences.

Above all theſe a Table to know the Epact, and what day of the Weeke, the firſt day of *March* hapned upon, by Inſpection continued to the yeare 1700.

All these between the Limbe and the Center.

ON the right edge a line of equal parts from the Center decimally sub-divided, being a line of 10 inches; also a Dyalling Tangent or Scale of 6 houres, the whole length of the quadrant not issuing from the Center.

On the left edge a Tangent of 60 deg. 26′ from the Center.

Also a Scale of Latitudes fitted to the former Scale of houres not issuing from the Center, and below it a small Chord.

The Uses of the Quadrant.

Lords-day	1657 25	63 1	68 26	74 3	☉	85 4	91 11	96 6	*anno* *epact*
Mon-day	58 6	☾	69 7	75 14	80 9	86 15	☽	97 17	*anno* *epact*
Tues-day	59 17	64 12	70 18	♂	81 20	87 26	92 22	98 28	*anno* *epact*
Wed-nesday	☿	65 23	71 29	76 25	82 1	☿	93 3	99 9	*anno* *epact*
Thurs-day	60 28	66 4	♃	77 6	83 12	88 7	94 14	♃	*anno* *epact*
Fri-day	61 9	67 15	72 11	78 17	♀	89 18	95 25	700 20	*anno* *epact*
Satur-day	62 20	♄	73 22	79 28	84 23	90 29	♄	701 1	*anno* *epact*

Dayes the ſame as the firſt of *March.*

March	1	8	15	22	29	*November*
Auguſt	2	9	16	23	30	*Auguſt*
May	3	10	17	24	31	*Jnuary*
October	4	11	18	25	0	*October*
April	5	12	19	26	00	*July*
Septem.	6	13	20	27	00	*December*
June	7	14	21	28	00	*February*

Perpetual Almanack.

Of the Uſes of the Projection.

BEfore this Projection can be uſed, the Suns declination is required,&c by conſequence the day of the moneth for the ready finding thereof there is repeated the ſame table that ſtands on the Back-ſide of this quadrant in each ruled ſpace,the uppermoſt figure ſignifies the yeare of the Lord, and the column it is placed in ſheweth upon what day of the Weeke the firſt day of March hapned upon in that yeare, and the undermoſt figure in the ſaid ruled ſpace ſheweth what was the Epact for that yeare and this continued to the yeare 1701 incluſive.

Example.

Looking for the yeare 1660 I find the figure 60 ſtanding in *Thurſday* Column, whence I may conclude that the firſt day of *March* that yeare will be T*hurſday*, and under it ſtands 28 for the *Epact* that yeare.

Of the Almanack.

HAving as before found what day of the Weeke the firſt day of *March* hapned upon, repaire to the Moneth you are in, and thoſe figures that ſtand againſt it ſhewes you what dayes of the ſaid moneth the Weeke day ſhall be, the ſame as it was the firſt day of *March.*

Example For the yeare 1660, having found that the first day of *March* hapned upon a *Thursday*, looke into the column against *June*, and *February*, you will find that the 7th, 14th, 21th and 28th dayes of those Moneths were *Thursdayes*, whence it might be concluded if need were that the quarter day or 24th day of *June* that yeare hapneth on the *Lords day*.

Of the Epact.

THe *Epact* is a number carried on in account from yeare to yeare towards a new change, and is 11 dayes, and some odde time besides, caused by reason of the Moons motion, which changeth 12 times in a yeare Solar, and runnes also this 11 dayes more towards a new change, the use of it serves to find the Moones age, and thereby the time of high Water.

To know the Moons age.

ADde to the day of the Moneth the *Epact*, and so many days more, as are moneths from *March* to the moneth you are in, including both moneths, the summe (if lesse then 30) is the Moones age, if more, subtract 30, and the residue in the Moons age (*prope verum.*)

Example.

The *Epact* for the year 1658 is 6, and let it be required to know the Moons age the 28 of *July*, being the fift moneth from *March* both inclusive

$$\begin{array}{r} 6 \\ 28 \\ \underline{5} \end{array}$$

The summe of these three numbers is 39

Whence rejecting 30, the remainder is 9 for the Moons age sought.

The former Rule serves when the Moneth hath 31 dayes, but if the Moneth hath but 30 Dayes or lesse, take away but 29 and the residue is her agee.

To find the time of the Moones comming to South.

MUltiply the Moones age by 4, and divide by 5, the quotient shewes it, every Unit that remaines is in value twelve minutes of time, and because when the Moon is at the full, or 15 dayes, old shee comes to South at the houre of 12 at midnight, for ease in multiplication and Division when her age exceedes 15 dayes reject 15 from it.

Example,

So when the Moon is 8 dayes old, she comes to South at 24 minutes past six of the clock, which being knowne, her rising or setting may be rudely guessed at to be six houres more or lesse before her being South, and her setting as much after, but in regard of the varying of her declination no general certaine rule for the memory can be given.

Here it may be noted that the first 15 dayes of the Moones age she commeth to the Meridian after the Sun, being to the Eastward of him, and the later 15 dayes, she comes to the Meridian before the Sun, being to the Westward of him.

To find the time of high Water.

TO the time of the Moones comming to South, adde the time of high water on the change day, proper to the place to which the question is suited, the summe shewes the time of high water.

For

Moones age Dayes.		Moon South Ho.	mi.	Tide London Ho.	Mi
0	15	12	——	3	00
1	16	12	48	3	48
2	17	1	36	4	36
3	18	2	24	5	24
4	19	3	12	6	12
5	20	4		7	00
6	21	4	48	7	48
7	22	5	36	8	36
8	23	6	24	9	24
9	24	7	12	10	12
10	25	8	00	11	00
11	26	8	48	11	48
12	27	9	36	12	36
13	28	10	24	1	24
14	29	11	12	2	12

For *Example*, There is added in a Table of the time of high Water at *London*, which any one may cast up by memory according to these Rules, it is to be noted, that Spring Tides, high winds, and the Moon in her quarters causes some variation from the time here expressed.

This Rule may in some measure satisfie and serve for vulgar use for such as have occasion to go by water, and but that there was spare roome to grave on the Epacts nothing at all should have been said thereof.

A Table shewing the houres and

Minutes to be added to the time of the Moons comming to South for the places following being the time of high Water on the change day.

	H. m.
Quinborough, Southampton, Portsmouth, Isle of Wight, Beachie, the Spits, Kentish Knocke, half tide at Dunkirke.	00 : 00
Rochester, Maulden, Aberdeen, Redban, West end of the Nowre, Black taile.	00 : 45
Gravesend, Downes, Rumney, Silly half tide, Blackness, Ramkins, Senihead.	1 : 30

 Dundee.

Dundee, St. Andrewes, Lixborne, St. Lucas, Bel Isle, Holy Isle.	2 : 15
London, Tinmouth, Hartlepoole, Whitby, Amsterdam, Gascoigne, Brittaine, Galizia.	3 : 00
Barwick, Flamborough head, Bridlington bay, Ostend, Flushing, Bourdeaux, Fountnesse.	3 : 45
Scarborough quarter tide, Lawrenas, Mountsbay, Severne, Kingsale, Corke-haven, Baltamoor, Dungarvan, Calice, Creeke, Bloy seven Isles.	4 : 30
Falmouth, Foy, Humber, Moonles, New-castle, Dartmouth, Torbay, Caldy Garnesey, St. Mallowes, Abrowrath, Lizard.	5 : 15
Plymouth, Weymouth, Hull, Lin, Lundy, Antwerpe, Holmes of Bristol, St. Davids head, Concalo, Saint Malo.	6 : 00
Bristol, foulnes at the Start.	6 : 45
Milford, Bridg-water, Exwater, Lands end, Waterford, Cape cleer, Abermorick, Texel.	7 : 20
Portland, Peterperpont, Harflew, Hague, St. Magnus Sound, Dublin, Lambay, Macknels Castle.	8 : 15
Poole, S. Helen, Man Isle, Catnes, Orkney, Faire Isles, Dunbar, Kildien, Basse Islands, the Casquers, Deepe at halfe tide.	9 :
Needles, Oxford, Laysto, South and North Fore-lands.	9 : 45
Yarmouth, Dover, Harwich, in the frith Bullen, Saint John de luce, Calice road.	10 : 30
Rye, Winchelsea, Gorend, Rivers mouth of Thames, Faire Isle Rhodes.	11 : 15

To

To find the Epact for ever.

IN Order hereto, firſt, find out the *Prime* Number divide the yeare of the Lord by 19 the reſidue after the Diviſion is finiſhed being augmented by an Unit is the *Prime* ſought, and if nothing remaine the *Prime* is an Unit.

To find the Epact.

MUltiply the *Prime* by 11, the product is the *Epact* ſought if leſſe then 30, but if it be more, the reſidue of the Product divided by 30 is the *Epact* ſought, there note that the *Prime* changeth the firſt of *January*, and the *Epact* the firſt of *March.*

Otherwiſe.

Having once obtained the *Epact* adde 11 ſo it the Summe if leſſe then 30 is the *Epact* for the next yeare if more reject 30, and the reſidue is the *Epact* ſought.

Caution.

When the *Epact* is found to be 29 for any yeare, the next yeare following it will be 11 and not 10, as the Rule would ſuggeſt.

A Table of the Epacts belonging to the reſpective Primes.

Pr.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Ep.	11	22	3	14	25	6	17	28	9	20	1	12	23	4	15	26	7	18	29

The Prime number called the *Golden Number*, is the number of 19 years in which ſpace the Moone makes all variety of her changes, as if ſhe change on a certain day of the month on a certain yeare ſhe ſhall not change the ſame day of the moneth again till 19 yeares after: and then it doth not happen upon the ſame houre of the day, yet the difference doth not cauſe one dayes variation in 300 yeares, as is obſerved by Mr. *Philips.*

The Uses of the Quadrant.

WIthout rectifying the Bead nothing can be performed by this Projection, except finding the Suns Meridian Altitude being shewn upon the Index, by the intersection of the Parallel of declination therewith.

Also the time when the Sun will be due East or West.

TRace the Parallel of Declination to the right edge of the Projection, and the houre it there intersects (in most cases to be duly estimated) shewes the time sought, thus when the Sun hath 21 deg. of North declination, we shall find that he will be due East or West, about three quarters of an houre past 4 in the afternoon, or a quarter past 7 in the morning. The declination is to be found on the Back-side of the quadrant by laying the thread over the day of the moneth.

To rectifie the Bead.

LAy the thread upon the graduated Index, and set the Bead to the observed or given Altitude, and when the Altitude is nothing or when the Sun is in the Horizon set the Bead to the Cypher on the graduated Index, which afterwards being carried without stretching to the parallel of Declination the threed in the Limbe shewes the Amplitude or Azimuth , and the Bead amongst the houres shewes the true time of the day.

Example.

Upon the 24th of April the Suns declination will be found to 16 deg. North.

Now to find his Amplitude and the time of his rising, laying the threed over the graduated Index, set the Bead to the beginning of the graduations of the Index, and bring it without stretching to the parallel of declination above being 16 d. and the threed in the

Limbe

limbe will lye over 26 deg. 18′ for the Suns Amplitude or Coast of rising to the Northward of the East, and the Bead amongst the houres sheweth 24 minutes past 4 for the time of Sun rising.

Which doubled gives the length of the night 8 houres 49 min.

In like manner the time of setting doubled gives the length of the day.

The same to find the houre and Azimuth let the given Altitude be 45 degrees.

HAving rectified the Bead to the said Altitude on the Index and brought it to the intersect, the parallel of declination the thread lyes over 50 degrees 48′.

For the Suns Azimuth from the South.

And the Bead among the houres shewes the time of the day to be 41 minutes past 9 in the morning, or 19 minutes past two in the afternoon.

Another Example wherein the operation will be upon the Reverted taile.

Let the altitude be 3 *deg.* 30′
And the declination 16 *deg. North as before.*

TO know when to rectify the Bead to the upper or neather Altitude will be no matter of difficulty, for if the Bead being set to the neather Altitude will not meet with the parallel of declination, then set it to the upper Altitude, and it will meet with Winter parallel of like declination, which in this case supplyes the turn.

So in this Example, the Bead being set to the upper Altitude of 3 deg. 30′ and carried to the Winter parallel of declination.

The thread in the Limbe will fall upon 68 deg. 28′ for the Suns Azimuth from the North, and the Bead among the houres shewes the time of the day to be either 5 in the morning or 7 at night.

Another Example.

Admit the Sun have 20 degr. of North Declination (as about the 9th of *May*) and his observed altitude were 56 deg. 20′ having

ving rectified the Bead thereto, and brought it to intersect the parallel of 20 deg. among the houres it shewes the time of the day to be 11 in the morning or 1 in the afternoon; and the Azimuth of the Sun to be 26 deg. from the South.

The Uses of the Projection.

TO find the Suns Altitude on all houres or Azimuths will be but the converse of what is already said, therefore one Example shall serve.

When the Sun hath 45 deg. of Azimuth from the South, And his Declination 13 deg. Northwards.

Lay the threed over 45 deg. in the Limbe, and where the threed intersects the Parallel of Declination thereto remove the Bead which carried to the Index without stretching, shewes 43 deg. 50′ for the Altitude sought.

Likewise to the same Declination if it were required to find the Suns Altitude for the houres of 2 or 10.

Lay the threed over the intersection of the houre proposed with the parallel of Declination, and thereto set the bead which carried to the Index shewes the Altitude sought namely 44 deg. 31′.

The same Altitude also belongs to that Azimuth the threed in the former Position lay over in the Limbe.

This Projection is of worst performance early in the morning or late in the evening, about which time Mr. *Daries* Curve is of best performance whereto we now addresse our selves.

Of the curved line and Scales thereto fitted.

This as we have said before was the ingenious invention of M. *Michael Dary* derived from the proportionalty of two like equiangled plain Triangles accommodated to the latitude of *London*, for the ready working of these two Proportions.

1 *For the Houre.*

As the Cosine of the Latitude, is to the secant of the Declination; So is the difference between the sine of the Suns proposed and Meridian Altitude.

To the versed sine of the houre from noone, and the converse, and so is the sine of the Suns Meridian Altitude, to the versed sine of the semidiurnal Arke.

2 *For the Azimuth.*

The Curve is fitted to find it from the South and not from the North, and the Proportion wrought upon it will be.

As the cosine of the Latitude, is to the Secant of the Altitude. So is the difference of the versed sines of the Suns (or Stars) distance from the elevated Pole, and of the summe of the Complements both of the Latitude and Altitude, to the versed sine of the Azimuth from the noon Meridian.

Which will not hold backward to find the Altitude on all Azimuths, because the altitude is a term involved, both in the second and third termes of the former proportion.

If the third terme of the former Proportion had not been a difference of Sines, or Versed sines, the Curved line would have been a straight-line, and the third term always counted from one point, which though in the use it may seem to be so here, yet in effect the third term for the houre is always counted from the Meridian altitude.

Here observe that the threed lying over 12 or the end of the Versed Scale, and over the Suns meridian altitude in the line of altitudes, it will also upon the curve shew the Suns declination, which by construction is so framed that if the distance from that point to the meridian altitude, be made the cosine of Latitude, the distance of the said point from the end of the versed Scale numbred with 12 shall be the secant of the declination to the same Radius, being both in one straight-line by the former constitution of the threed, and instead of the threed you may imagine a line drawn over the quadrant, then by placing the threed as hereafter directed

it will with this line & the fitted scales constitute two equiangled plaine triangles, upon which basis the whole work is built.

In the three first Proportions following relating to time, the Altitude must alwayes be counted upwards from O in the line of Altitude, and the Declination in the Curve upwards in Summer, downwards in Winter.

1 *To find the time of the Suns rising and setting by the Curve.*

We have before intimated that the suns Declination is to be found on the back of the quadrant, having found it, lay one part of the thread over o deg. in the Line of Altitude, and extending it, lay the other part of it over the Suns Declination counted from O in the Curve, and the thread upon the Versed scale shewes the time of Suns rising and setting, which being as much from six towards noon in Winter as towards mid-night in Summer, the quantity of Declination supposed alike both wayes on each side the Equinoctial, the thread may be layd either way from O in the Curve to the Declination.

Example.

When the Sun hath 20 deg. of Declination, the thread being laid over 20 deg. in the Curve and O in the Altitude on the left edge shewes that the Sun riseth/setteth 1 houre 49′ before/after six in the Summer and riseth/setteth as much after/before six in the Winter.

2 *The Altitude and Declination of the Sun being given to find the houre of the day.*

Count the Altitude from O in the Scale of Altitudes towards the Center, and thereto lay the thread, then count the Declination from O in the Curve, if North upwards towards the Center, if South downwards towards the Limbe.

And lay the thread extended over it, and in the Versed Scale it shewes the time of the day sought.

Example

Example.

The Altitude being 24 d. 46′ and the Declination 20 d. North counting that upwards in the Scale of Altitudes, and this upward in the curve, and extending the through thread, it will intersect the Versed Scale at 7 and 5, shewing the houre to be either 7 in the morning, or 5 in the afternoon.

Another Example for finding when twliight begins.

Let the Suns Declination be 13 deg. North, the Depression supposed 18 degr. under the Horizon.

In stead of the case propounded, suppose the Sun to have 13 deg. of South Declination, and Altitude 18 deg. above the Horizon accordingly extending the thread through 18 in the Altitudes counted upward from O in the line of Altitudes and through 13 deg. counted downward in the Curve from O, and upon the Versed Scale, the thread will shew that the Twilight begins at 28 minutes past 2 in the morning, and at 32 min. past 9 at night.

3 *The Converse of the last Proposition is to find the Suns altitudes on all houres.*

EXtend the thread over the houre proposed in the versed Scale and also over the Declination in the Curve counted upward if North, downward if South.

And in the Scale of Altitudes it shewes the Altitude sought.

Example.

If the Sun have 13 deg. of North Declination his Altitude for the houre of 7 in the morning, or 5 in the after-noon will be found to be 19 deg. 27′.

In the following Propositions the altitude must alwayes be counted from O in the Curve downwards, and the Declination in the line of altitudes, if North downward, if South upwards.

4 *To find the Suns Amplitude or coast of rising and setting.*

Example.

If the Sun had 20 deg. of Declination the thread being laid to O in the Curve, and to 20 in the line of altitudes or Declinations, either upwards or downwards the thread will lye 33 deg. 21′ from 90 in the Versed Scale, for the quantity of the Suns Coast of rising or setting from the true East or West in Winter Southward, in Summer Norhward.

5 *The Suns altitude and Declination being proposed to find his Azimuth.*

Count the altitude from O in the Curve downward, and the declination in the Winter upon the line of Declinations from O upwards, in Summer downwards, and the thread extended sheweth the Azimuth sought, on the Versed Scale.

Example.

So when the Sun hath 18 deg. 37′ of North Declination, as as about 19 *July*, if his altitude were 39 deg. the Suns Azimuth would be found to be 69 deg. from the South.

6 *The Converse of the former Proposition will be to find the Suns Altitude on all Azimuths.*

The Instrument will perform this Proposition though the Portion for finding the Azimuth cannot be inverted.

Lay the thread to the azimuth in the Versed Scale, and to the Declination in the Scale on the left edge, and upon the Curve it will intersect the altitude sought.

Example.

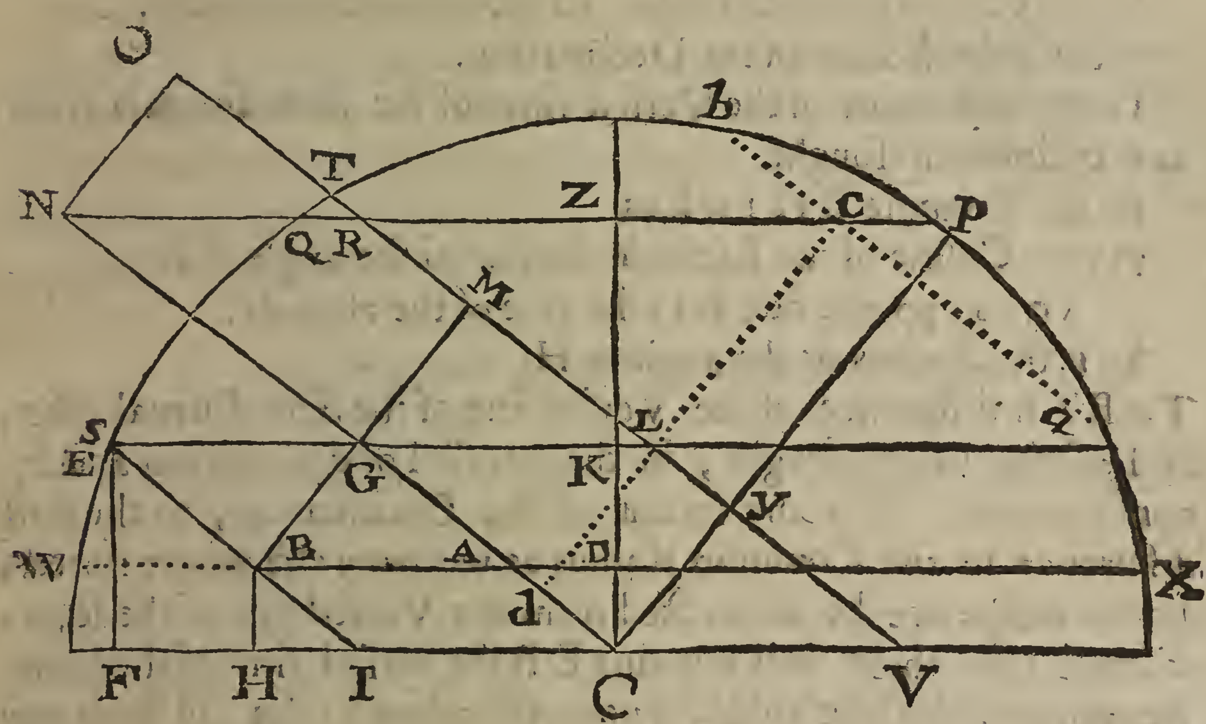
If the Sun had 16 deg. 13′ of South Declination, as about the 27th of *October*, if his Azimuth were 39 deg. from the South the altitude agreeable thereto would be found to be 14 deg.

These

These Uses being understood; if the houre and altitude or the azimuth and altitude were given to find the Declination, the manner of performance cannot lurke.

Of the Houre and Azimuth Scale on the right Edge of the Quadrant.

This Scale being added by my selfe, and derived from Proportions in the Analemma, I shall first lay them down, and then apply them.



In the former Scheme draw F C V the Horizon, Z C the Axis of the Horizon, C P the axis of the Spheare G C continued to N the Equator, O L a parallel of North, and E I a parallel of South Declination, W X a parallel of winter altitude, S L a parallel of altitude lesse then the Complement of the latitude, N Z P a parallel of greater altitude, and from the points E and B let fall the perpendiculars E F and B H, and from the points B G and N let fall the perpendiculars B G, G M, and N O which will be the sines of the Suns declination, by this meanes there will be divers right lined right angled plaine Triangles consti-

tuted from whence are educed, the Proportions following to calculate the Suns houre or Azimuth.

Note, first, that T V is the Versed sine of the Semidiurnal arke in Summer, and E I in Winter, and Y V the sine of the houre of rising before six in Summer, equal to the distance of I from the Axis continued in Winter, which may be found in the Triangle C Y V, but the Proportion is.

As the Cotangent of the latitude, To Radius.

So the Tangent of the Suns Declination,

To the sine of his ascentional difference, being the time of his rising from six, thus we may attaine the Semidiurnal arke.

Then for the houre in the Triangle B H I it holds.

As the Cosine of the latitude, to the sine of the altitude.

So is the Secant of the Declination.

To the difference of the Versed sines of the Semidiurnals arke, and of the houre sought.

In the Triangle B H I it leys.

As the Cosine of the Latitude the sine of the angle at I.

To its opposite side B H the sine of the altitude.

So is the Radius or the angle at H.

To B I h difference of the Versed sine of the Semidiurnal arke, and of the houre sought, in the parallel of declination and by consequence, so is the secant of the Declination, to the said difference in the Common Radius as we have else where noted, if this difference be subtracted from the Versed sine of the semidiurnal arke there will remaine E B the versed sine of the houre from noon, the like holds, if perpendiculars be let fall from any other parallel of Declination, from the same Scheme it also followes.

As the Cosine of the Latitude,

Is to the secant of the Declination.

So is the sine of the Meridian Altitude.

To the versed sine of the semidiurnal arke.

Here observe the like Proportion between the two latter terms, as between the two former which may be of use on a Sector.

If

If the Scheme be considered not as fitted to a peculiar question for finding the houre, but as having three sides to find an angle, it will be found upon such a consideration in relation to the change of sides, that the Proportion for the Azimuth following is no other then the same Proportion applyed, to other sides of the Triangle, and so we need have no other trouble to come by a Proportion for the Azimuth, but it also followes from the same Scheme.

In the Triangles C D A and C K G, and C Z N the first operation will be to find A D, and G K, and N R in all which the Proportion will hold.

As the Radius to the Tangent of the Latitude.

Or as the Cotangent of the Latitude to Radius.

So is the Tangent of the Altitude, to the said respective quantities, which when the Altitude is lesse then the Complement of the Latitude, are the sins of the Suns Azimuth from the Vertical belonging to the proposed Altitudes when the Sun is in the Equinoctial, or hath no declination.

The next proportion will be.

As the Cosine of the Latitude, Is to the Secant of the Altitude.

So is the Sine of the declination.

To the difference sought being a 4 Proportional.

Hereby we may find A B in the Winter Triangle A G B which added to A D, the summe is the sine of the Azimuth from the Vertical consequently W B, is the Versed sine of the Azimuth, from the noon Meridian.

Also we find G L in the Summer triangle L M G, when the Altitude is lesse then the Complement of the Latitude, which added to S G the summe S L is the Versed sine of the Azimuth from the South.

Likewise we may find N R in the Triangle R O N, and by subtracting it from N Z, there will remaine R Z, and consequently Q R the versed sine of the Azimuth from the Meridian in Summer when the Altitude is greater then the Co-latitude.

And for Stars that come to the Meridian between the Zenith, and the Elevated Pole, we may find N c, in the Triangle N c d

where it holds, as the ſine of the Angle at N, the complement of the Latitude, to its oppoſite ſides c d, the prickt line, the ſine of the Declination: ſo is the Radius to N c, the parallel of altitude the Azimuth ſought.

The latter Proportion lyes ſo evident, it need not be ſpoken to, if what was ſaid before for the houre be regarded, and the former Proportion lyes.

As the Coſine of the Latitude, the ſine of the Angle at A.
To its Oppoſite ſide D C, the ſine of the altitude.
So is the ſine of the Latitude, the angle at C.
To its oppoſite ſide A D in the parallel of altitude.
And in ſtead of the Coſine, and ſine of the Latitude.
We may take the Radius, and the Tangent of the Latitude.

Another Analogy will be required to reduce it to the common Radius.

As the Coſine of the Altitude to Radius.
So the fourth before found in a parallel.
To the like quantity to the Common Radius.

Theſe Analogies or Proportions being reduced into one, by multiplying the termes of each Proportion, and then freed from needleſſe affection will produce the Proportion at firſt delivered.

The Uſes of the ſaid Scale.

WE have before noted, that if two termes of a Proportion be fixed, and naturall lines thereto fitted of an equal length, that if any third term be ſought in the former line, the fourth term will be found in the other line by inſpection, as ſtanding againſt the third.

So here, in this Scale which conſiſts of two lines, the one an annexed Tangent, the other a line of Sines continued both wayes, the Radius of the Sines being firſt fitted, the Tangent annexed muſt be of ſuch a Radius, as that 38 deg. 28′, of it may be equall in length to the Radius of the Sine to which it is adjoyned,

and

and then looking for the Declination in the Tangent just against it stands the time of rising, from six or ascentional difference, or the Semidiurnal arke, if the same be accounted from the other end as a Versed Sine.

So if the Suns Altitude be given, and accounted in the Tangent, just against it stands the Suns Azimuth, when he is in the Equinoctial upon the like altitude, and thus the point N will be found in the Tangent at the altitude, when it is more then the Colatitude.

1. *An Example for finding the time of the Sun rising.*

If the Declination be 13 deg. looke for it in the annexed Tangent, and just against it in the houre Scale stands 16 deg. 53′ the ascentional difference in time 1 houre 7 ½ min. shewing that the Sun riseth so much before, and setteth so much after 6 in Summer, and in Winter riseth so much after, and setteth before 6, for this arke may be found on either side of six where the declination begins each way.

2 *To find the time of the day.*

To perform this Proposition wee divide the other Proportion into two, by introducing the Radius in the Middle.

As the Radius is to the Secant of the Declination.
So is the sine of the altitude to a fourth.

Again.

As the Cosine of the Latitude to Radius.
So the fourth before found.
To the difference of the Versed Sines of the Semidiurnal arke, and of the houre sought.

The former of these Proportions must be wrought upon the quadrant, the latter is removed by fitting the Radius of the Sines that gives the answer, equal in length to the Cosine of the latitude.

Wherefore to find the time of the day, lay the thread to the Secant of the declination in the limbe, and from the sine of the altitude take the nearest distance to it, and because the Secant is

made

made, but to halfe the Common Radius, set downe one foot of this extent at the Declination in the annexed Tangent, and enter the said extent twice forward, and it will shew the time of the Day.

Example.

Let the Declination be supposed 23 deg. 31′ North, and the Altitude 38 deg. 59′ the nearest distance from the Sine thereof, to the thread laid over the Secant of 32 deg. 31′ will reach being turned twice over from 32 d. 31′ in the annexed Tangent neerest the Center to 33 deg. 45′ in the Sines, *alias* to 56 d. 15′ counted as a Versed Sine shewing the time of the day to be a quarter past 8 in the morning, or three quarters past three in the afternoon.

3 *To find the Suns Altitude on all houres.*

Take the distance between the houre and the Declination in the fitted Scale, and enter it downe, the line of Sines from the Center, then laying the thread over the Cosine of the Declination in the Limbe, the nearest distance to it shall be the sine of the Altitude sought.

Example.

Thus whee the Sun hath 13 deg. of South Declination, count it in that part of the annexed Tangent nearest the Limbe, if then it were required to find the Suns Altit. for the houres of 10 or 2 by the former Prescriptions the Altitude would be found 10 d. 25′

4 *To find the Suns Amplitude.*

Take the Sine of the Declination from the line of the Sines, and apply it to the fitted Scale where the annexed Tangent begins, and either way it will reach to the Sine of the Amplitude.

Example.

So when the Sun hath 15 deg: of Declination his Amplitude will be found to be 24 deg. 35′.

5 To

5 *To find the Azimuth or true Coast of the Sun.*

Here we likewise introduce the Radius in the latter Proportion.

1 In Winter lay the thread to the Secant of the Altitude in the Limbe, and from the sine of the Declination, take the nearest distance to it, the said extent enter twice forward from the Altitude in the annexed Tangent, and it will reach to the Versed Sine of the Azimuth from the South.

Example.

So when the Sun hath 15 deg. of South Declination, if his Altitude be 15 deg. the nearest distance from the sine thereof to the thread laid over the Secant of 15 degrees, shall reach in the fitted Scale from the annexed Tangent of 15 deg. being twice repeated forward to the Versed sine of 39 deg. 50′ for the Suns Azimuth from the South.

2 In Summer when the Altitude is lesse then 40 deg. enter the former extent from the sine of the Declination to the thread laid over the Secant of the Altitude twice backward from the Altitude in the annexed Tangent, and it will reach to the Versed sine of the Azimuth from the South.

Example.

So if the Sun have 15 deg. of North Declination, and his Altitude be 30 deg. the prescribed extent doubled shall reach from the annexed Tangent of 30 deg. to the Versed sine of 75 deg. 44′. for the Suns Azimuth from the South.

3 In Summer when the Altitude is more then 40 deg. and lesse then 60 deg. apply the extent from the sine of the Declination to the thread, laid over the Secant of the Altitude, once to the Discontinued Tangent placed a Crosse the quadrant from the Altitude backwards minding how farre it reaches, just against the

like arke in the annexed Tangent ſtands the Verſed ſine of the Azimuth from the South.

4 When the Altitude is more then 60 deg. this fitted Scale is of worſt performance, however the defect of the Secant might be ſupplyed by Varying the Proportion.

6 To find the Suns Altitude on all Azimuths.

JUſt againſt the Azimuth propoſed ſtands the Suns altitude in the Equator ſuitable thereto, which was the firſt Arke found by Calculation when we treated of this ſubject, and the ſecond arke is to be found by a Proportion in ſines wrought upon the quadrant.

This quadrant is alſo particularly fitted for giving the houre, and Azimuth in the equal limbe.

The ſine of 90 deg. made equal to the ſine if 51 deg. 32′ gives the altitude of Suns or Stars at ſix, for if the thread be laid over the Declination counted in the ſaid ſine, it ſhewes the Altitude ſought in the limbe, ſo when the Sun hath 13 deg. of Declination his Altitude or Depreſſion at 6 is 10 deg. 9′.

It alſo gives the Vertical Altitude if the Declination be counted in the limbe, ſeeke what arke it cuts in that particular ſine, when the Sun hath 13 deg. of Declination, his Vertical Altitude or Depreſſion is 16 deg. 42′.

To find the houre of the Day.

HAving found the Altitude of the Sun or Stars at ſix, take the diſtance between the ſine thereof in the line of Sines, and the Altitude given, and entring one foot of that extent at the Declination in the Scale of entrance laying the thread to the other foot according to neareſt diſtance, it will ſhew the houre from ſix in the limbe.

Example.

When the Sun hath 13 deg. of Declination his Altitude, or De-

Depreſſion at ſix will be 10 deg. 9′ if the Declination be North, and the Altitude of the Sun be 24 deg. 5′ the time of the day will be halfe an houre paſt 7 in the morning, or as much paſt 4 in the afternoon.

In winter when the Sun hath South Declination as alſo for ſuch Stars as have South Declination, the ſine of their Altitude muſt be added to the ſine of their Depreſſion at ſix, and that whole extent entred as before.

When the Sun hath the ſame South Declination, if his Altitude be 11 deg. 7′ the time of the day will be half an houre paſt 8 in the morning, or 30 min. paſt 3 in the afternoon.

To find the Azimuth of the Sun or Stars.

LAy the thread over their Altitude in the particular ſine fitted to the Latitude, and in the equal Limbe it ſhewes a fourth Arke.

When the Declination is North, take the diſtance in the line of Sines between that fourth Arke and the Declination, and enter one foot of that extent at the Altitude in the Scale of entrance, laying the thread to the other foot, and in the equal Limbe it ſhewes the Azimuth from the Eaſt or Weſt.

Example.

When the Altitude is 44 deg. 39′ the Arch found in the equal Limbe will be 33 deg. 20′ then if the Declination be 23 deg. 31′ North, the diſtance in the line of ſines between it and the ſaid Arke being entred at 44 deg. 39′ in the Scale of entrance the thread being laid to the other foot will ſhew the Azimuth to be 20 deg. from the Eaſt or Weſt.

But if the Declination be South, adde with your Compaſſes the ſine thereof to the ſine of the fourth Arke, and enter that whole extent as before, and the thread will ſhew the Azimuth in the equal limbe.

Example. When the Altitude is 12 d. 13′ the fourth Arch will be found to be 9 degrees 32 minutes, then admit the Declination to be 13 degrees South, whereto adding the Sine of the fourth Arke, the whole will be equall to the sine of 22 deg. 41 minutes, and this whole extent being entred at 12 deg. 13′ in the Scale of entrance lay the thread to the other foot according to nearest distance, and it will intersect the equal Limbe at 40 deg. and so much is the Suns Azimuth from the East or West.

Because the Scale of entrance could not be continued by reason of the Projection, the residue of it is put on an little Line neare the Amanack the use whereof is to lay the thread to the Altitude in it when the Azimuth is sought, and in the Limbe it shewes at what Arke of the Sines the point of entrance will happen which may likewise be found by pricking downe the Co-altitude on the line of Sines out of the fitted houre Scale on the right edge.

How to find the houre and Azimuth generally in the equal limb either with or without Tangents or Secants hath been also shewed, and how that those two points for any Latitude might be there prickt and might be taken off, either from the Limbe, or from a line of Sines, or best of all by Tables, for halfe the natural Tangent of the Latitude of *London*, is equal to the sine of 39 deg. And half the Secant thereof equal to the sine of 53 d. 30 Against which Arkes of the Limbe the Tangent and Secant of the Latitude are graduated, but of this enough hath been said in the Description of the small quadrant.

Of the Quadrat and Shadowes.

THe use thereof is the same as in the small quadrant onely if the thread hang over any degree of the Limb lesse then 45d. to take out the Tangent thereof out of the quadrat count the Arch from the right edge of the quadrant towards the left, and lay the thread over it, the pricks are repeated in the Limbe to save this trouble for those eminent parts.

Of the equal Limbe.

WE have before shewed that a Sine, Tangent and Secant may be taken off from it, and that having a Sine or Secant with the Radius thereof the correspondent Arke thereto might be found, & that a Chord might be taken off from Concentrick Circles or by helpe of a Bead, but if both be wanting enter the Semidiameter or

Radius

Radius whereto you would take out a Chord twice downe the right edge from the Center, and laying the thread over halfe the and laying the thread over halfe the Arch propofed, take the nearest distance to it, and thus may a chordbe taken out to any number of degrees lesse then a Semicircle.

It hath been asserted also that the houre and Azimuth might be found generally without Protraction by the sole helpe of the Limb with Compasses and a thread.

Example for finding the houre.

THe first work will be to find the point of entrance take out the Cosine of the Latitude by taking the nearest distance to the thread laid over the said Arke from the concurrence of the Limbe with the right edge, and enter it down the right edge line and take the nearest distance to the thread laid over the complement of the Declination counted from the right edge, this extent entred down the right edge finds the point of entrance, let it be noted with a mark. Next to find the fine point take out the fine of the Declin. & enter it dowh the right edge, & from the point of termination, take the nearest distance to the thread laid over the ark of the Latit. counted from the right edge, this extent enter from the Center and it finds the fine point, let it be noted with a marke.

Thirdly, take out the fine of the Altitude & in Winter add it in lenght to the fine point, in Summer enter it from the Center & take the distance between it & the fine point which extent entred upon the point of entrance, if the thread be laid to the other foot shewes the the houre from *6* in the equal limb before or after it, as the Sine of the Altitude fell short or beyond the fine point.

Example. In the latitude of 39 d. the Sun having 23 d. 31′ of North Declination, and Altitude 51 deg. 32′ the houre will be found to be 33 deg. 45′ from six towards noon.

Note the point of entrance and fine point Vary not, till the Declination Vary.

After the same manner may the Azimuth be found in the limb, by proportions delivered in the other great quadrant. Also both or any angle when three sides are given may be found by the last generalProportion in the small quadrant which finds the halfe Versed fine of the Arke fought, which would be too tedious to insist upon & are more proper to be Protracted with a line of Chords,

To find the Azimuth univerſally.

THe Proportion uſed on the ſmal quadrant for finding it in the equal limbe (wherein the firſt Operation for the Vertical Altitude was fixed for one day ,) by reaſon of its Excurſions will not ſerve on a quadrant , for the Sun or Stars when they come to the Meridian between the Zenith and the elevated Pole, but the Proportion there uſed for finding the houre applyed to other ſides will ſerve for the Azimuth Univerſally , and that is

As the Radius , Is to the ſine of the Latitude ,
So is the ſine of the Altitude,
To a fourth ſine.

Again.

As the Coſine of the Altitude,
Is the Secant of the Latitude.

Or,

As the Coſine of the Latitude ,
Is the Secant of the Altitude.

So In Declinations towards the Elevated Pole is the difference, but towards the Depreſſed Pole the ſumme of the fourth ſine, and of the ſine ſine of the Declination.

To the ſine of the Azimuth from the Vertical.

In Declinations towards the Depreſſed Pole , the Azimuth is alwayes obtuſe, towards the elevated Pole if the Declination be more then the fourth Arch it is acute , if leſſe obtuſe.

Example for the Latitude of the Barbados 13 deg.

Altitude 27 deg. 27′.
Declination 20 deg. North.

Lay the thread to 27 deg. 27′ in the Limbe, and from the ſine of 13 deg. take neareſt diſtance to it which enter on the line of Sines from the Center, and take the diſtance between the limited point, and the ſine of 20 deg. the Declination, this latter extent enter twice downe the line of the Sines from the Center, and

and take the neareſt diſtance to the thread laid over the Secant of 27 deg. 27′ this extent enter at the ſine of 77 deg. the Complement of the Latitude, and laying the thread to the other foot it will lye over 16 deg. in the equal Limbe, the Suns Azimuth to the Northwards of the Eaſt or Weſt.

Otherwaies.

Another Example for the ſame Latitude and Declination, the Altitude being 52 deg. 27′ lay the thread to it in the Limbe, and take the neareſt diſtance to it from the ſine of 13 deg. as before, and enter it downe the line of ſines from the Center, and from the point of the limitation take the diſtance to the ſine of 20 deg. the Suns Declination, this latter extent enter once downe the line of ſines from the Center, and take the neareſt diſtance to the Thread laid over the Secant of the Altitude 52 deg. 27′ then lay the thread to 77 deg. the Complement of the Latitude in the leſſer ſines, and enter the former extent between the Scale and the thread, and the foot of the Compaſſes ſheweth 16 deg. as before, for the Suns Azimuth to the Northward of the Vertical, that the Sun may have the ſame Azimuth, upon two ſeveral Altitudes hath been ſpoken to before, and how to do this without Secants hath been ſhewne.

Two ſides with the Angle comprehended to find the third ſide.

Divers wayes have been ſhewed for doing of this before, I ſhall adde one more requiring no Verſed ſines nor Tangents.

1 If both the ſides be leſſer then quadrants, and the Angle at liberty.

Or,

2 If one of the ſides be greater then a quadrant, and the Angle included acute, it will hold.

As the Radius, To the Coſine of one of the including ſines.

So is the Coſine of the other, To a fourth ſine.

Again

Again.

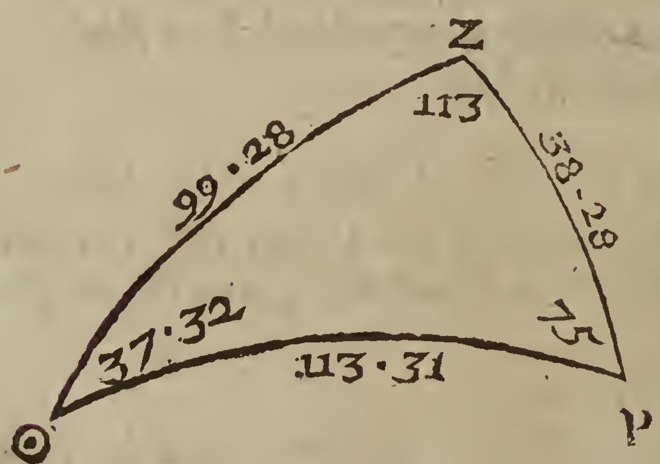
As the Cosecant of one of the including Sides
So is the Sine of the other,
So is the Cosine of the angle included,
To a seventh Sine.

The difference between the fourth and the seventh Sine, is the Cosine of the Side sought.

1 In the first case if the angle given be obtuse, and the seventh Sine greater then the fourth Sine, the Side sought is greater then a quadrant in other cases lesse.

If in the second case the seventh Sine be lesse then the fourth, the side sought is greater then a quadrant in other cases lesse.

In this second case when one of the includers is greater then a quadrant, and the angle obtuse resolve the opposite Triangle by the former Rules, or the summe of the fourth and seventh Sine shall be the Cosine of the side sought in this case greater then a quadrant. We have before noted that the Cosine of an Arke greater then a quadrant is the Sine of that Arkes excesse above 90 deg. this no other then the converse of the Proportion for the houre demonstrated from the Analemma, in the Triangle O Z P.



Let there be given the Sides O P 113 deg. 31′ the side Z P 38 deg. 28′ and the angle comprehended Z P O 75 to find the Side O Z.

Operation.

Lay the thread to 51 deg. 32′ in the Limbe, and from 13 deg. 31′ in

in the Sines take the nearest distance to it which measured from the Center will reach to the sine of 18 deg. 12 minutes the fourth Sine.

Again, laying the thread to 23 deg. 31′ in the Limbe, from the Sine of 15 deg. take the nearest distance to it, then lay the thread to the Secant of 51 deg. 32′ and enter the said extent between the Scale and the thread, the distance between the resting foot, and the Sine of 18 deg. 12 minutes before found measured from the Center is equal to the Sine of 9 deg. 32′ being the Cosine of the side sought which in this instance because the seventh Sine is lesse then the fourth sine is greater then a quadrant, and consequently must have 90 deg. added thereto, therefore the side O Z is 99 deg. 28 minutes if the question had been put in this Latitude what depression the Sun should have had under the Horizon at the houres of 5 or 7 in the Winter Tropick it would have been found 9 deg. 28′ and this is such a Triangle as hath but one obtuse Angle yet two sides greater then quadrants, and how to shunne a Secant, and a parallel entrance hath been shewed elsewhere.

Of the Stars on the Projection, and in other places of the fore-side of the quadrant.

SUch only are placed on the Projection as fall between the Tropicks being put an according to their true Declinations, and in that respect might have stood any where in the parallel of Declination, but in regard we shall also find the time of the night by them with Compasses, they are also put on in a certain Angle from the right edge of the quadrant, to find the quantity of the Angle for Stars of Northerly declination, get the difference of the Sines of the Stars Altitude six houres from the Meridian, and of its Meridian Altitude, and find to the Sine of what Arch the said difference is equal, against that Arch in the Limbe, let the Star be graduated in its proper declination, but for Stars of Southwardly Declination, get the summe of the Sines of their Depression at six and of their Meridian Altitude, and find what Arke in the Sines corresponds thereto as before.

We have put on no Stars of Southwardly Declination that will fall beyond the Winter Tropick, but some of Northerly Declination falling without the Summer Tropick, are put on that are

placed without the Projection towards the Limbe.

All these Stars must be graduated against the line of Sines at their respective Altitudes or Depressions at the Stars houre of Six from the Meridian, and must have the same letter set to them in both places, as also upon the quadrant of 12 houres of Ascension on the Back-side where they are put on according to their true Ascension with their Declinations and Ascensional differences graved against them with the former Letter, and such of them as have more then 12 houres of right Ascension have the Character *plus* + affixed, denoting that if there be 12 houres of Ascension added to that Ascension they stand against, the summe is their whole true right Ascension.

To find the quantity of a Stars houre from the Meridian by the Projection.

SEt the Bead upon the Index of Altitude to the Stars observed Altitude, and bring it to the parallel of Declination the Star is graved in, so will it shew among the houre lines, that Stars houre from the Meridian, and the thread in the Limbe will shew the Stars Azimuth.

Example.

Admit the Altitude of *Arcturus* be 52 deg. the houre of that Star from midnight, if the Altitude increase will be 7′ past 10 *ferè*, and the Azimuth of that Star will be 47 deg. 43′ to the Eastwards of the South.

The houre and Azimuth of any Star within the Tropicks, may be also found by the fitted Scale on the right edge of the quadrant, or by the Curve, after the same manner as for the Sun, using the Stars Declination as was done for the Suns, or in the equall limb as we shewed for the Sun, which may well serve for most of the Stars in the Hemisphere.

Otherwise with Compasses according to the late suggested placing of them.

To find the houre of any Star from the Meridian that hath North Declination.

TAke the distance between the Star point in the line of Sines, and its observed Altitude, and laying the thread over the Star where it is graved on or below the Projection, enter the former extent paralelly between the thread and the Scale, and it shewes the Stars houre from six in the sines towards noone, if the Altitude fell beyond the Star point, otherwise towards midnight.

Example.

For the Goat Star let its Altitude be 40 deg. and past the Meridian, the houre of that Star will be 44′ from six, for the Compasses fall upon the sine of 11 deg. 4′ the houre is towards noon Meridian, because the Altitude is greater then 34 deg. the point where the Star is graved, the thread lying over the Star intersects, the Limbe at 25 deg. 47′ if the distance between the Star, and its Altitude be entred at the sine of that Arke, and the thread laid to the other foot, the houre will be found in the equal Limbe the same as before.

For Stars of Southwardly Declination.

BEcause the Star point cannot fall the other way beyond the Center of the quadrant, therefore the distance between the Star point, and the Center must be increasing by adding the sine of the Stars Altitude thereto, which will fall more outwards towards the Limbe, and then that whole extent is to be entred as before.

Example.

The *Virgins Spike* hath 9 deg. 19′ of South Declination the Depression of that Star at six will be found by help of the particular sine to be 7 deg. 17′ and at that Arke in the sines the Star is graved, if the Altitude of that Star were 20 deg. the sine thereof added to the Star will be equal to the sine of 29 deg. 6′ this whole

extent entred at the sine of 37 deg. 52′ the Arke of the Limbe against which the Star is graved, and the thread laid to the other foot, the houre of that Star if the Altitude increase will be 19′ past 9.

To find the true time of the night.

THis must be done by turning the Stars houre into the Suns houre or common time, either by the Pen as hath been shewed before, which may be also conveniently performed by the back of this quadrant, for the thread lying over the day of the moneth sheweth the Complement of the Suns Ascension in the Limbe.

Or with Compasses on the said quadrant of Ascensions.

THe thread lying over the day of the moneth, take the distance between it and the Star on the said quadrant, the said extent being applyed, the same way as it was taken the Suns foot to the Stars houre shall reach from the Stars houre to the true houre of the night, and if one of the feet of the Compasses fall off the quadrant, a double remedy is elswhere prescribed.

Example.

If on the 12th of *January* the houre of the *Goat Star* was 16′ past 5 from the Meridian, the true time sought would be 49′ past 1 in the morning.

Example.

If upon the third of *January*, the houre of the *Virgins Spike*, were observed to be 19′ past 9, the true time sought would be 45′ past 2 in the morning.

To find the time of a Stars rising and setting.

THe Ascentional difference is graved against the Star, the *Virgins Spike* hath 48′ of Ascentional difference, that is to say, that Stars houre of rising is at 48′ past 6, and setting at 12′ past 5, And the true time of that Stars rising upon the third of *January*, will be at 23′ past 10 at night, and of its setting at 47′ past 8 in the morning, found by the former directions.

Of the rest of the lines on the back of this quadrant.

THey are either such as relate to the motion of the Sun or Stars, or to Dialling, or such as are derived from Mr *Gunters* Sector.

The Tangent of 51 deg. 32′ put through the whole Limbe is peculiarly fitted to the Latitude of *London*, and will serve to find the time when the Sun will be East or West, as also for any of the Stars that have lesse Declination then the place hath Latitude.

Lay the thread to the Declination counted in the said Tangent, and in the Limbe it shewes the houre from 6 if reckoned from the right edge.

Example.

When the Sun hath 15 deg. of North Declination the time of his being East or West will be 12 deg. 17′ in time about 49′ before or after six, *fere*.

The Suns place is given in the Ecliptick line by laying the thread over the day of the moneth in the quadrant of Ascensions, of which see *page* 16 & 17 of the small quadrant.

Of the lines relating to Dialling.

SUch are the Line of Latitudes, and Scale of houres, of which before, and the line Sol in the Limbe, of which I shall say nothing at present, it is onely placed there in readinesse to take off any Arke from it, according to the accustomed manner of taking off lines from the Limbe to any assigned Radius.

The

The requisite Arkes of an upright Decliner will be given by the particular lines on the Quadrant for the Latitude without the trouble of Proportionall worke.

1 *The substiles distance from the Meridian.*

ACcount the Plaines declination as a sine in the fitted hour Scale on the right edge of the fore-side, and just against it in the annexed Tangent, stands the substiles distance from the meridian.

If an upright Plaine decline 30 deg. the substiles distance will be 21 deg. 41 minutes.

2. *The Stiles height.*

Count the Complement of the Plaines Declination in the said fitted houre scale as a sine and apply it with Compasses to the line of sines issuing from the Center, for the former Plaine the stiles height will be found 32 deg. 37′.

3 *The Inclination of Meridians.*

Account the stiles height in the annexed tangent of the fitted hour Scale, and just against it in the sine stands the Complement of the Inclination of meridians which for the former plaine will be found to be 36 deg. 25′

4 *The Angle of* 12 *and* 6.

Account the Plaines Declination in the Limbe on the Backside from the right edge, and lay the thread over it, and in the particular Tangent it shewes the Angle between the Horizon and six 32 deg. 9′ in this Example the Complement whereof is the Angle of 12 and 6, namely 57 deg. 51 min.

Also

Alſo the requiſite Arkes of a direct Eaſt or Weſt, reclining or inclining Dial may be found after the ſame manner for this Latit.

1 *The ſubſtiles diſtance.*

ACcount the Plaines $^{Re}_{in}$clination in the Limbe on the Backſide from the left edge, and in there lay the thread, and in the particular Tangent it ſhewes the Arke ſought.

So if an Eaſt or Weſt plain recline or incline 60 deg. the ſubſtiles diſtance will be found to be 32 deg. 12′.

2 *The ſtiles height.*

Account the $^{Re}_{in}$clination in the particular Sine on the foreſide and in the Limbe it ſhewes the ſtiles height, which for the former Example will be found to be 42 deg. 41′.

3 *The inclination of Meridians.*

The Proportion is, As the Sine of the Latitude, to Radius.
So is the ſine of the ſubſtiles diſtance.
To the ſine of the inclination of Meridians, when the ſubſtiles diſtance is leſſe then the Latitude of the place it may be found in the particular ſine on the foreſide, by the interſection of the thread, and for this Example will be 42 deg. 53′.

4 *The Angle of 12 and 6.*

Account the Complement of the $^{Re}_{in}$clination in the peculiar hour Scale as a ſine, and juſt againſt it in the annexed Tangent ſtands the Complement of the Angle ſought, in this Example the Angle of 12 and 6 is 68 deg. 20′.

In other Latitudes the Operations muſt be performed by Proportional worke with the Compaſſes.

Of the Lines derived from Mr. Gunters Sector.

Such are the Lines of superficies Solids, &c.

Of the Line of Superficies or Squares.

THe chiefe uses of this Line joyntly with the Line of Lines in the Limbe, is when a square number is given to find the Root thereof, or a Root given to find the square number thereto, these Lines placed on a quadrant will perform this some what better then a Sector, because it is given by the Intersection of the thread without Compasses, the properties of the quadrant casting these lines large where on a Sector they would be narrow.

To find the square Root of a number.

The Root being given to find the Square Number of that Root.

IN extracting the square Root pricks must be set under the first third, fift, and seventh figure, and so forward and as many pricks as fall to be under the square number given, so many figures shall be in the Root, and accordingly the line of lines, and superficies must vary in the number they represent, I am very unwilling to spend any time about these kind of Lines, as being of small performance, and by my self and almost by all men accounted meere toyes.

If a number be given in the superficies, the thread in the lines sheweth the Root of it, and the contrary, if a number be given in the lines the thread laid over it intersects the Square thereof.

The performance thereof by these lines is so deficient that I shall give no Example of it.

When a number is given to find the square thereof, if not to large the Reader may correct the last figure of it by multiplying it in his memory.

To three numbers given to find a fourth in a Duplicated Proportion.

That is to worke a Proportion between Numbers and Squares.

Example.

If the Diameter of a Circle whose *Area* is 154 be 14, what shall the Diameter of that Circle be whose *Area* is 616.

Example.

Lay the thread over 616 in the superficies, and from 14 in the equal parts, take the nearest distance to it, then lay the thread to 154 in the superficies, and enter the former extent between the thread and the Scale, and the foot of the Compasses will rest upon 28 the diameter sought.

To find a Proportion between two or more like superficies.

ADmit there be two Circles, and I would know what Proportion their *Areas* bear to each other, in this case the proper use of a Line of superficies would be to have it on a ruler, and to measure the lengths of their like sides, for Circles the lengths of their Diameters upon it, and then I say, the numbers found on the superficies beare such Proportion each to other as the *Areas* or superficial contents, and for small quantities may be done on the quadrant by entring downe the larger extent of the Compasses on the Line of Lines from the Center, and mind the point of limitation, enter then the other extent on the point of limitation, and lay the thread to the other foot, find what number it cuts in the superficies, and the greater shall beare such Proportion to the lesser as 100, &c. the length of the whole line doth to the parts cut.

The Proportion that two superficies beare each to other is the same that the squares of their like sides, and therefore their sides may be measured either in foot or inch measure, and then the Squares taken out as before shewed.

The line of superficies serves for the reducing of Plots to any proportion.

ADmit a Plot of a piece of ground being cast up containes 364 Acres, and it were required to draw another Plot which being cast up by the same Scale should containe but a quarter so much, and let one side of the said Plot be 60 inches, against 60 in the lines, the square of it will be found to be 3600, and the fourth part hereof would be 900, which account in the superficies and you will find the Square Root of it to be 30, and so many inches must be the like side of the lesser Plot if being cast up by the same Scale it should containe but $\frac{1}{4}$ of what it did before.

If the line of Superficies were on a streight ruler, then to perform such a Proposition as this, would be to measure therewith the side of the Plot given, minding what number it reaches to in the Superficies, the fourth part of the said Number being reckoned on the Superficies, and thence taken shall be the length of the side in the Proportion required.

Of the Line of Solids.

IF a number be duly estimated in the said line, and the thread laid over it, it will in the line of lines shew the cube Root of that number, and the converse the Root being assigned, the Cube may be found, but by reason of the sorry performance of these Lines I shall spend no time about it, if this line be placed on a loose Ruler, and the like sides of two like Solids be measured therewith, those Solids shall beare such Proportion in their contents each to other as the measured lengths on the Solids.

Three Numbers being given to find the fourth in a Duplicated Proportion.

Example.

IF a Bullet of 4 inches Diameter weigh 9 pound, what shall a Bullet of 8 inches Diameter weigh? Answer 72 pounds.

In-

In this caſe let the whole line of Solids repreſent 100, alwayes the Solid content whether given or ſought, muſt be accounted in the line of Solids, and the Sides or Diameters in the Equall parts.

Lay the thread to 9 in the line of Solids, and from 8 in the inches take the neareſt diſtance to it, enter one foot of that extent at 4 in the inches, and lay the thread to the other foot: and it will lye over 72 in the Solids for the weight of the Bullet ſought.

An Example of the Converſe.

If a Bullet whoſe Diameter is 4 Inches weigh 9 pound, another Bullet whoſe weight is 40 pound, what ſhall be the Diameter of it.

Lay the thread to 40 in the Solids, and from 4 Inches in the lines take the neareſt diſtance to it.

Then lay the thread to 9 in the Solids, and enter the ſaid extent at the equal Scale, ſo that the other foot turned about may but juſt touch the thread, and it it will reſt at $6\frac{1}{2}$ Inches neareſt, which is the Diameter ſought.

Of the Line of inſcribed Bodies.

This Line hath theſe letters ſet to it.

D		*Dodecahedron*
S		
I	*Signifying the Sides of a*	*Icoſahedron*
C		*Cube*
O		*Octohedron*
T		*Tetrahedron*

And the Letter S Signifieth the Semidiameter of a Sphere, the uſe whereof are to find the Sides of the five Regular Bodies that may be inſcribed in a Sphere.

Example.

A joyner being to cut the 5 Regular Bodies desires to know the lengths of the sides of the said 5 Regular Bodies that may be inscribed in a Sphere where Diameter is 6 inches.

Lay the thread over S and take 3 inches out of the line of equal parts or Inches, and enter that extent so that one foot resting on the said Scale of inches, the other turned about may but just touch the thread, the resting point thus found, I call the point of entrance, from the said point take the nearest distances to the thread laid over the Letters.

		Inch.	Dec. parts
D	*And measure those Extents on the Line of Inches, and you will find them to reach to*	2	.13
I		3	.15
C		3	.45
O		4	.23
T		4	.86

Which are the Dimensions of the respective sides of those Bodies to which the Letters belong.

The uses of the Lines of quadrature, Segments, Mettals and Equated Bodies, I leave to the Disquisition of the *Reader*, when he shall have occasion to put them in practice, which I think will be seldome or never, and wherein the assistance of the Pen will be more commendable.

These lines were added to this quadrant to fill up spare room, and to shew that what ever can be done on the Sector, may be performed by them on a quadrant.

A Table

A TABLE

Of the Latitude of the most eminent Places in *England*, *Wales*, *Scotland* and *Ireland*.

	d.	m.		d.	m.
Bedford	52	8	*Reading*	51	28
Barwick	55	54	*Salisbury*	51	4
Bristol	51	27	*Shrewsbery*	52	47
Buckingham	52		*Stafford*	52	52
Cambridge	52	12	*Stamford*	52	38
Canterbury	51	17	*Truero*	50	30
Carlisle	55		*Warwick*	52	20
Chichester	50	48	*Winchester*	51	3
Chester	53	16	*Worcester*	52	14
Colchester	51	58	*Yorke.*	53	58
Derby	52	58			
Dorchester	50	40	WALES	d.	m.
Durham	54	50	*Anglezey*	53	28
Exceter	50	43	*Barmouth*	52	50
Gilford	51	12	*Brecknock*	52	1
Gloucester	51	53	*Cardigan*	52	12
Hartford	51	49	*Carmarthen*	51	56
Hereford	52	7	*Carnarvan*	53	16
Huntington	52	19	*Denbigh*	53	13
Ipswich	52	8	*Flint*	53	17
Kendal	54	23	*Llandaffe*	51	35
Lancaster	54	10	*Monmouth*	51	51
Leicester	52	40	*Montgomeroy*	51	56
Lincolne	53	14	*Pembrooke*	51	46
London	51	32	*Radnor*	52	19
Northampton	52	14	*St. David*	52	00
Norwich	52	42			
Nottingham	53		The ISLANDS.	d.	m
Oxford	51	46	*Garnzey*	49	30

	d.	m.
Jersey	49	12
Lundy	51	22
Man	54	24
Portland	50	33
Wight Isle.	50	39
SCOTLAND.	d.	m.
Aberdean	57	32
Dunblain	56	21
Dunkel	56	48
Edinburgh	55	56
Glascow	55	52
Kintaile	57	44
Orkney Isle	60	6
St. Andrewes	56	39
Skirassin	58	36
Sterling.	56	12
IRELAND.	d.	m.
Antrim	54	38
Arglas	54	10
Armach	54	14
Caterlagh	52	41
Clare	52	34
Corke	51	53
Droghedah	53	38
Dublin	53	13
Dundalk	53	52
Galloway	53	2
Youghal	51	53
Kenny	52	27
Kildare	53	00
Kings towne	53	8
Knock fergus	54	37
Kynsale	51	41
Lymerick	52	30
Queens towne	52	52
Waterford	52	9
Wexford.	52	18

A Table

A Table of the right Ascensions and Declinations of some of the most principal fixed Stars for some yeares to come.

	R. Ascension.		*Declination.*			*Magnitude*
	H	m	D.	m.		
Pole Star	00	31	87	34	N	2
Andromedas Girdle	00	50	33	50	N	2
Whales Belly	01	35	12		S	3
Rams head	1	48	21	49	N	3
Whales mouth	2	44	2	42	N	2
Medusas head	2	46	39	35	N	3
Perseus right side	2	59	48	33	N	2
Buls eye	4	16	15	46	N	1
Goat	4	52	45	37	N	1
Orions left foot	4	58	8	38	S	1
Orions left shoulder	5	6	5	59	N	3
First, in Orions girdle	5	15	00	35	S	3
Second, in Orions girdle	5	19	1	27	S	3
Third, in Orions girdle	5	23	2	9	S	3
Orions right shoulder	5	36	7	18	N	2
The Wagoner	5	39	44	56	N	2
Bright foot of the Twins	6	18	16	39	N	3
Great Dog	6	30	16	13	S	1
Castor or Apollo	7	12	32	30	N	2
The little Dog	7	22	6	6	N	2
Pollux or Hercules	7	24	28	48	N	2
Hidra's heart	9	10	7	10	S	1
Lions heart	9	50	13	39	N	1
Lions Neck	9	50	21	41	N	3
Great Beares rump	10	40	58	43	N	2
Lions back	11	30	22	4	N	2
Lions tail	11	31	16	30	N	1
The Virgins girdle	12	38	5	20	N	3
First in the great Bears taile next the rump	12	38	57	51	N	2
Vindemiatrix	12	44	15	51	N	3
Virgins Spike	13	7	9	19	S	1

Middle-

Names.	*R. Ascension.*		*Declination.*	*Magnitude*
	H	m	D. m.	
Middlemoſt in the Great Beares tail	13	10	56 45 N	2
Laſt in the end of the Great Beares tail	13	34	51 05 N	2
Arcturus	14	00	21 03 N	1
South Ballance	14	32	14 33 S	2
Brighteſt in the Crown	15	24	27 43 N	3
North Ballance	14	58	08 03 S	3
Serpentaries left hand	15	56	02 46 S	3
Scorpions heart	16	08	25 35 S	1
Serpentaries left knee	16	18	09 46 S	3
Serpentaries right knee	16	49	15 12 S	3
Hercules head	16	59	14 51 N	3
Serpentaries head	17	19	12 52 N	3
Dragons head	17	48	51 36 N	3
Brighteſt in the Harp	18	25	38 30 N	1
Eagle or Vultures heart	19	34	08 00 N	2
Upper horn of Capricorn	19	58	13 32 S	3
Swans tail	20	30	44 05 N	2
Left ſhoulder of Aquarius	21	13	07 02 S	3
Pegaſus mouth	21	27	08 19 N	3
Right ſhoulder of Aquarius	21	48	01 58 S	3
Fomahant	22	39	31 17 S	1
Pegaſus upper Wing, or *Marchab*	22	48	13 21 N	2
Pegaſus Lower Wing.	23	55	33 25 N	2

A Table

Mr. Sutton knowing that ſome of the Tables of *Declination* and *Right Aſcenſion* in our *Engliſh Books* are antiquated and removed forward, took the pains to Calculate a new Table of *Right Aſcenſions* and *Declinations* to ſerve for the future, in regard I was not at leiſure to accompliſh it; which followeth.

A Table of the Suns Right Afcenfion and

Dayes.	*January* ☉ R. A. H.	*January* ☉ R. A. M.	*January* ☉ Decl. D.	*January* ☉ Decl. M.	*February* ☉ R. A. H.	*February* ☉ R. A. M.	*February* ☉ Decl. D.	*February* ☉ Decl. M.	*March* ☉ R. A. H.	*March* ☉ R. A. M.	*March* ☉ Decl. D.	*March* ☉ Decl. M.
1	19	35	21	46	21	42	13	49	23	28	3	27
2	19	39	21	36	21	46	13	29	23	32	3	03
3	19	43	21	25	21	50	13	08	23	36	2	39
4	19	47	21	14	21	54	12	48	23	39	2	16
5	19	51	21	03	21	58	12	28	23	43	1	52
6	19	56	20	52	22	02	12	06	23	46	1	29
7	20	00	20	40	22	06	11	45	23	50	1	05
8	20	04	20	27	22	10	11	24	23	53	0	41
9	20	09	20	15	22	14	11	03	23	57	0	18
10	20	13	20	01	22	17	10	41	0	01	*North*	6
11	20	17	19	48	22	21	10	19	0	05	0	30
12	20	22	19	34	22	25	9	57	0	08	0	53
13	20	26	19	20	22	29	9	35	0	12	1	17
14	20	30	19	05	22	33	9	13	0	15	1	41
15	20	34	18	50	22	36	8	51	0	19	2	04
16	20	38	18	35	22	40	8	26	0	23	2	28
17	20	42	18	19	22	44	8	06	0	26	2	51
18	20	46	18	03	22	48	7	43	0	30	3	15
19	20	50	17	47	22	52	7	20	0	33	3	38
20	20	54	17	30	22	55	6	57	0	37	4	01
21	20	58	17	13	22	59	6	34	0	41	4	24
22	21	03	16	56	23	03	6	11	0	44	4	48
23	21	07	16	39	23	06	5	48	0	48	5	11
24	21	11	16	21	23	10	5	24	0	52	5	34
25	21	15	16	03	23	13	5	01	0	55	5	57
26	21	19	15	44	23	17	4	37	0	59	6	19
27	21	23	15	26	23	21	4	14	1	03	6	42
28	21	27	15	07	23	25	3	51	1	06	7	04
29	21	31	14	48					1	10	7	27
30	21	35	14	28					1	14	7	49
31	21	38	14	09					1	17	8	11

Dayes.	April.				May				June.			
	☉ R.A.		☉ Decl.		☉ R.A.		☉ Decl.		☉ R.A.		☉ Decl.	
	H.	M.	D.	M.	H.	M.	D.	M.	H.	M.	D.	M.
1	1	21	8	33	3	14	18	03	5	19	23	11
2	1	25	8	55	3	18	18	18	5	23	23	15
3	1	29	9	17	3	22	18	33	5	27	23	19
4	1	33	9	38	3	26	18	48	5	31	23	22
5	1	36	9	51	3	30	19	02	5	36	23	24
6	1	40	10	21	3	34	19	16	5	40	23	26
7	1	44	10	42	3	38	19	29	5	44	23	28
8	1	47	11	03	3	42	19	42	5	48	23	29
9	1	51	11	24	3	46	19	55	5	52	23	30
10	1	54	11	44	3	50	20	08	5	56	23	31
11	1	58	12	05	3	54	20	20	6	00	23	31½
12	2	02	12	24	3	58	20	32	6	04	23	31
13	2	06	12	45	4	02	20	44	6	08	23	30
14	2	10	13	04	4	06	20	55	6	12	23	29
15	2	13	13	24	4	10	21	05	6	17	23	28
16	2	17	13	43	4	14	21	16	6	21	23	26
17	2	21	14	02	4	18	21	26	6	25	23	24
18	2	25	14	21	4	22	21	36	6	29	23	21
19	2	29	14	40	4	26	21	45	6	33	23	18
20	2	32	14	58	4	30	21	54	6	38	23	14
21	2	36	15	16	4	34	22	02	6	42	23	11
22	2	40	15	34	4	38	22	11	6	46	23	06
23	2	44	15	52	4	42	22	19	6	50	23	01
24	2	48	16	09	4	46	22	26	6	54	22	56
25	2	51	16	27	4	50	22	33	6	58	22	51
26	2	55	16	43	4	54	22	40	7	02	22	45
27	2	59	17	00	4	58	22	46	7	06	22	39
28	3	03	17	16	5	02	22	52	7	10	22	32
29	3	07	17	32	5	06	22	57	7	14	22	25
30	3	10	17	48	5	11	23	02	7	19	22	17
31					5	15	23	07				

Dayes.	July				August				September			
	☉ R. A.		☉ Decl.		☉ R. A.		☉ Decl.		☉ R. A.		☉ Decl.	
	H.	M.	D.	M.	H.	M.	D.	M.	H.	M.	D.	M.
1	7	23	22	09	9	25	15	16	11	19	4	28
2	7	27	22	01	9	29	14	58	11	23	4	6
3	7	31	21	52	9	33	14	39	11	26	3	42
4	7	35	21	43	9	37	14	21	11	30	3	19
5	7	39	21	34	9	40	14	02	11	33	2	56
6	7	43	21	24	9	44	13	43	11	37	2	33
7	7	47	21	14	9	48	13	24	11	41	2	10
8	7	51	21	04	9	51	13	04	11	44	1	46
9	7	55	20	53	9	55	12	45	11	48	1	23
10	7	59	20	42	9	58	12	25	11	51	0	59
11	8	03	20	30	10	02	12	05	11	55	0	36
12	8	07	20	18	10	06	11	45	11	59	0	12
13	8	11	20	06	10	10	11	25	12	02	South	11
14	8	15	19	54	10	14	11	04	12	06	0	35
15	8	19	19	41	10	17	10	43	12	09	0	58
16	8	23	19	28	10	21	10	22	12	13	1	22
17	8	27	19	14	10	25	10	01	12	17	1	46
18	8	31	19	00	10	28	9	40	12	20	2	09
19	8	35	18	46	10	32	9	18	12	24	2	33
20	8	39	18	32	10	35	8	57	12	27	2	56
21	8	43	18	17	10	39	8	35	12	31	3	19
22	8	47	18	02	10	43	8	14	12	35	3	43
23	8	51	17	46	10	46	7	52	12	38	4	06
24	8	55	17	31	10	50	7	30	12	42	4	30
25	8	58	17	15	10	53	7	07	12	45	4	53
26	9	02	16	59	10	57	6	45	12	49	5	16
27	9	06	16	42	11	01	6	22	12	53	5	39
28	9	10	16	25	11	04	6	00	12	57	6	02
29	9	14	16	08	11	08	5	37	13	01	6	26
30	9	17	15	51	11	11	5	14	13	04	6	49
31	9	21	15	33	11	15	4	51				

Dayes.	*October*				*November*				*December.*			
	☉ *R. A.*		☉ *Decl.*		☉ *R. A.*		☉ *Decl.*		☉ *R. A.*		☉ *Decl.*	
	H.	*M.*	*D.*	*M.*	*H.*	*M.*	*D.*	*M.*	*H.*	*M.*	*D.*	*M.*
1	13	08	7	11	15	07	17	38	17	15	23	08
2	13	12	7	34	15	11	17	54	17	20	23	13
3	13	15	7	57	15	15	18	10	17	25	23	17
4	13	19	8	19	15	19	18	26	17	29	23	20
5	13	22	8	42	15	23	18	41	17	34	23	23
6	13	26	9	04	15	27	18	56	17	38	23	26
7	13	30	9	26	15	31	19	11	17	42	23	28
8	13	34	9	48	15	36	19	26	17	47	23	29
9	13	38	10	10	15	40	19	40	17	51	23	30
10	13	41	10	31	15	45	19	53	17	56	23	31
11	13	45	10	53	15	49	20	07	18	00	23	31½
12	13	49	11	14	15	53	20	19	18	05	23	31
13	13	53	11	36	15	58	20	32	18	09	23	30
14	13	57	11	57	16	02	20	44	18	14	23	29
15	14	00	12	18	16	07	20	56	18	19	23	27
16	14	04	12	38	16	11	21	08	18	24	23	25
17	14	08	12	59	16	15	21	19	18	28	23	22
18	14	12	13	19	16	19	21	29	18	33	23	19
19	14	16	13	39	16	23	21	39	18	37	23	15
20	14	20	13	59	16	28	21	49	18	41	23	11
21	14	24	14	19	16	32	21	58	18	45	23	07
22	14	28	14	38	16	36	22	08	18	49	23	02
23	14	32	14	57	16	40	22	16	18	54	22	56
24	14	36	15	16	16	44	22	24	18	58	22	50
25	14	39	15	35	16	49	22	32	19	03	22	43
26	14	43	15	53	16	53	22	39	19	07	22	36
27	14	47	16	11	16	57	22	46	19	11	22	29
28	14	51	16	29	17	02	22	52	19	16	22	21
29	14	55	16	47	17	06	22	58	19	20	22	13
30	14	59	17	04	17	11	23	03	19	25	22	04
31	15	03	17	21					19	30	21	55

A Rectifying Table for the Suns Declination.

	Years		Years		Years	
	1657 1661 1665 1669 1673		1659 1663 1667 1671 1675		1660 1664 1668 1672 1676	
Moneths	*min.*		*min.*		*min.*	
January	3	s	2	a	5	a
	4	s	3	a	7	a
	5	s	4	a	9	a
February	5	s	5	a	10	a
	5	s	5	s	11	a
	6	s	5	a	11	a
March	6	s	5	a	13	s
	5	a	5	s	12	a
	5	a	5	s	12	a
April	5	a	5	s	11	a
	5	a	5	s	10	a
	4	a	4	s	9	a
May	4	a	4	s	8	a
	3	a	3	s	6	a
	2	a	2	s	4	a
June	1	a	1	s	2	a
	0	s	0	a	0	s
	1	s	1	a	3	s

	Years		Years		Years	
	1657 1661 1665 1669 1673		1659 1663 1667 1671 1675		1660 1664 1668 1672 1676	
Moneths	*min.*		*min.*		*min.*	
July	2	s	2	a	5	s
	3	s	3	a	7	s
	4	s	4	a	9	s
August.	5	s	5	a	10	s
	5	s	5	a	11	s
	6	s	5	a	12	s
Septẽber	6	s	5	a	13	s
	6	a	5	s	13	a
	6	a	5	s	12	a
October	6	a	5	s	12	a
	5	a	5	s	11	a
	4	a	5	s	9	a
Novem.	3	a	4	s	7	a
	2	a	3	s	5	a
	1	a	2	s	3	a
Decemb.	0	a	1	s	1	a
	1	s	0	a	1	s
	2	s	1	a	3	s

The use of the Rectifying Table.

NOte that the minutes under the reſpective years is to be added or ſubſtracted to or from the Suns Declination in the former Table, as is noted with the letter a or s: and alſo note that the firſt figure in each moneth ſtands for the firſt 10 dayes of the moneth, and the ſecond for the ſecond 10 days, & the third for the laſt 10 dayes, except in *March* or *September*, which in *March* will be the firſt 9 dayes only, and in *September* the firſt 12 dayes.

Example.

I would know the Suns Declination the 15 day of *May* 1668. Now becauſe this day of the moneth falls in the ſecond 10 dayes, I look in the Table under the year 1668, and right againſt *May* you ſhall finde that in the ſecond place of the moneth ſtands 6 a, which ſhews me that I muſt adde 6 minutes to the Suns Declination in the former Table 21 degrees 5 min. that ſtands againſt the 15 day of *May*, and then I find that the Sun will have 21 deg. 11 min. of North Declination, and ſo for the reſt, which will never differ above two minutes from the truth, but ſeldome ſo much, and for the moſt part true.

Note that the former Table of the Suns Declination is fitted exactly for the year 1666. by the Rules Mr. Wright *gives in his* Correction of Errours, *and from his Tables, and may indifferently ſerve for the years* 1658, 1662. 1670, 1674, *without any ſenſible errour, and the* T*able of Right Aſcenſions will not vary a minute of time in many years.*

FINIS.

Errours in the Horizontal Quadrant.

PAge 5 line 6 in an Italian letter ſhould not have been diſtinct, nor in another letter from the former line. page 5. line 9. for quarter, read half. p. 5. l. 13. *r.* of a quadrant. p. 11. l. 7. *r.* 63*d.* 26′. p. 19. l. 7. *r.* the ſame day to. p. 23. l. 17. *r.* and ends at 32′ paſt 9. p. 27. l. 7. *for* N R, *r.* N Z. p. 28. l. 4. *r.* in the parallel. p. 30. l. 9, & l. 10. *r.* 23*d.* 31′. p. 38. l. 4. *r.* Is to the ſine. p. 50. l. 5. *r.* whereof the Diameter.

AN APPENDIX Touching *REFLECTIVE DIALLING.*

By JOHN LYON:

Professor of this, or any other part of the Mathematicks, *neer* Sommerset *House in the* Strand.

LONDON,
Printed *Anno Domini,* 1658.

DIRECT DIALLING

By a Hole or Nodus.

To draw a Dial under any window that the Sun shines upon by help of a thread fastened in any point of the direct Axis found in the Ceiling, and a hole in any pane of glasse, or a knob or Nodus upon any side of the window or window-post.

Constructio.

First, draw on pastboard or other material, an Horizontal Dial for the Latitude proposed.

Then by help of the Suns Azimuth, which may be found by help of a general Quadrant, at any time, or by knowing the true hour of the day with the help of the said Horizontal Dial: and draw that true Meridian from the hole or Nodus proposed, both above on the Cieling, and below on the walls and floor of the Room; so that if a right line were extended from the said hole or Nodus by any point in any of those lines, it would be in the meridian Circle of the World.

To finde a point in the direct Axis of the world, which will ever fall to be in the said Meridian, in which point the end of a thread is to be fastened.

FIrst, fix the end of a thread or small silk in the center of the Hole or Nodus, and move the other end thereof up or down in the said meridian formerly drawn on the Cieling or wall, untill by applying the side of a Quadrant to that thread, it is found to be elevated equal to the Latitude of the place; so is that thread directly scituated parallel to the Axis of the world, and the point where the end of that thread toucheth the meridian either on the Cieling or wall, is that point in the direct Axis sought for, wherein fix one end of a thread, (which thread will be of present use in projecting of hour-points in any place proposed, then:

To find the Hour-points either under the window, or any other convenient place in the Room.

Place the center of the said Horizontal Dial in the Center of the Hole or Nodus; also scituate the said Dial exactly parallel to the Horizon, and the meridian of the said Dial in the meridian of the world, which (as before) may easily be done, if at that instant you know the true hour of the day.) Then take the thread whose end is fixed in a point in the direct Axis, and move it to and fro, until the said thread doth interpose between your eye, and the hour-line on the said Horizontal Dial which you intend to draw, and then keeping your eye at that scituation, make a point or mark in any place where you please, or under the window, so that the said thread or string may interpose between that point or mark so made, and your eye, as aforesaid; which said point so found will shew the true time of the day at that hour all the year long, the Sun shining thereon, so will that point, together with the said thread, serve to shew the hour, instead of an hour-line.

In like manner, the said thread fixed in the Axis may be again moved to and fro, until the said thread doth interpose between the

eye

eye and any other hour-line desired on the said Horizontal Dial and then (as before) make another point or mark in any place at pleasure, or under the said window, by projecting a point from the eye, so that the said thread also interpose between that point to be made and the eye, so will that point so found shew the true time of the day for the same hour that did the hour line on the said Horizontal Dial, which was shadowed by the said thread.

In like manner may be proceeded (by help of that thread, and the several hour-lines on the said Horizontal Dial) to finde the other hour-points which must have the same numbers set to them as have the hour lines on the said Horizontal Dial.

Otherwise to make a Dial from a hole in any pane of glasse in a window, and to graduate the hour-lines below on the Sell, or Beam, or on the ground; that hole is supposed to be the center of the Horizontal Dial, and being true placed, the stile thereof, if supposed continued, will run into the point in the Meridian of the Cieling before found, where a thread is to be fixed; then let one extend a thread fastned in the center of the Horizontal Dial parallelly to the Horizon, over each respective hour-line, and holding it steady, let another extend the thread fastened in the Meridian, in the Cieling along by the edges of the former Horizontal thread, and so this latter thread will finde divers points on the ground, through which if hour-lines be drawn, and the Sun shine through the hole in the pane of Glasse before made, the spot of the Sun on the ground shall shew the time of the day.

For the points that will be thus found on the Beam or Transome, the thread fixed in the Cieling, or instead of it a piece of tape there fixed must be moved so up and down, that the spot of the Sun may shine upon it; and being extended to the Transome or Beam graduated with the hour-lines, as before directed, it there shews the time of the day. Here note, that it will be convenient to have that pane of Glasse darkened through which that spot is to shine.

In like manner may a Dial be made from a nail head, a knot in a string tied any where a crosse, or from any point driven into the

bas

bar of a window, and the hour-lines graduated upon the Transome or board underneath.

To make a Reflected Dial on the Ceiling of the Room is onely the contrary of this, by ſuppoſing the Horizontal Diall with its ſtile to be turned downwards, and run into the true meridian on the ground, where the thread is to be fixed, and to be extended along by the former Horizontal thread (held over the reſpective hours as before) upward, to find divers points in the Cieling, as ſhall afterwards be ſhewed.

Of Dials to ſtand in the Weather.

These may be alſo made by help of an Horizontal Dial.

DRive two nails or pins into the wall, on which the edge of a Board of competent breadth may reſt, then to hold up the other ſide of the Board, drive two hooks into the wall above, whereto with cord or line the outſide of the Board may be ſuſtained, and this Board being Horizontal, place the Horizontal Dial its Meridian-line in the true Meridian of the world. If a Plain look towards the South, the ſtile of the Horizontal Dial continued by a thread from the center will run into the Plain, which note to be the center of the new Dial, as alſo that line is the new ſtile, which muſt be ſupported with ſtayes, when you fix it up.

By a thread from the center laid over every hour-line on the Horizontal Dial, croſs the Horizontal line of the Plain, which note with the ſame hours the Horizontal Dial hath.

The hour-lines on the Plain are to be drawn from the center before found through thoſe points, and ſo cut off by the Dial, or continued at pleaſure.

If the Center of the Dial be aſſigned before you begin the work, in ſuch Caſes you may remove the Horizontal Dial up and down, keeping it ſtill to the true poſition or hour, till you finde the Axis or ſtile run into the Center.

But

But if the Plain look into the East or West, then possibly the Axis of the Horizontal Dial will not meet with the Plain: in such Cases you must fix a board so, that it may receive the Axis, (the board being perpendicular to the Plain) this stile or Axis is to be fastened to the Plain by two Rests, the hour-lines may be drawn by the eye, or shadowed out by a Light: Bring the thread that represents the Axis or stile into any hour-point (on the Horizontal Dial) by your eye or shadow; at the same time the thread or shadow making marks on the Plain, shews where the hour-lines are to passe.

After the same manner any hour-line is to be drawn over any irregular or crooked Plain. Further observe, that any point in the middle, or neer the end of the stile will as well shew the hour of the Day, as the whole stile.

Of Refracted Dials.

IF you stick up a pin or stick, or assign any point in any concave Boul or Dish, to shew the hour, and make that the center of the Horizontal Dial, assigning the meridian-line on the edges of the Boul, point out the rest of the hour-lines also on the edges of the Boul, and taking away the Horizontal Dial, elevate a string or thread from the end of the said pin fastned thereto over the Meridian-line equal to the Elevation of the Pole or the Latitude of the place; then with a candle, or if you bring the thread to shade upon any hour-point formerly marked out on the edges of the Boul, at the same time the shade in the Boul is the hour line.

And if the Boul be full of water, or any other liquor, you may draw the hour-lines, which will never shew the true hour, unlesse filled with the said Liquor again.

Reflected Dialling.

To draw a Reflected Dial on any Plain or Plains, be they never ſo Gibous, and Concave, or Convex, or any irregularity whatſoever, the Glaſs being fixed at any Reclination at pleaſure, (provided it may caſt its Reflex upon the places propoſed.) Together with all other neceſſary lines or furniture thereon, viz. *the Parallels of Declination, the Azimuth lines, the Parallels of Altitude (or proportions of ſhadows) the Planetary Hour-lines, and the* Cuſpis *of thoſe Houſes which are above the Horizon, &c.*

1. *If the Glaſſe be placed Horizontal upon the Tranſome of a window, or other convenient place:*

How upon the Wall or Cieling whereon that Glaſſe doth reflect to draw the Hour-lines thereon, although it be never ſo irregular, or in any form whatſoever.

CONSTRUCTIO.

FIrſt, draw on Paſtboard or other Material an Horizontal Dial for the Latitude propoſed.

Then by help of the Azimuth, or at the time when the Sun is in the Meridian; or by knowing the true hour of Day, whereby may be drawn ſeveral lines on the Cieling, Floor, and Walls of the Room: ſo as in reſpect of the center of the Glaſſe they may be in the true Meridian-circle of the World: For if right lines were extended from the center of the ſaid Glaſſe by any point, though elevated in any of thoſe lines ſo drawn, it would be directly in the Meridian Circle of the World.

Now

Now all Reflective Dialling is performed from that principle in *Opticks*, which is, *That the angle of Incidence is equal to the angle of Reflection.* And as any direct Dial may be made by help of a point found in the direct Axis, so may any Reflected Dial be also made by help of any point found in the Reflected Axis.

And in regard the reflected Axis for the most part will fall above the Horizon of the Glasse without the window, so that no point there can be fixed, therefore a point must be found in the said Reflected Axis continued below the Horizontal of the said Glasse, until it touch the ground or floor of the Room in some part of the Meridian formerly drawn, which point will be the point in the reversed Axis desired, and may be found, as followeth.

One end of the thread, being fixed at or in the center of the said Glasse, move the other end thereof in the meridian formerly drawn below the said Glasse, until the said reversed Axis be depressed below the Horizon, as the direct Axis was elevated above the Horizon, which may be done by applying the side or edge of a Quadrant to the said thread, and moving the end thereof to and fro in the said meridian, until the thread with a plummet cut the same degree as the Pole is above the Horizontal Glasse, and then that point where the end of the thread toucheth the Meridian either on the floor or wall of the room, is the point in the reflected reversed Axis sought for.

Now if the Reversed Axis cannot be drawn from the Glasse by reason of the jetting of the window or other impediment, that point in the reverse Axis may be found by a line parallel thereto, by fixing one end of it on the Glasse, and the other end in the meridian, so as that it may be parallel to the floor or wall in which the reversed Axis-point will fall, and finde the Axis point from that other end of the lath: so if the same Distance be set from that point backward in the Meridian on the floor, as is the Lath, the point will be found in the Reversed Axis desired.

Thus having found a point in the reflected reversed Axis; it is not hard, by help whereof and the Horizontal Dial, to draw the reflected hour-lines on any Cieling or Wall, be it never so concave or convex.

To do which: First note, that all straight lines in any projecti-on on any Plain, do always represent great Circles in the Sphere, such are all the hour-lines.

Place the center of this Horizontal Dial in the center of the Glasse, the hour-lines of the said Dial being horizontal, and the Meridian of the said Dial in the Meridian of the world, which may be done by plumb lines let fall from the meridian on the Cie-ling: Then fix the end of a thread or silk in the said center of the Dial or Glasse, and draw it directly over any hour-line on the Dial which you intend to draw, and at the further side of the room, and there let one hold or fasten that thread with a small nail.

Then in the point formerly found on the reversed Axis on the floor, fix another thread there (as formerly was done in the cen-ter of the Diall) then take that thread, and make it just touch the thread (on the hour-line of the Horizontal Dial extended) in any point thereof, it matters not whereabouts, and mark where the end of that thread toucheth the wall or Cieling, and there make some mark or point.

Then again move the same thread higher or lower at pleasure, till it, as formerly touch the said same hour thread, and mark a-gain whereabouts on the wall or Cieling, the end of the said thread also toucheth. In like manner may be found more points at pleasure, but any two will be sufficient for the projecting or drawing any hour-line on any plain, how irregular soever. For if you move a thread, and also your eye to and fro, un-til you bring the said thread directly between your eye and the points formerly found, you may project thereby as many points as you please at every angle of the Wall or Cieling, whereby the reflected hour-line may be exactly drawn.

Again, in like manner remove the said thread fastned in the center of the Horizontal Dial, (which also is the center of the Glasse) on any other hour-line desired to be drawn, and as be-fore fasten the other end of the thread, by a small nail, or other-wise at the further side of the room, but so that the said thread may lie just on the hour-line proposed to be drawn on the Hori-

zontal.

zontal Dial. Then (as before) take the thread fastened in the point on the reflected Axis, and bring it to touch the thread of the hour-line in any part thereof, and mark where the end of that thread toucheth the said Wall or Cieling: Then again (as before) move the said thread so, as that it only touch the said thread of the hour-line in any other part thereof, and also mark where the end of that thread toucheth the said Wall or Cieling: So is there found two points on the Wall or Cieling, being in the reflected hour-line desired, by help of which two points the whole hour-line may be drawn; for if (as before) a thread be so scituated, that it may interpose between the eye and the said two points found, you may make many points at pleasure, whereunto the said thread may also interpose, which for more conveniency may be made at every angle or bending of the Wall or Cieling, be they never so many: So that if lines be drawn from point to point, that said reflected hour-line will be also exactly drawn.

In like manner may the other hour-lines be drawn so, that the Reflex or spot of the Sun from the said Horizontal Glasse scituated in the said window (as before) shining amongst the said reflected hour-lines drawn on the wall or Cieling, will exactly shew the hour of the day desired.

Now if lines be drawn round about the said Room, equal to the Horizon of the said Glasse, it will shew when the Sun is in or neer the Horizon.

To draw the Æquator and Tropicks on any Wall or Cieling to any Horizontal reflecting Glasse.

1 *To draw the Reflected Æquator or Equinoctial-line on the Wall or Cieling, which represents a great Circle.*

TAke the thread fixed in the Center of the Glasse, and move the end thereof to and fro in the meridian line drawn on the Cieling, untill by help of a Quadrant the said thread be elevated equal to the complement of the Latitude, (which will be alwayes perpendicular to the reversed Axis) marking in the me-

Meridian where the end of that thread falls, then on that point and the said meridian line on the Cieling erect a perpendicular line, which line may be continued on any plane whatsoever, and is the reflected Equinoctial line desired.

Note that all great Circles are right lines, & are alwayes drawn or projected from a right line.

2. *To draw the* Tropicks. *Note, that all Parallels of Declination are lesser Circles, and are Conick Sections.*

First, make or take out of some Book a Table of the Suns Altitude for each hour of the day, calculated for the place or Latitude proposed, when the Sun is in either of the Tropicks. Then take the thread fixed in the center of the Glasse, and by applying one side of a quadrant to the said thread, and moving one end of it to and fro in the hour-line proposed, elevate the said thread answerable to the Suns height in that hour, when he is in that Tropick you desire to draw, and mark where the end of that thread so elevated toucheth in that hour-line proposed. So may you in like manner finde a several point in each hour-line for the Suns height in that Tropick, whereby a line may be drawn on the Wall or Cieling from point to point formerly made in the said hour-lines, which the Tropick desired.

In like manner may any parallel of Declination be drawn: If there be first calculated a Table of the Suns altitude at all hours of the day, when the Sun hath any Declination proposed, whereby may be drawn either the Parallels of the Suns place, or the parallels of the length of the day.

To draw the parallels of Declination to any Reflected Glasse most easily, by help of a Trigon *first made on past board or other material.*

Fix the Trigon to the reflected reversed Axis, so that the center of the Trigon may be in the center of the Glasse, then will the Equinoctial on the Trigon be perpendicular to the said Axis: then take the thread fixed in the center of the Glasse, and lay it

along

along either of the Tropicks or other parallels of Declination required, which is drawn on the ſaid Trigon, which thread muſt be continued ſo, that the end thereof may touch any hour-line, and on that hour-line mark the point of touch, the thread being ſtill laid on the ſame parallel of declination on the Trigon: in the ſame manner finde a point in each hour-line. Laſtly, draw a line by thoſe points ſo found, which will be the Tropick-line or other parall l of declination, as the thread was laid on, on the Trigon.

To draw the Azimuth-lines on any Wall or Cieling to any Horizontal reflecting Glaſſe. Note that all Azimuths are great Circles.

FIrſt, find a vertical point, either above to the Zenith, or below to the Nadir of the Glaſſe (by ſome called a perpendicular or plumb line) and mark in what point it cuts the floor of the room, which point I call the reflected vertical point, wherein the end of a thread is to be fixed: For by a point found in the reflected Axis of the Horizon the Azimuths may be drawn, as by a point found in the reflected Axis of the Equinoctial the hour-lines may be drawn.

Then on paſtboard or other material draw the points of the Compaſſe or other degrees, and fix the center thereof in the center of the Glaſſe, and the meridian thereof in the meridian of the world, as was ſhewn in drawing the hour-lines, being careful to place it horizontal.

Then take the thread fixed in the place of the glaſſe, and draw it over any Azimuth, which is deſired to be drawn, and at the further ſide of the Room faſten that thread with a ſmall nail as it was in drawing the reflected hour-lines: Then take the thread whoſe end is faſtened in the ſaid reflect vertical point, and bring that thread ſo as juſt to touch the ſaid horizontal thread, and augment it, until the end thereof touch the wall or Cieling, and there make a mark or point. In like manner, move the ſaid thread, whoſe end is faſtened in the ſaid vertical point, higher or lower at

plea-

pleasure, till as formerly it touch the said horizontal thread, and mark again whereabouts the end thereof toucheth the said Wall or Cieling: Now by help of these two points found in the reflected Azimuth line, the whole Azimuth line may be drawn; for if (as before in drawing the Hour-lines) a thread be so scituated, that it may interpose between the eye and the said two points, you may make many points at pleasure, to which the said thread so situated may also interpose, which may be made at every angle or bending of the wall or Cieling (as before) whereby the reflected Azimuth-line desired may be drawn. In like manner may the other reflected Azimuth lines be drawn.

Also there may be lines drawn parallel to the Horizon round about the room, by help of the thread fixed in the center of the Glasse, and a Quadrant for the elevation thereof, which will shew the Suns altitude at any appearance thereof.

Thus have I shewed the drawing of a Reflected Dial from an Horizontal Glasse, with all the usual furniture thereon, though the wall or place on which it is to be drawn be never so gibous or irregular, or in what shape soever.

Now the Glasse may be exactly situated Horizontal, if you draw a reflected parallel for the present day, and know also the true hour, and so place the Glasse, that the spot or reflex of the Sun may fall thereon on the Cieling, for there is no way by an Instrument to do it, the Glasse is so small.

Of Reclining Reflecting Glaſſes.

Reflected Dialling from any Reclining Glaſſe.

I ſhall now ſhew how to draw any Reflected Dial, with all the Furniture (that poſſible may be) the Glaſs being ſet at any poſſible Reclination.

In the drawing of which there is principally to be conſidered,

1 *The Reflected Horizon.* | Note, the Horizon & Meridian are two great circles.
2 *The Reflected Meridian.*

1 *To draw the Reflected Horizon according to the ſituation of any reclining Glaſſe whatſoever.*

FIrſt, let two pieces of nealed wire be faſtened on the window on each ſide of the ſaid Glaſſe, the ends thereof being without the room in the air, at whoſe ends let there be faſtned a thread which may be pulled ſtraight at pleaſure, by bending of the wire, then bend thoſe wires upward or downward, until the thread faſtened at the end of each wire be exactly horizontal with the center of the Glaſſe, which may be tried by a quadrant: Then I tie a ſtring or thread croſs the room, in ſuch ſort that I may from moſt part of the thread ſee the reflecting glaſs, and therein the ſaid horizontal thread without the room: Then on the ſaid thread croſs the room, I tie a ſlipping knot to move to and fro at pleaſure, which knot I move to and fro on the ſaid thread, until by looking in the ſaid Glaſſe I finde from my eye the ſaid knot and part of the horizontal thread without, all as it were in a right line, the one interpoſing the ſight of the other.

Then

Then being careful to keep the knot in that position, fasten one end of a thread in the place of the center of the reclining reflecting glasse, and bring that thread so, as just to touch the aforesaid knot, augmenting that thread, until the end thereof touch the wall or Cieling, and there make a mark or point; so is there one point found on the Wall or Cieling in the Reflected Horizon of the World. Then I begin again, and remove the position of that thread (which went overthwart the Room) either higher or lower at pleasure, still having regard that I may from the most part of the said thread see the Reflecting Glasse, and therein the same horizontal thread without the room. Then, as before, I move the said knot on the said thread to & fro, until (as before) by looking in the said Glasse I find from my eye the said knot, and part of the Horizontal thread both in one right line, the one interposing the sight of the other; and by the said knot I bring that thread, whose end is fastened in the center of the said glasse, and keeping it just to touch the said knot, I continue it, until the end thereof touch the Wall or Cieling, as before, and there I make another mark or point; so is there two points found in the said reflected Horizon on the wall or Cieling. By which said two points, if a thread (as before) be so scituated, that it may interpose between the eye and the said two points, there may be many points made to be in the same interposition of the thread, which (as before) may be made at every bending or angle of the Wall or Cieling, whereby the reflected Horizon desired may be drawn, by drawing a line from point to point round about the Room; Which wil be the true reflected Horizon according to the situation of the glasse.

2 *To draw the Reflected Meridian, according to the situation of any Reclining Glasse whatsoever.*

First, take a lath or thin piece of wood of any convenient length at pleasure, as some one and an half, or two foot long, and at each end thereof make a hole, the one to hang a thread and plummet, and the other is to put a small nail therin to fasten it in some part of the window over the center of the Glasse, so that

that the thread and plummet may hang without the room: then by help of the Suns Azimuth you may draw the meridian line, (as before) as if the Glasse were horizontal, and move the lath with the thread and plummet at the end of it to and fro, until the thread and plummet be in the direct meridian of the world with the center of the Glasse. Then (as before) tie a thread crosse the room, in such sort that from or by some part of the said thread both the Reclining glasse and the thread to which the plummet is fastened may be seen at one time. Then (as before) on the said thread, which crosses the room, I tie a slipping knot, which I move to and fro on the said string, until by looking in the said Glasse I find from my eye the said knot and some part of the perpendicular thread without, all as it were in one right line, the one shadowing or interposing the sight of the other, being then very careful to keep that knot in the same position, then take the thread (whose end whereof being fastened in the said center of the Glasse) and bringing it just to touch the said knot, I augment that thread, until the end thereof touch the said wall or Cieling, and the said thread also touch the knot, as before: then in that place where the end of the said thread toucheth the wall or Cieling, I make a mark, which mark or point will be directly in the reflected meridian of the world, according to the situation of that Glasse. Then again I remove that thread (overthwart the room) on which the said knot is, either higher or lower then it formerly was at pleasure, still having regard that from some part of the said thread within, you may see both the Reclining Glasse, and the perpendicular thread without at one time; and (as before) move the said slipping knot on the said thread, until by looking in the said Reclining Glasse, you see the said knot and some part of the perpendicular thread without in one right line, so as the one shadows or hinders the sight of the other, (as before) which knot then must not be removed from its situation, then take that thread (whose end is fastened in the Glasse) and bring it to touch that knot, the end of the said thread being continued to touch the wall or Cieling: so is that point of touch on the Cieling another

point found in the Reflected Meridian of the world. So is there two points found in the said Reflected Meridian, on the wall or Cieling; by which, if a thread (as before) be so situated, that it may interpose between the eye and the said two points, many points thereby in the said reflected Meridian may be made at every bending or angle of the wall or Cieling, whereby the Reflected meridian desired may be drawn, by drawing a line from point to point obliquely in the Room, which will be the true Reflected Meridian of the world, according to the situation of that Glasse.

Now this Reflected Horizon and Meridian being first drawn, they will be of great use in drawing the Hour-lines, together with all the furniture that possibly can be drawn on any Diall.

To draw the Reflected Hour-lines to any Reclining Glasse on any plane whatsoever, that the Sun will be reflected on: By help of an ordinary Horizontal Dial for that Latitude.

FIrst, extend several threads from the center of the Glasse to the extremity of the Reflected Horizon in the Room (which for more conveniency and use may be the several hour-lines, and may also serve as a bed to situate the Horizontal Diall on the Reflected Horizon) having regard to situate the center of the Dial on the center of the Glasse, and the Meridian of that Dial on the Reflected Meridian of the World: Then to finde the point in the Reflected reversed Axis on the floor of the Room; Take a thread, one end thereof being fastened in the center of the Glasse, and move the other end thereof to and fro in the reflected meridian under the Reflected Horizon, until by help of a Quadrant the said thread is found to be depressed under the reflected Horizon, equal to the latitude of the place, and where the end of the

the ſaid thread interſects or meets the Reflected Meridian either on the floor or wall, that point is the reflected reverſed Axis, as was required. In which point faſten one end of a thread, which thread will be of great uſe in drawing the reflected hour-lines on any wall or Cieling whatſoever. Now if this thread, whoſe end is faſtened in a point on the reflected reverſed Axis, be taken and brought to touch any part of any one of the threads of the hour-lines (produced to and faſtened in the reflected Horizon) the ſaid thread being continued ſo, as the end thereof may touch the wall or Cieling, and alſo any part of the ſaid thread touch the hour-line or thread propoſed; that point on the wall or Cieling is in the reflected hour-line deſired to be drawn: Alſo the other point in the ſame reflected hour-line may be found; If the ſaid thread, whoſe end is faſtened in the Reflected Axis, be brought to touch ſome other part of the ſame hour-thread propoſed; ſo that when (as before) the end of the ſaid thread toucheth the wall or Cieling, ſome part of that thread may alſo touch the hour-line deſired, which point of touch on the wall or Cieling, is alſo another point in the ſaid reflected hour-line deſired. By which two points ſo found (as before) the reflected hour-line may be drawn by a thread, projecting by thoſe points from the eye, as it was formerly directed in drawing the reflected hour-lines to an Horizontal Glaſſe.

To draw the Reflected Equinoctial line, and alſo the Tropicks on any wall or Cieling, to any Reclining Reflecting glaſſe.

1 *To draw the reflected Equinoctial line on the Wall or Cieling.*

TAke that thread, whoſe end is faſtened in the center of the reclining glaſſe, and move the other end thereof to and fro in the ſaid Reflected meridian formerly drawn, until (by help of a quadrant) the ſaid thread is elevated above the reflected Horizon formerly drawn, equal to the Complement of the Latitude,

(which as before will be alwayes perpendicular to the reversed Axis) and make a point in the said reflected meridian, where the end of the said thread toucheth; then on that point and the said reflected meridian on the Cieling, raise a perpendicular line, which is the Reflected Equinoctial line desired.

2. To *draw the reflected* Tropicks, *or other Parallels of Declination.*

FIrst, (as before) make or take out of some Book a Table of the Suns Altitude for each hour of the day, calculated for the place or Latitude proposed, when the Sun is in either of the Tropicks, or other parallel of Declination: then take that thread, whose end is fastened in the center of the Glasse, move the other end thereof to and fro in the hour-line proposed, until by applying one side of a quadrant to the said thread you find the said thread elevated above the reflected Horizon answerable to the Suns height in that hour proposed, when he is in that Tropick or degree of Declination proposed. Which altitude required will be found in the foresaid Table for that end calculated, which said thread being of the elevation above the reflected Horizon, as the said Table directeth: then mark where the end of the thread (so elevated) toucheth the Wall or Cieling in that hour-line: so is one point found in the reflected parallel of Declination desired to be drawn. In like manner, find in the said Table in the same parallel or degree of declination what altitude the Sun hath at the next hour, and elevate the said thread, whose end is fastened in the center of the Glasse, equal to the Suns altitude in that hour above the said reflected Horizon, by help of the said Quadrant, and where the other end of the said thread falleth in the hour-line proposed, make another mark or point. And so in like manner make the points (belonging to that parallel of Declination) in the remaining hour-lines, according to the several Altitudes found in the said Table of Altitudes: Then drawing by hand a line to passe through those several points so found, as before, which line is the reflected-

ed

ed parallel of the Suns declination desired. In like manner may be drawn all or any other parallel of Declination, which may have respect to the Suns place, or the length of the day, as shall be desired.

Or,

To draw the said reflected Tropicks, or other parallels of Declination, without any Tables calculated, only, by help of a Trigon *first made on pastboard or other material.*
Note that all Parallels are lesser Circles.

FIrst (as formerly is shewd in drawing the parallels of Delination to a Reflecting Horizontal Glasse) fasten the Trigon on the reflected reversed Axis, so that the center of the Trigon may be in the center of the Glasse, then also will the Equinoctial on the Trigon be perpendicular to the said reflected reversed Axis: then take the thread fixed in the center of the said Glasse, (which is also in the center of the Trigon) and lay it upon that parallel of Declination, drawn on the said Trigon, whose reflected parallel is required to be drawn on the plane or Cieling: then move the Trigon, the thread lying on the said parallel, until the end of the said thread touch any hour-line on the said wall or Cieling, in which point of touch on that hour-line make a mark, so will that point be in the reflected parallel of Declination desired. In like manner, move the said Trigon, still keeping the thread on the same parallel, until the end of that thread touch another hour-line on the said plane or Cieling, and there also make another mark. And so in like manner find a point in each hour-line through which that reflected parallel must passe; then drawing a line to passe through those several points on the said plane or Cieling, which line is the reflected parallel of the Suns Declination desired.

In like manner may be drawn any other reflected parallel of Declination required.

To draw the reflected Azimuth-lines to any reclining Glasse, on any plane whatsoever that the Sun-beams will be reflected on. Here note that Azimuths are great Circles.

FIrst, know that the reflected vertical point in the Axis of the Reflected Horizon, will alwayes be found in the reflected meridian. And look how many degrees the reflected Horizon differs from the direct Horizon, so many must the reflected Axis of the Horizon differ from the direct Axis of the Horizon: Hence the reflected vertical point, whereby the reflected Azimuth-lines are drawn, may be thus found.

Take that thread whose end is fixed in the center of the Glasse, and move the other end thereof to & fro in the reflected meridian, until by applying one side of a quadrant thereto, you find the said thread depressed just 90 degrees, or perpendicular under the reflected Horizon; then make a mark or point where the other end of the said thread toucheth the said reflected Meridian on the Wall, Ground, or Floor of the Room, which point so found is the reflected vertical point desired, in which point fasten one end of a thread:

Then on pastboard or other material draw the points of the Compasse or other degrees, placing the center thereof in the center of the Glasse, and the meridian thereof in the reflected meridian of the world, which said pastboard must be also situated in the reflected Horizon just as the Horizontal Dial was formerly directed to be situated for drawing the reflected hour-lines: And as the threads from the center fastened in the reflected Horizon were also the hour-lines on the Horizontal Diall, whereby the reflected hour-lines were drawn. So now the threads from the center fastened in the Reflected Horizon may be the Horizontal Azimuth lines, whereby the reflected Azimuth-lines may be drawn: Or if that thread which fastned in the center of the glass be drawn exactly over any Azimuth-line, the end whereof being

being faſtened by a nail or other means in the reflected Horizon on the other ſide of the Room, there may ſeveral points be found in the wall or Cieling, through which the reflected Azimuth line muſt paſſe, as followeth :

Take that thread, one end of which is faſtened in the ſaid vertical point, and bring it juſt to touch the Azimuth thread formerly faſtened, and continue it until the end thereof touch the wall or Cieling, (and alſo the thread it ſelf touch the ſaid Azimuth it ſelf, as before) in which point of touch on the wall or Cieling make a mark, through which point that reflected Azimuth-line muſt paſſe. Then move the ſaid ſtring faſtened in the ſaid vertical point, ſo that it may juſt touch the ſaid thread again, but in another place: then as before, continue that thread, untill the end thereof touch the wall or Cieling again, as before, and there make another mark, through which the ſaid reflected Azimuth line muſt alſo paſſe; In like manner may more points be found for your further guide, in drawing that Azimuth-line. But two points being found will be ſufficient.

To draw any reflected line by any two points given over any plane whatſoever, without projecting by the eye.

FAſten two threads in the place of the center of the ſaid reclining Glaſſe, drawing the ſaid threads ſtraight, faſtening each of the other ends in the two reflected Azimuth-points formerly found on the wall or Cieling. Then ſituate a thread croſs or thwart the room, ſo as it may croſſe thoſe other threads from the center, neer at right angles, and alſo juſt touch both of them in that ſituation. By which ſaid thread croſſe the room may any number of points in the ſaid reflected Azimuth-line to be drawn, be found at pleaſure : For if the end of another thread be alſo faſtened in the center of the ſaid Glaſſe, making the other end thereof to touch the wall or Cieling, but ſo that it may alſo juſt touch the ſaid thread, which is faſtened croſſe the room, which

point

point of touch on the ſaid wall or Cieling is another point in the ſaid reflected Azimuth line required to be drawn. In like manner may more points be found at every angle or bending of the wall or Cieling for the exacter drawing the reflected Azimuth line required, which doth find points, whereby is drawn the ſame reflected Azimuth line (or other lines) as was formerly done by a thread ſo ſituated, that it may interpoſe between the eye and any two points aſſigned on the wall or Cieling.

In like manner, if the thread faſtened on the further ſide of the room were removed on another Azimuth line on the ſaid paſtboard, and then faſten it again on the further ſide of the room (as before) you may by help of the ſaid thread faſtened in the ſaid vertical point find ſeveral points on the wall or Cieling, through which that Azimuth-line will paſſe; So may you either by this or the former way draw what Azimuth lines you pleaſe, either in points of the Mariners Compaſſe or degrees, as you pleaſe, by drawing it firſt on paſtboard, as before is directed.

And note generally, that ſuch relation the point found on the floor or ground in the reflected reverſed Axis, hath to the hour-lines drawn on the Horizontal Dial, in drawing the reflected hour-lines; The ſame hath the Reflected vertical point found on the floor or ground, to the Azimuths drawn on the paſtboard in drawing the reflected Azimuth-lines.

To draw the reflected parallels of the Suns altitude, or proportions of ſhadows to any reclining Glaſſe on any Plane whatſoever, that the Sun-beams will be reflected on.
Here note, that parallels of Altitude are leſſer Circles, therefore are not repreſented by a right line.

First, know generally that what reſpect the parallels of Declination have to the hour-lines, ſuch have the parallels of Altitude to the Azimuths.

For if one end of a thread be faſtened in the place of the center

ter of the reclining Glasse, and the other end moved to and fro in any reflected Azimuth line, until the said thread be elevated any number of degrees proposed above the reflected Horizon (the Elevation of which thread being found, by applying a *Quadrant* thereto, and making a mark or point where the end of the said thread toucheth the said reflected Azimuth drawn on the wall or Cieling, that point so found is the point through which that Almicanter or reflected parallel of the Suns altitude must passe.

In like manner, remove the other end of the said thread fastned in the center of the Glasse to another reflected Azimuth-line, and (as before) move it higher or lower, untill by applying the edge of a quadrant to that thread, you find the said thread above the reflected Horizon the same number of degrees first proposed, and at the end of the said thread in that Reflected Azimuth-line drawn on the wall or Cieling I make another mark or point, through which the same Reflected Almicanter or parallel of Altitude must also passe: And so in like manner I find a point on each reflected Azimuth-line, through which the same parallel of Altitude must passe. Then drawing by hand a line to passe through these several points so found, as before, that line is the Reflected parallel of the Suns Altitude proposed. In like manner may be drawn all the other parallels of Altitude desired, which will shew the Suns altitude or the Proportion of any shadow to its altitude, at any appearance of the Suns reflex thereon.

To draw the Jewish or old unequal hour-lines to any Reclining Glasse on any plane whatsoever that the Sun-beams will be reflected on. Here note that the Jewish hour-lines are great Circles.

FIrst, (by the Rules formerly given) draw two reflected parallels of Declination of 16*d.* 55′, the one being neer the Summer, and the other neer the Winter-Tropick: for when the Sun hath that Declination, the day is 15 hours long in the Summer, and 9 in the winter: Then (as is formerly directed) situate

a thread juſt between the eye, and thoſe three points in the ſaid Reflected Dial, as is expreſſed in the inſuing Table, ſo may you thereby draw all or any of thoſe Jewiſh hour-lines deſired, which will at any appearance of the ſpot by the reflex of the Glaſſe amongſt thoſe hour-lines, ſhew how many of the equal hours is paſt ſince Sun-riſing, as was deſired. Now in this Latitude of 51 deg. 30′, If the parallels of the Suns declination be drawn, both when the day is 9 and 15 hours long, that is, when it is 16*d.* 55′, any of thoſe Jewiſh hour-lines will interſect the common hour-lines, either upon the hours, half hours, or quarters. And ſuch a declination may be found, that it ſhall ſo do in any Latitude deſired.

Unequal Hours.	15 H. M	*Equ.* H.	9 H.M.	*Unequ. hours*	15 H. M.	*Equ.* H.	9 H.M.
0	4 30	6	7 30				
1	5 45	7	8 15	7	1 15	1	0 45
2	7 00	8	9 00	8	2 30	2	1 30
3	8 15	9	9 45	9	3 45	3	2 15
4	9 30	10	10 30	10	5 00	4	3 00
5	10 45	11	11 15	11	6 15	5	3 45
6	12 00	12	12 00	12	7 30	6	4 30

To draw the Circles of Poſition to any reclining Glaſſe on any plane whatſoever, that the Sun-beams will be reflected on.

NOte that all Circles of Poſition are great Circles of the Sphere, and do alwayes interſect each other in that point of the Reflected meridian which toucheth the Reflected Horizon, which may be called the common interſection; which ſaid Circles of Poſition are reckoned upon the Reflected Equinoctial both wayes from the ſaid meridian down to the ſaid Horizon: The Horizon Eaſtward being the Cuſpis of the firſt Houſe, and the Horizon Weſtward being the Cuſpis of the ſeventh Houſe; and the Reflected meridian the cuſpis of the tenth Houſe. So that thoſe

thoſe Meridian-planes, whoſe Reclination is 60 degrees Weſtwards, (being meaſured from the meridian in the Equinoctial) lies in the Cuſpis of the eighth Houſe, and 30 degr. Weſtward lies in the Cuſpis of the ninth houſe, and 30 deg. Eaſtward in the Cuſpis of the eleventh Houſe, and 60 deg. Eaſtward in the Cuſpis of the twelfth Houſe: which are all the Houſes above the Horizon.

Now to draw any Circle of Poſition, or the Cuſpis of any Houſe on any Cieling or wall to any reclining Glaſſe is done as followeth:

Firſt, faſten a thread, in ſuch ſort, within the Room, as that it may interpoſe between the eye and the ſaid common point of interſection on the wall or Cieling, and alſo between that point where the reflected hour-line of 4 (being 60 deg. Weſtward from the ſaid Meridian) interſects the reflected Equinoctial alſo on the Cieling, whereby points may be made at every bending or angle of the wall or Cieling, to which the thread ſo ſituated may alſo interpoſe, by which points the Reflected Cuſpis of the eighth Houſe may be drawn. In like manner may the Cuſpis of any other Houſe above the Horizon, as the *9th.* or *10th.* which is the Meridian (or *Medium Cœli*) or *11th.* or *12th.* be drawn alſo. For if (as before) the ſaid thread be again ſo faſtened within the Room, as that it may alſo interpoſe between the eye and the ſaid common point of interſection, and alſo thoſe points where the reflected hour-line of 2 (being 30 deg. Weſtward from the ſaid meridian) do cut the reflected Equinoctial, whereby may be drawn the reflected Cuſpis of the ninth Houſe. Or where the Reflected hour-line of 10 (being alſo 30 deg. Eaſtward from the meridian) do alſo cut the ſaid reflected Equinoctial, whereby may be drawn the Cuſpis of the *11th.* Houſe. Or where the reflected hour-line of 8 (being 60 deg. Eaſtward from the meridian) do alſo cut the ſaid reflected Equinoctial, whereby may be drawn the Cuſpis of the *12th.* Houſe. The Horizon alwayes being the Cuſpis of the firſt and ſeventh Houſes, and the meridian the Cuſpis of the

the tenth house or *Medium Cœli*: wherein generally it is to be noted, That in all planes which cut the common Intersection of the meridian and Horizon, (as doth the Horizontal, and also all meridian planes both Direct and Reclining) these Circles of Position are all parallel to the meridian, and therefore parallel each to other. For look what respect the hour-lines in all Direct or Reclining Polar Planes, or Direct meridian Planes have to the Axis of the World: Such respect have the Circles of Position, in all Horizontal, or Direct meridian or Reclining meridian Planes, to the Axis of the Prime vertical: For as the hour-lines in the first are all parallel to the Axis of the Equinoctial, in whose Poles they meet: So the Circles of Position in the second are all parallel to the Axis of the Prime Vertical, in whose Poles they also meet.

The reason why Glasses reflect a double Spot, is because they are polisht on both sides, which may be remedied with a Pumex-*stone. Those that desire to read more of this Subject may see what is written by* Kircher, in primitiis Gnomicæ Catoptricæ, *and since him by* Magnan *and others,*

VALE.

FINIS.